1

Circuit Variables

Assessment Problems

AP 1.1 To solve this problem we use a product of ratios to change units from dollars/year to dollars/millisecond. We begin by expressing \$10 billion in scientific notation:

$\$100 \text{ billion } = \$100 \times 10^9$

Now we determine the number of milliseconds in one year, again using a product of ratios:

$$\frac{1 \text{ year}}{365.25 \text{ days}} \cdot \frac{1 \text{ day}}{24 \text{ hours}} \cdot \frac{1 \text{ hour}}{60 \text{ mins}} \cdot \frac{1 \text{ min}}{60 \text{ secs}} \cdot \frac{1 \text{ sec}}{1000 \text{ ms}} = \frac{1 \text{ year}}{31.5576 \times 10^9 \text{ ms}}$$

Now we can convert from dollars/year to dollars/millisecond, again with a product of ratios:

$$\frac{\$100 \times 10^9}{1 \text{ year}} \cdot \frac{1 \text{ year}}{31.5576 \times 10^9 \text{ ms}} = \frac{100}{31.5576} = \$3.17/\text{ms}$$

AP 1.2 First, we recognize that 1 ns = 10^{-9} s. The question then asks how far a signal will travel in 10^{-9} s if it is traveling at 80% of the speed of light. Remember that the speed of light $c = 3 \times 10^8$ m/s. Therefore, 80% of c is $(0.8)(3 \times 10^8) = 2.4 \times 10^8$ m/s. Now, we use a product of ratios to convert from meters/second to inches/nanosecond:

$$\frac{2.4 \times 10^8 \text{ m}}{1\text{s}} \cdot \frac{1 \text{ s}}{10^9 \text{ ns}} \cdot \frac{100 \text{ cm}}{1 \text{ m}} \cdot \frac{1 \text{ in}}{2.54 \text{ cm}} = \frac{(2.4 \times 10^8)(100)}{(10^9)(2.54)} = \frac{9.45 \text{ in}}{1 \text{ ns}}$$

Thus, a signal traveling at 80% of the speed of light will travel 9.45″ in a nanosecond.

AP 1.3 Remember from Eq. (1.2), current is the time rate of change of charge, or $i = \frac{dq}{dt}$ In this problem, we are given the current and asked to find the total charge. To do this, we must integrate Eq. (1.2) to find an expression for charge in terms of current:

$$q(t) = \int_0^t i(x)\,dx$$

We are given the expression for current, i, which can be substituted into the above expression. To find the total charge, we let $t \to \infty$ in the integral. Thus we have

$$q_{\text{total}} = \int_0^\infty 20e^{-5000x}\,dx = \frac{20}{-5000}e^{-5000x}\Big|_0^\infty = \frac{20}{-5000}(e^{\infty} - e^{0})$$
$$= \frac{20}{-5000}(0-1) = \frac{20}{5000} = 0.004\ \text{C} = 4000\,\mu\text{C}$$

AP 1.4 Recall from Eq. (1.2) that current is the time rate of change of charge, or $i = \frac{dq}{dt}$. In this problem we are given an expression for the charge, and asked to find the maximum current. First we will find an expression for the current using Eq. (1.2):

$$i = \frac{dq}{dt} = \frac{d}{dt}\left[\frac{1}{\alpha^2} - \left(\frac{t}{\alpha} + \frac{1}{\alpha^2}\right)e^{-\alpha t}\right]$$
$$= \frac{d}{dt}\left(\frac{1}{\alpha^2}\right) - \frac{d}{dt}\left(\frac{t}{\alpha}e^{-\alpha t}\right) - \frac{d}{dt}\left(\frac{1}{\alpha^2}e^{-\alpha t}\right)$$
$$= 0 - \left(\frac{1}{\alpha}e^{-\alpha t} - \alpha\frac{t}{\alpha}e^{-\alpha t}\right) - \left(-\alpha\frac{1}{\alpha^2}e^{-\alpha t}\right)$$
$$= \left(-\frac{1}{\alpha} + t + \frac{1}{\alpha}\right)e^{-\alpha t}$$
$$= te^{-\alpha t}$$

Now that we have an expression for the current, we can find the maximum value of the current by setting the first derivative of the current to zero and solving for t:

$$\frac{di}{dt} = \frac{d}{dt}(te^{-\alpha t}) = e^{-\alpha t} + t(-\alpha)e^{\alpha t} = (1-\alpha t)e^{-\alpha t} = 0$$

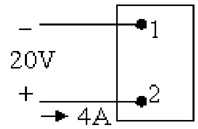
Since $e^{-\alpha t}$ never equals 0 for a finite value of t, the expression equals 0 only when $(1 - \alpha t) = 0$. Thus, $t = 1/\alpha$ will cause the current to be maximum. For this value of t, the current is

$$i = \frac{1}{\alpha}e^{-\alpha/\alpha} = \frac{1}{\alpha}e^{-1}$$

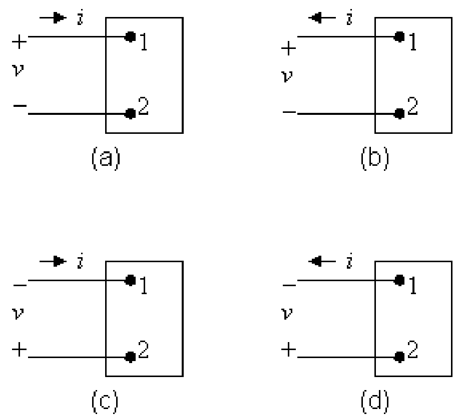
Remember in the problem statement, $\alpha = 0.03679$. Using this value for α,

$$i = \frac{1}{0.03679}e^{-1} \cong 10\ \text{A}$$

AP 1.5 Start by drawing a picture of the circuit described in the problem statement:



Also sketch the four figures from Fig. 1.6:



[a] Now we have to match the voltage and current shown in the first figure with the polarities shown in Fig. 1.6. Remember that 4A of current entering Terminal 2 is the same as 4A of current leaving Terminal 1. We get

(a) $v = -20\,\text{V}, \qquad i = -4\,\text{A};$ (b) $v = -20\,\text{V}, \qquad i = 4\,\text{A}$

(c) $v = 20\,\text{V}, \qquad i = -4\,\text{A};$ (d) $v = 20\,\text{V}, \qquad i = 4\,\text{A}$

[b] Using the reference system in Fig. 1.6(a) and the passive sign convention, $p = vi = (-20)(-4) = 80\,\text{W}$. Since the power is greater than 0, the box is absorbing power.

[c] From the calculation in part (b), the box is absorbing 80 W.

AP 1.6 Applying the passive sign convention to the power equation using the voltage and current polarities shown in Fig. 1.5, $p = vi$. From Eq. (1.3), we know that power is the time rate of change of energy, or $p = \frac{dw}{dt}$. If we know the power, we can find the energy by integrating Eq. (1.3). To begin, find the expression for power:

$$p = vi = (10{,}000e^{-5000t})(20e^{-5000t}) = 200{,}000e^{-10{,}000t} = 2 \times 10^5 e^{-10{,}000t}\ \text{W}$$

Now find the expression for energy by integrating Eq. (1.3):

$$w(t) = \int_0^t p(x)\,dx$$

Substitute the expression for power, p, above. Note that to find the total energy, we let $t \to \infty$ in the integral. Thus we have

$$w = \int_0^\infty 2 \times 10^5 e^{-10{,}000x}\,dx = \frac{2 \times 10^5}{-10{,}000} e^{-10{,}000x}\Big|_0^\infty$$

$$= \frac{2 \times 10^5}{-10{,}000}(e^{-\infty} - e^0) = \frac{2 \times 10^5}{-10{,}000}(0 - 1) = \frac{2 \times 10^5}{10{,}000} = 20\ \text{J}$$

AP 1.7 At the Oregon end of the line the current is leaving the upper terminal, and thus entering the lower terminal where the polarity marking of the voltage is negative. Thus, using the passive sign convention, $p = -vi$. Substituting the values of voltage and current given in the figure,

$$p = -(800 \times 10^3)(1.8 \times 10^3) = -1440 \times 10^6 = -1440\ \text{MW}$$

Thus, because the power associated with the Oregon end of the line is negative, power is being generated at the Oregon end of the line and transmitted by the line to be delivered to the California end of the line.

Problems

P 1.1 To begin, we calculate the number of pixels that make up the display:

$$n_{\text{pixels}} = (1280)(1024) = 1{,}310{,}720 \text{ pixels}$$

Each pixel requires 24 bits of information. Since 8 bits comprise a byte, each pixel requires 3 bytes of information. We can calculate the number of bytes of information required for the display by multiplying the number of pixels in the display by 3 bytes per pixel:

$$n_{\text{bytes}} = \frac{1{,}310{,}720 \text{ pixels}}{1 \text{ display}} \cdot \frac{3 \text{ bytes}}{1 \text{ pixel}} = 3{,}932{,}160 \text{ bytes/display}$$

Finally, we use the fact that there are 10^6 bytes per MB:

$$\frac{3{,}932{,}160 \text{ bytes}}{1 \text{ display}} \cdot \frac{1 \text{ MB}}{10^6 \text{bytes}} = 3.93 \text{ MB/display}$$

P 1.3 We can set up a ratio to determine how long it takes the bamboo to grow 10 μm First, recall that 1 mm = $10^3\mu$m. Let's also express the rate of growth of bamboo using the units mm/s instead of mm/day. Use a product of ratios to perform this conversion:

$$\frac{250 \text{ mm}}{1 \text{ day}} \cdot \frac{1 \text{ day}}{24 \text{ hours}} \cdot \frac{1 \text{ hour}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{250}{(24)(60)(60)} = \frac{10}{3456} \text{ mm/s}$$

Use a ratio to determine the time it takes for the bamboo to grow 10 μm:

$$\frac{10/3456 \times 10^{-3} \text{ m}}{1 \text{ s}} = \frac{10 \times 10^{-6} \text{ m}}{x \text{ s}} \qquad \text{so} \qquad x = \frac{10 \times 10^{-6}}{10/3456 \times 10^{-3}} = 3.456 \text{ s}$$

P 1.6 Our approach is as follows: To determine the area of a bit on a track, we need to know the height and width of the space needed to store the bit. The height of the space used to store the bit can be determined from the width of each track on the disk. The width of the space used to store the bit can be determined by calculating the number of bits per track, calculating the circumference of the inner track, and dividing the number of bits per track by the circumference of the track. The calculations are shown below.

$$\text{Width of track } = \frac{1 \text{ in}}{77 \text{ tracks}} \frac{25{,}400\mu\text{m}}{\text{in}} = 329.87\mu\text{m/track}$$

$$\text{Bits on a track } = \frac{1.4 \text{ MB}}{2 \text{ sides}} \frac{8 \text{ bits}}{\text{byte}} \frac{1 \text{ side}}{77 \text{ tracks}} = 72{,}727.273 \text{ bits/track}$$

Circumference of inner track $= 2\pi(1/2'')(25{,}400\mu\text{m/in}) = 79{,}796.453\mu\text{m}$

Width of bit on inner track $= \dfrac{79{,}796.453\mu\text{m}}{72{,}727.273\text{ bits}} = 1.0972\mu\text{m/bit}$

Area of bit on inner track $= (1.0972)(329.87) = 361.934\mu\text{m}^2$

P 1.9 First we use Eq. (1.2) to relate current and charge:

$$i = \frac{dq}{dt} = 20\cos 5000t$$

Therefore, $dq = 20\cos 5000t\,dt$

To find the charge, we can integrate both sides of the last equation. Note that we substitute x for q on the left side of the integral, and y for t on the right side of the integral:

$$\int_{q(0)}^{q(t)} dx = 20\int_0^t \cos 5000y\,dy$$

We solve the integral and make the substitutions for the limits of the integral, remembering that $\sin 0 = 0$:

$$q(t) - q(0) = 20\frac{\sin 5000y}{5000}\bigg|_0^t = \frac{20}{5000}\sin 5000t - \frac{20}{5000}\sin 5000(0) = \frac{20}{5000}\sin 5000t$$

But $q(0) = 0$ by hypothesis, i.e., the current passes through its maximum value at $t = 0$, so $q(t) = 4\times 10^{-3}\sin 5000t$ C $= 4\sin 5000t$ mC

P 1.12 Assume we are standing at box A looking toward box B. Then, using the passive sign convention $p = vi$, since the current i is flowing into the $+$ terminal of the voltage v. Now we just substitute the values for v and i into the equation for power. Remember that if the power is positive, B is absorbing power, so the power must be flowing from A to B. If the power is negative, B is generating power so the power must be flowing from B to A.

[a] $p = (20)(15) = 300$ W 300 W from A to B

[b] $p = (100)(-5) = -500$ W 500 W from B to A

[c] $p = (-50)(4) = -200$ W 200 W from B to A

[d] $p = (-25)(-16) = 400$ W 400 W from A to B

P 1.17 **[a]** To find the power at 625 μs, we substitute this value of time into both the equations for $v(t)$ and $i(t)$ and multiply the resulting numbers to get $p(625\,\mu\text{s})$:

$$v(625\,\mu\text{s}) = 50e^{-1600(625\times10^{-6})} - 50e^{-400(625\times10^{-6})} = 18.394 - 38.94 = -20.546\text{ V}$$

$$i(625\,\mu\text{s}) = 5\times10^{-3}e^{-1600(625\times10^{-6})} - 5\times10^{-3}e^{-400(625\times10^{-6})}$$

$$= 0.0018394 - 0.003894 = -0.0020546\text{ A}$$

$$p(625\,\mu\text{s}) = (-20.546)(-0.0020546) = 42.2\text{ mW}$$

[b] To find the energy at 625 μs, we need to integrate the equation for $p(t)$ from 0 to 625 μs. To start, we need an expression for $p(t)$:

$$p(t) = v(t)i(t) = (50)(5\times10^{-3})(e^{-1600t} - e^{-400t})(e^{-1600t} - e^{-400t})$$

$$= \frac{1}{4}(e^{-3200t} - 2e^{-2000t} + e^{-800t})$$

Now we integrate this expression for $p(t)$ to get an expression for $w(t)$. Note we substitute x for t on the right side of the integral.

$$w(t) = \frac{1}{4}\int_0^t (e^{-3200x} - 2e^{-2000x} + e^{-800x})dx$$

$$= \frac{1}{4}\left[\frac{e^{-3200x}}{-3200} + \frac{e^{-2000x}}{1000} - \frac{e^{-800x}}{800}\right]\Bigg|_0^t$$

$$= \frac{1}{4}\left[\frac{e^{-3200t}}{-3200} + \frac{e^{-2000t}}{1000} - \frac{e^{-800t}}{800} - \left(\frac{1}{-3200} + \frac{1}{1000} - \frac{1}{800}\right)\right]$$

$$= \frac{1}{4}\left[\frac{e^{-3200t}}{-3200} + \frac{e^{-2000t}}{1000} - \frac{e^{-800t}}{800} + 5.625\times10^{-4}\right]$$

Finally, substitute $t = 625\,\mu$s into the equation for $w(t)$:

$$w(625\,\mu\text{s}) = \frac{1}{4}[-4.2292\times10^{-5} + 2.865\times10^{-4} - 7.5816\times10^{-4} + 5.625\times10^{-4}]$$

$$= 12.137\,\mu\text{J}$$

[c] To find the total energy, we let $t \to \infty$ in the above equation for $w(t)$. Note that this will cause all expressions of the form e^{-nt} to go to zero, leaving only the constant term 5.625×10^{-4}. Thus,

$$w_{\text{total}} = \frac{1}{4}[5.625\times10^{-4}] = 140.625\,\mu\text{J}$$

P 1.24 **[a]** We can find the time at which the power is a maximum by writing an expression for $p(t) = v(t)i(t)$, taking the first derivative of $p(t)$ and setting it to zero, then solving for t. The calculations are shown below:

$$p = 0 \quad t < 0, \qquad p = 0 \quad t > 40\text{ s}$$

$$p = vi = (t - 0.025t^2)(4 - 0.2t) = 4t - 0.3t^2 + 0.005t^3\text{ W} \qquad 0 < t < 40\text{ s}$$

$$\frac{dp}{dt} = 4 - 0.6t + 0.015t^2 = 0$$

Use a calculator to find the two solutions to this quadratic equation:

$t_1 = 8.453$ s; $\qquad t_2 = 31.547$ s

Now we must find which of these two times gives the minimum power by substituting each of these values for t into the equation for $p(t)$:

$$p(t_1) = (8.453 - 0.025(8.453)^2)(4 - 0.2 \cdot 8.453) = 15.396\text{ W}$$

$$p(t_2) = (31.547 - 0.025(31.547)^2)(4 - 0.2 \cdot 31.547) = -15.396\text{ W}$$

Therefore, maximum power is being delivered at $t = 8.453$ s.

[b] The maximum power was calculated in part (a) to determine the time at which the power is maximum: $p_{\text{max}} = 15.396$ W (delivered)

[c] As we saw in part (a), the other "maximum" power is actually a minimum, or the maximum negative power. As we calculated in part (a), maximum power is being extracted at $t = 31.547$ s.

[d] This maximum extracted power was calculated in part (a) to determine the time at which power is maximum: $p_{\text{maxext}} = 15.396$ W (extracted)

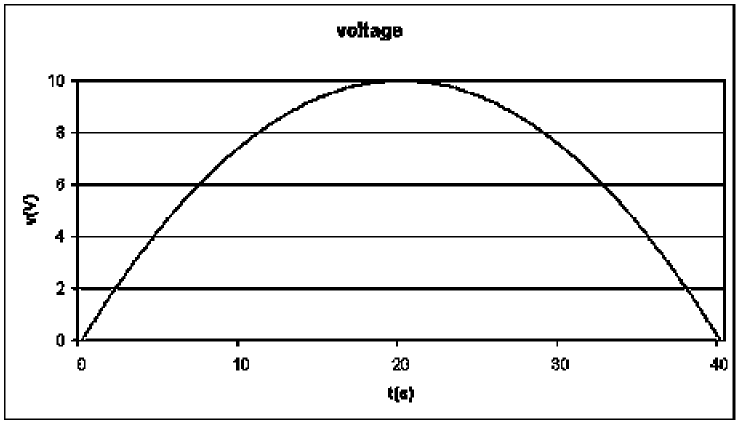
[e] $w = \int_0^t p\,dx = \int_0^t (4x - 0.3x^2 + 0.005x^3)\,dx = 2t^2 - 0.1t^3 + 0.00125t^4$

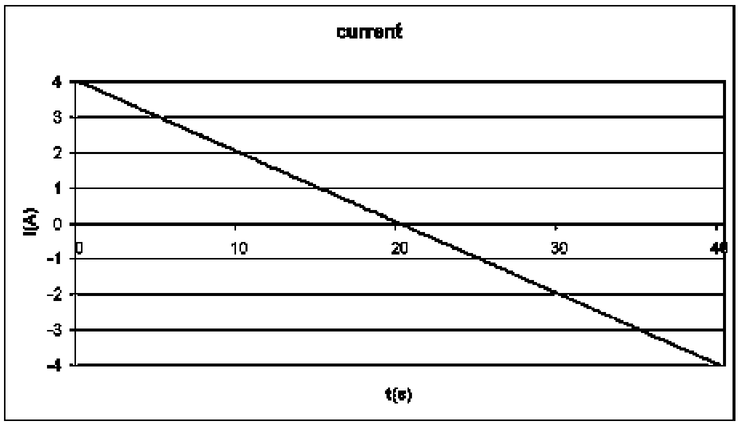
$$w(0) = 0\text{ J} \qquad w(30) = 112.50\text{ J}$$

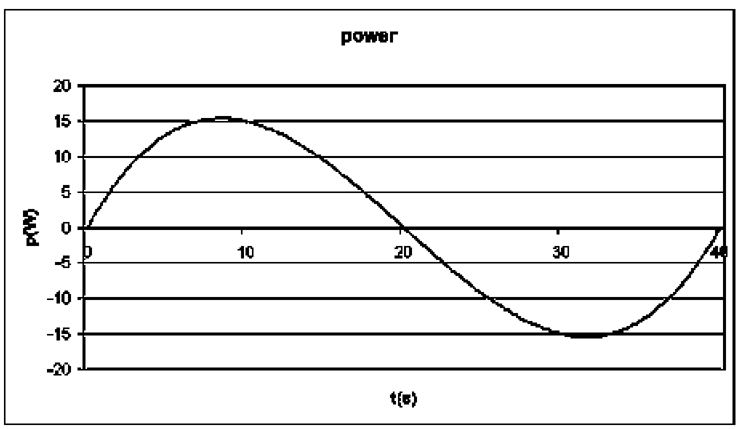
$$w(10) = 112.50\text{ J} \qquad w(40) = 0\text{ J}$$

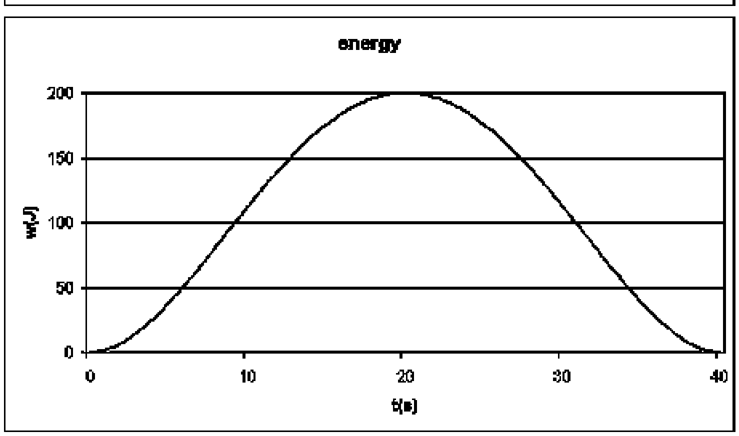
$$w(20) = 200\text{ J}$$

To give you a feel for the quantities of voltage, current, power, and energy and their relationships among one another, they are plotted below:


voltage
v(V)
t(s)
10
8
6
4
2
0
0
10
20
30
40


current
i(A)
t(s)
4
3
2
1
0
-1
-2
-3
-4
0
10
20
30
40


power
p(W)
t(s)
20
15
10
5
0
-5
-10
-15
-20
0
10
20
30
40


energy
w(J)
t(s)
200
150
100
50
0
0
10
20
30
40

P 1.26 We use the passive sign convention to determine whether the power equation is $p = vi$ or $p = -vi$ and substitute into the power equation the values for v and i, as shown below:

$$
\begin{aligned}
p_\mathrm{a} &= -v_\mathrm{a}i_\mathrm{a} = -(-18)(-51) = -918 \text{ W} \\
p_\mathrm{b} &= v_\mathrm{b}i_\mathrm{b} = (-18)(45) = -810 \text{ W} \\
p_\mathrm{c} &= v_\mathrm{c}i_\mathrm{c} = (2)(-6) = -12 \text{ W} \\
p_\mathrm{d} &= -v_\mathrm{d}i_\mathrm{d} = -(20)(-20) = 400 \text{ W} \\
p_\mathrm{e} &= -v_\mathrm{e}i_\mathrm{e} = -(16)(-14) = 224 \text{ W} \\
p_\mathrm{f} &= v_\mathrm{f}i_\mathrm{f} = (36)(31) = 1116 \text{ W}
\end{aligned}
$$

Remember that if the power is positive, the circuit element is absorbing power, whereas is the power is negative, the circuit element is developing power. We can add the positive powers together and the negative powers together — if the power balances, these power sums should be equal:

$\sum P_\mathrm{dev} = 918 + 810 + 12 = 1740$ W;

$\sum P_\mathrm{abs} = 400 + 224 + 1116 = 1740$ W

Thus, the power balances and the total power developed in the circuit is 1740 W.