MECH 431
Dynamic System Analysis

Exam II

Thursday, Sept 14, 2006

Duration: 60 minutes

Open Book Exam

Write clearly your derivations and answers on the question sheet

Name:

ID#:

Problem I [40 Pts]

A loudspeaker produces sound waves by the movement of a diaphragm in response to an electrical input. In the cross-sectional view shown below $e_i(t)$ is the input and the output is the displacement x. A coil of wire with N turns and radius a is attached to the diaphragm. Let $\alpha=2\pi aN\mathcal{B}$, where $\mathcal{B}$, is the flux density in the air gap of the permanent magnet

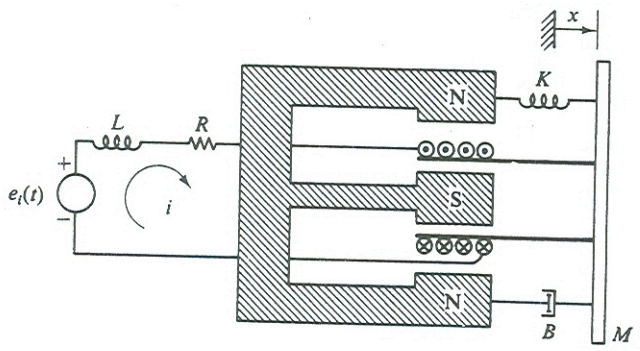
a. Verify that the equations

$$\dot{x} = v$$
$$\dot{v} = \frac{1}{M}(-Kx - Bv + \alpha i)$$
$$\frac{di}{dt} = \frac{1}{L}[-\alpha v - Ri + e_i(t)]$$

represent a valid state-variable model

b. Find the transfer function and the input output differential equation.

c. Rewrite the input output equation for the case L=0 and find expressions for the damping ratio ζ and the undamped natural frequency ω_n.



Problem 2 [30 Pts]

Given the system with the transfer function

$$H(s) = \frac{Y(s)}{X(s)} = \frac{5}{s^2 + s + 9}$$

where the input

$$x(t) = 3cos(0.5t)$$

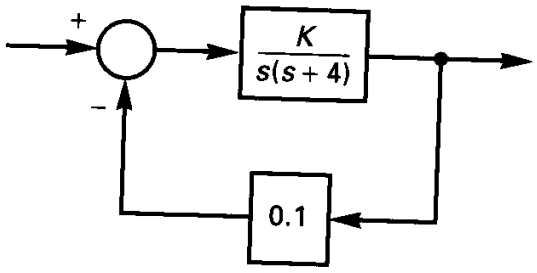
is applied at t=0.

1. Find the steady-state system response.
2. After how much time t can we consider the system to be at steady state.

Problem 3 [30 Pts]

Consider the system shown below.

1. Find the equivalent transfer function for the system
2. Find the range of K for which the system is
   - **(a)** Underdamped
   - **(b)** Critically damped
   - **(c)** Overdamped.



System for Problem 3

Laplace Transforms

Name	Time function, $f(t)$	Laplace transform, $F(s)$
Unit impulse	$\delta(t)$	1
Unit step	$u(t)$	$\frac{1}{s}$
Unit ramp	t	$\frac{1}{s^2}$
nth-Order ramp	t^n	$\frac{n!}{s^{n+1}}$
Exponential	e^{-at}	$\frac{1}{s+a}$
nth-Order exponential	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
Sine	$\sin bt$	$\frac{b}{s^2+b^2}$
Cosine	$\cos bt$	$\frac{s}{s^2+b^2}$
Damped sine	$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
Damped cosine	$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$
Diverging sine	$t\sin bt$	$\frac{2bs}{(s^2+b^2)^2}$
Diverging cosine	$t\cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$