

American University of Beirut

MATH 201

Calculus and Analytic Geometry III

Fall 2008-2009

quiz # 1

1) (15 points) Find the limit of the following sequences:

a) $\frac{2 \ln^3 n}{\sqrt{n+1} + 5}$ b) $\frac{n \sin(1/n)}{2 + n^{2/n}}$ c) $n \sqrt{3^n + 2^n}$

2) (40 points, 8 points each) Determine if the following series converges or diverges. Justify your answers

a) $\sum_{n=1}^{+\infty} \left(\frac{n+2}{n+1}\right)^{n^2}$

b) $\sum_{n=0}^{+\infty} \frac{1}{2^n(n+1)}$

c) $\sum_{n=0}^{+\infty} \frac{\cos(n^2 + 1)}{n^3 + 3^n}$

d) $\sum_{n=1}^{+\infty} \frac{(1 - \frac{1}{n})^n}{\sqrt{n+10}}$

e) $\sum_{n=2}^{+\infty} \frac{1}{(\ln n)^{\ln n}}$

3) (15 points) Find the sum of the series $\sum_{n=2}^{+\infty} \left[\frac{3^{n-2}}{2^{2n}} + \ln(1 - 1/n^2) \right]$

4) (20 points) Find the interval of convergence of the power series $\sum_{n=1}^{+\infty} \frac{1}{\sqrt{2+n^2}} \cdot (x+1)^n$.

For what value(s) of x for which the series converges (i) absolutely? (ii) conditionally?

5) a) (4 points) Using the fact that $(\tan^{-1} x)' = \frac{1}{1+x^2}$, find the Maclaurin series expansion of $\tan^{-1} x$

b) (3 points) express $\pi/4$ as a sum of an alternating series.

c) (3 points) what is the error done in approximating $\pi/4$ with $1 - 1/3 + 1/5$?