American University of Beirut

Faculty of Engineering and Architecture

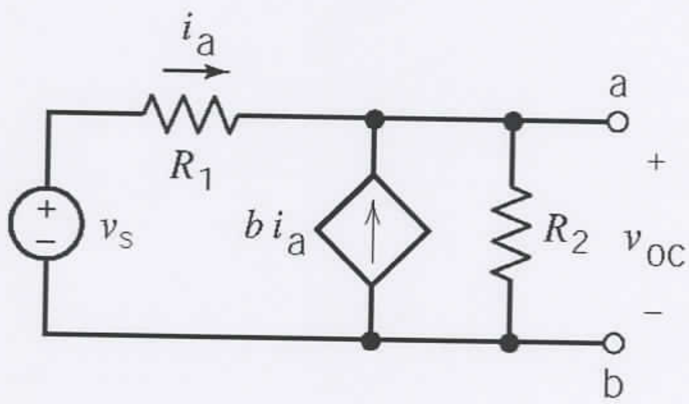
Department of Electrical and Computer Engineering

EECE 210 – Electric Circuits-Spring term 2011

Quiz II – Solution

Problem 1

Find V_{Th} with respect to the terminals a-b in the circuit shown below when $v_S = 15$ V, $R_1 = R_2 = 150\ \Omega$, and $b = 1$.



Solution

$$V_s = R_1 i_a + R_2(1+b)i_a \Rightarrow i_a = \frac{V_s}{R_1 + R_2(1+b)}$$

$$V_{Th} = R_2(1+b)i_a = \frac{R_2(1+b)V_s}{R_1 + R_2(1+b)}$$

Problem 2

Find R_{Th} with respect to the terminals a-b for the circuit in problem 1.

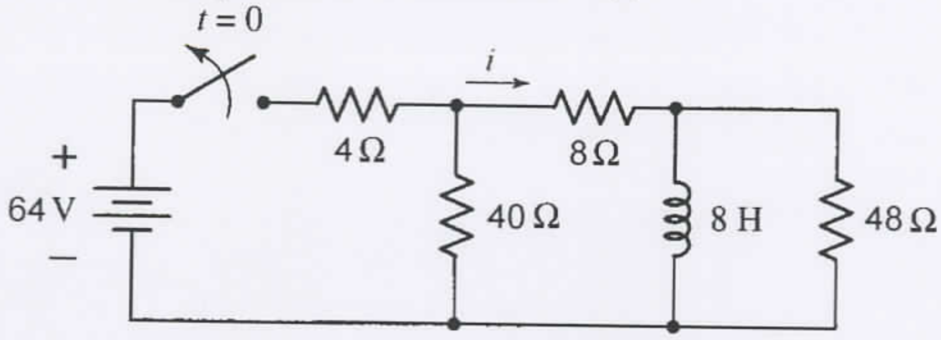
Solution

$$i_{sc} = (1+b)i_a = (1+b)\frac{V_s}{R_1}$$

$$R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{R_2(1+b)V_s}{R_1 + R_2(1+b)} * \frac{R_1}{(1+b)V_s} = \frac{R_1 R_2}{R_1 + R_2(1+b)}$$

Problem 3

The switch in the circuit shown below has been closed for a very long time before it opens at $t = 0$. Find the energy stored in the inductor at $t = 0^+$.



Solution

As the switch has been closed for a long period of time, the inductor acts as a short circuit. Accordingly, we can remove the 48Ω resistor and the current in the inductor is the same as the one in the 8 Ω resistor.

The total resistance is then given by: $R_T = (8 // 40) + 4 = \frac{8 * 40}{48} + 4 = \frac{20}{3} + 4 = \frac{32}{3}\Omega$

The total current (the current in the 4Ω resistor) is then given by: $I_4 = \frac{64 * 3}{32} = 6A$

The current in the 8Ω resistor is then: $I_8 = 6 * \frac{40}{48} = 5A$

The total energy stored in the inductor is given by: $W_L = \frac{1}{2} L (I_8)^2 = 4 * 25 = 100J$

Problem 4

In the circuit of problem 3, Find $i(t)$ at $t = 0.333$ s.

Solution

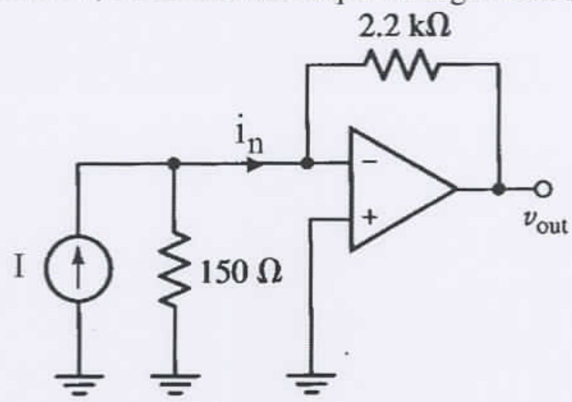
$$R_{eq} = (40 + 8) // 48 = 24\Omega$$

$$\tau = \frac{L}{R} = \frac{8}{24} = 0.33333$$

$$i(t) = \frac{I_0}{2} e^{-\frac{t}{\tau}} \Rightarrow i(0.333) = \frac{5}{2} e^{-1} = 0.92A$$

Problem 5

Consider the figure shown below, determine the output voltage when I=1 mA



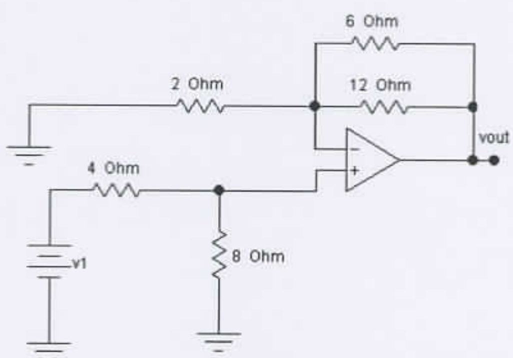
Solution

v_p=0 Volts, then v_n=0 and no currents flow in the 150 Ω resistor. Accordingly,

$$i_n = I = \frac{0 - v_{out}}{2.2 \times 10^3} \Rightarrow v_{out} = -2200I$$

Problem 6.

Consider the Figure shown below, determine the output voltage given that v1=1 volts.



Solution

Using voltage divider rule, $v_p = \frac{8}{8+4} v_1 \Rightarrow \frac{2}{3} v_1 = v_p$

Determine v_{out} as a function of v_n.

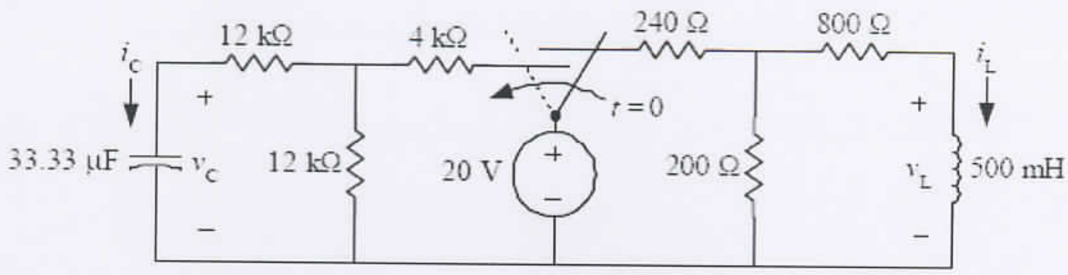
$$i_n = 0 \Rightarrow \frac{v_{out} - v_n}{6} + \frac{v_{out} - v_n}{12} = \frac{v_n}{2}$$

$$\frac{v_{out}}{4} = \frac{3v_n}{4} \Rightarrow v_{out} = 3v_n$$

$$v_n = v_p \Rightarrow v_{out} = 3v_n = 3\left(\frac{2}{3}\right)v_1 \Rightarrow v_{out} = 2v_1$$

Problem 7

For the circuit shown below, the switch has been in its position for a long period of time.



What is the value of the current in the inductor i_L at $t = 0^-$?

Solution

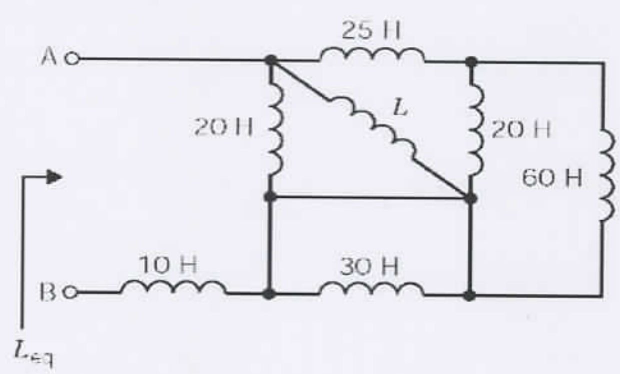
The inductor, and before the switch is open, acts as a short circuits.

$$i_{20} = \frac{20}{240 + (200 // 800)} = 50\text{mA}$$

$$i_L = i_{20}\frac{200}{200 + 800} = 10\text{mA}$$

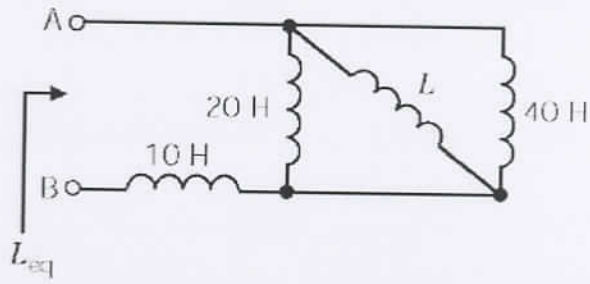
Problem 8

The figure shown below is equivalent to a single inductor having an equivalent inductance of 15 Henries. Determine L



Solution

The 25 H inductor is in series with a parallel combination of 20H and 60 H inductors. The inductance of the equivalent inductor is: 25+((60*20)/(60+20))=40 H. The 30 H inductor is in parallel with a short circuit, which is equivalent to as short circuit. After making these simplifications, we have



$L_{eq} = L_1 + 10,$ where $\frac{1}{L_1} = \frac{1}{20} + \frac{1}{40} + \frac{1}{L} \Rightarrow L_1 = \frac{40L}{3L+40}$

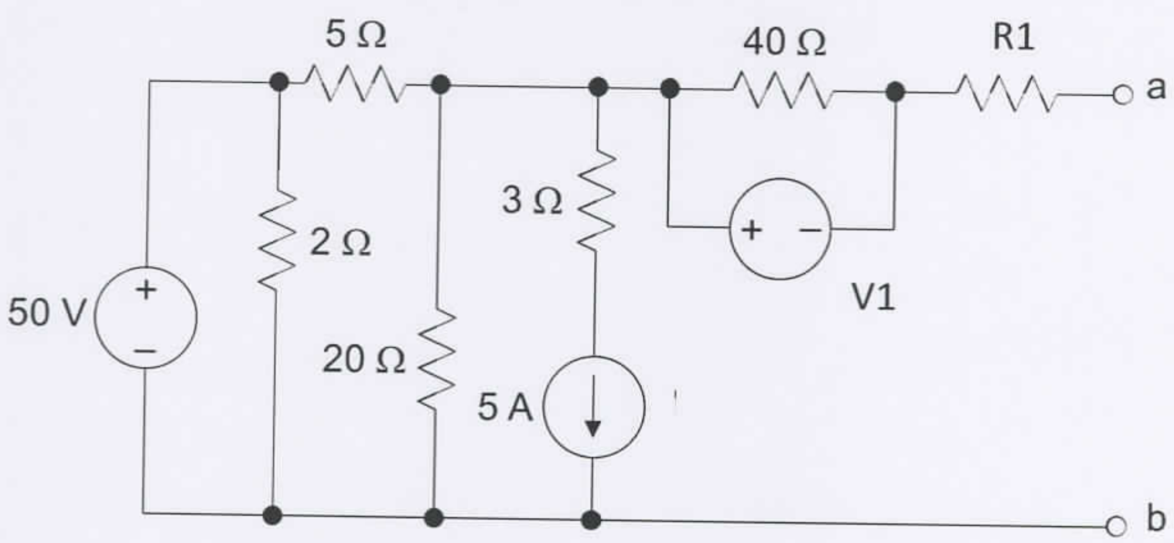
Substituting, we obtain

$$L_{eq} = \frac{40L}{3L+40} + 10 \Rightarrow 3LL_{eq} + 40L_{eq} = 40L + 30L + 400$$

$$L = \frac{400 - 40L_{eq}}{3L_{eq} - 70}$$

Problem 9

Find the Thevenin equivalent voltage seen between terminals ab if V1 = 3V and R1 = 10Ω.



Solution

Let V2 be the node voltage to the left of the 5 Ω resistor and V3 to the right. Using Nodal analysis, we have: V2 = 50 Volts and $\frac{V_3}{20} + \frac{V_3 - 50}{5} + 5 = 0$

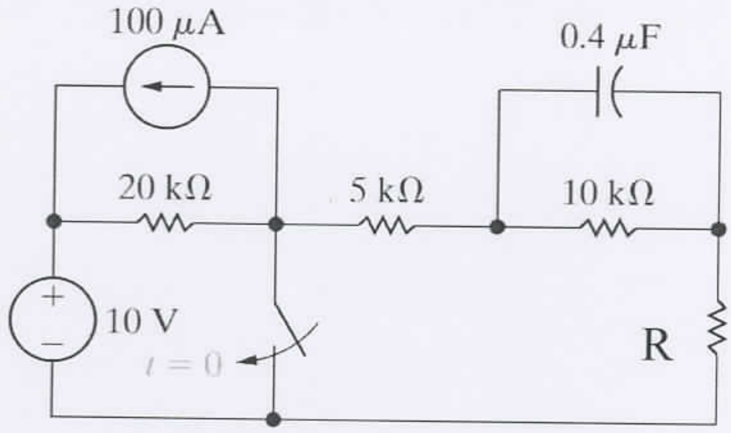
Rearranging the above equation, we obtain

$$V_3 + 4V_3 - 200 + 100 = 0 \Rightarrow 5V_3 = 100 \Rightarrow V_3 = 20 \text{ Volts}$$

$$V_3 - V_{ab} = V_1 \Rightarrow V_{ab} = V_3 - V_1 = 20 - V_1$$

Problem 10

The switch was open for a long time before it closes at t=0. Find the voltage across the capacitor after $t = 1\tau$ (one time constant after closing the switch) given that $R = 10\ k\Omega$.



Solution

First, let us determine the initial voltage across the capacitor when the switch is open.

1. Convert the current source in parallel with the 20 KΩ resistir to a voltage source of 2 V in series with this resistor
2. Combine the two voltage sources into one source of 8 Volts
3. Use voltage divider rule to obtain the voltage across the 10 KΩ resistor

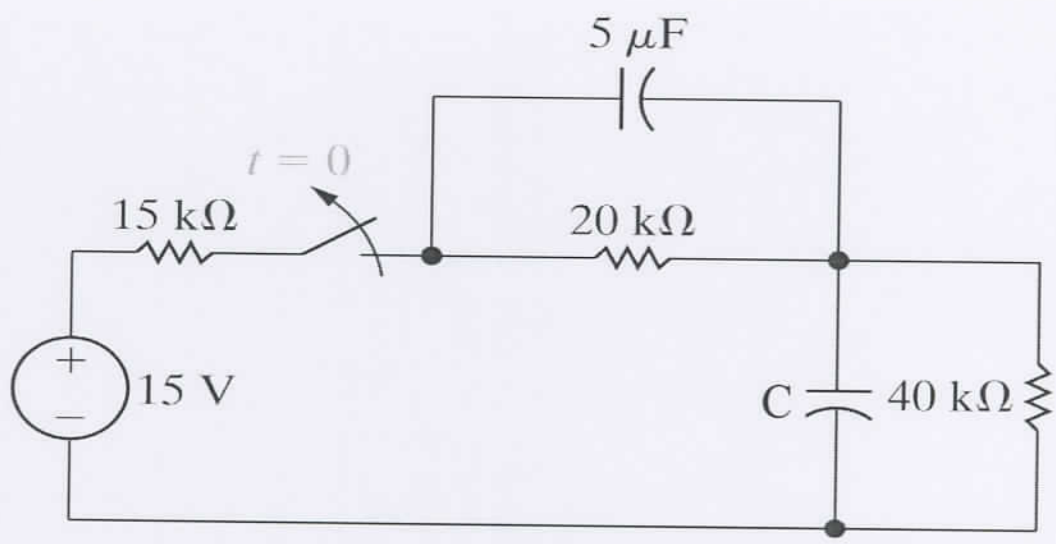
$$R_{10} = \frac{8*10}{10+5+20+R_1} = \frac{80}{35+R_1} = V_c(0)$$

When the switch is open, the capacitor will discharge through the 10 KΩ resistor.

$$V_c(t) = \frac{80}{35+R_1}e^{-\frac{t}{\tau}} = \frac{80}{35+R_1}e^{-1}$$

Problem 11

The switch was closed for a long time. It opens at t=0. Find the total energy stored in the capacitors at t=0, given that C = 1μF.



Solution

When the switch is closed for a very long period of time, both capacitors acts as open circuits and the voltage across the 5μF and the capacitor C are the voltages across the 20 KΩ and the 60KΩ resistors, respectively.

$$V_{40} = \frac{15*40}{15+20+40} = \frac{15*40}{75} = 8 \text{ Volts}$$

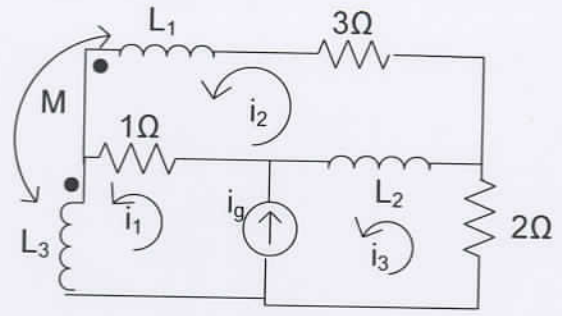
$$V_{20} = \frac{15*20}{15+20+40} = \frac{15*20}{75} = 4 \text{ Volts}$$

Energy stored in the capacitors is given by

$$E = \frac{1}{2}(5*10^{-6})(4^2) + \frac{1}{2}(C*10^{-6})(8^2) \text{ Joules}$$

Problem 12

Assume in the following circuit L_1=3 H, L_2=2 H, L3=26H, and M=5H. Find the mesh equation around i2.



Solution

$$3i_2 + L_1\frac{di_2}{dt} - M\frac{di_1}{dt} + 1(i_2 - i_1) + L_2\frac{d}{dt}(i_2 - i_3) = 0$$

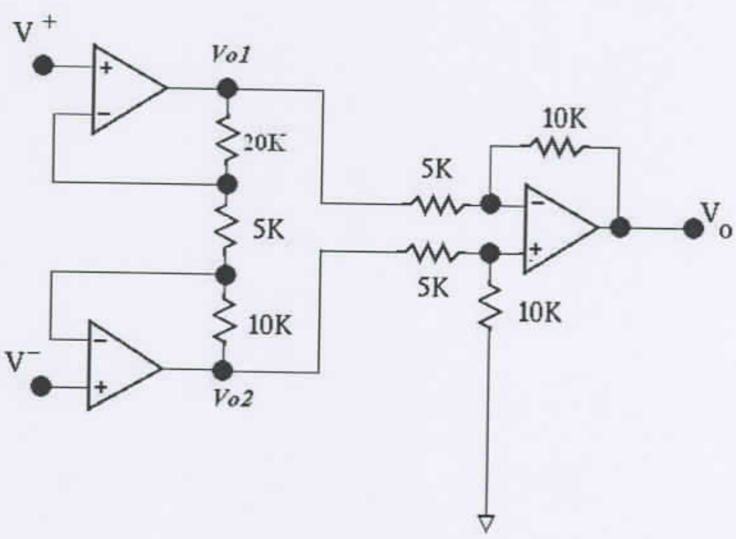
$$i_g = i_1 - i_3$$

$$3i_2 + L_1\frac{di_2}{dt} - M\frac{di_1}{dt} + 1(i_2 - i_1) + L_2\frac{d}{dt}(i_2 - i_1 + i_g) = 0$$

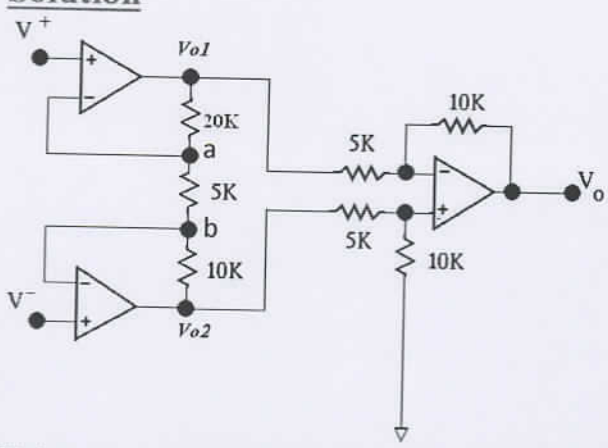
$$L_2\frac{di_g}{dt} + (L_1 + L_2)\frac{di_2}{dt} - (M + L_2)\frac{di_1}{dt} - i_1 + 4i_2 = 0$$

Problem 13

In the circuit shown below, find the output voltage V_0 if V^+=2V V^-=1V assuming that all operational amplifiers are ideal and operating in their linear region.



Solution



Voltage at node (a) is equal to $V^+ = 2$ Volts and the voltage at node b is equal to V^-=1 volts. Therefore, the current in the 5K resistor (which is the same in the 10K and 20K resistors) is given by:

$$v_a - v_b = 5*10^3*i \Rightarrow i = 0.2\,\text{mA}$$

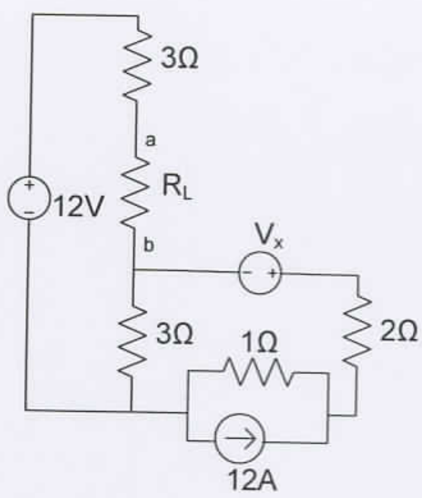
$$v_{01} - v_a = 20*10^3*0.2*10^{-3} \Rightarrow v_{01} = 4+2 = 6 \text{ volts}$$

$$v_b - v_{02} = 10*10^3*0.2*10^{-3} \Rightarrow v_{02} = 1-2 = -1 \text{ volts}$$

$$V_0 = \frac{10}{5}(v_{02} - v_{01}) = 14 \text{ volts}$$

Problem 14

In the circuit shown below, calculate the maximum power that can be dissipated by R_L if Vx=6V.



Solution

Maximum power transfer is obtained when the load is equal to R_{TH}. Accordingly, we need to determine the Thevenin equivalent

$$R_{Th} = (2+1)//3+3 = 4.5\Omega$$
$$V_{Th} = 12 - V_{3\Omega}$$

$V_{3\Omega}$ is the voltage across the 3Ω resistor which is obtained by converting the current source parallel to a resistor to a voltage source in series with the resistor, add the two voltage sources, and use voltage divider rule.

$$V_{3\Omega} = \frac{(12-V_x)*3}{6} = \frac{(12-V_x)}{2}$$

$$V_{Th} = 12 - \frac{(12-V_x)}{2} = \frac{12+V_x}{2}$$

Maximum power transfer is given by:

$$P = \frac{V_{Th}^2}{4R_{Th}} = \frac{(12+V_x)^2}{4*4*4.5} = \frac{(12+V_x)^2}{4*18}$$

$$\text{For } V_x = 6 \text{ volts} \Rightarrow P = \frac{(18)^2}{4*18} = 4.5\,\text{Watts}$$