Simple Resistive Circuits

Assessment Problems

AP 3.1



Start from the right hand side of the circuit and make series and parallel combinations of the resistors until one equivalent resistor remains. Begin by combining the 6Ω resistor and the 10Ω resistor in series:

 $6\,\Omega+10\,\Omega=16\,\Omega$

Now combine this 16Ω resistor in parallel with the 64Ω resistor:

$$16\,\Omega \| 64\,\Omega = \frac{(16)(64)}{16+64} = \frac{1024}{80} = 12.8\,\Omega$$

This equivalent 12.8Ω resistor is in series with the 7.2Ω resistor:

 $12.8\,\Omega+7.2\,\Omega=20\,\Omega$

Finally, this equivalent 20Ω resistor is in parallel with the 30Ω resistor:

$$20\,\Omega \| 30\,\Omega = \frac{(20)(30)}{20+30} = \frac{600}{50} = 12\,\Omega$$

Thus, the simplified circuit is as shown:



[a] With the simplified circuit we can use Ohm's law to find the voltage across both the current source and the 12Ω equivalent resistor:

 $v = (12 \Omega)(5 A) = 60 V$

[b] Now that we know the value of the voltage drop across the current source, we can use the formula p = -vi to find the power associated with the source:

$$p = -(60 \text{ V})(5 \text{ A}) = -300 \text{ W}$$

Thus, the source delivers 300 W of power to the circuit.

[c] We now can return to the original circuit, shown in the first figure. In this circuit, v = 60 V, as calculated in part (a). This is also the voltage drop across the 30Ω resistor, so we can use Ohm's law to calculate the current through this resistor:

$$i_A = \frac{60 \text{ V}}{30 \Omega} = 2 \text{ A}$$

Now write a KCL equation at the upper left node to find the current i_B :

$$-5 A + i_A + i_B = 0$$
 so $i_B = 5 A - i_A = 5 A - 2 A = 3 A$

Next, write a KVL equation around the outer loop of the circuit, using Ohm's law to express the voltage drop across the resistors in terms of the current through the resistors:

$$\begin{aligned} -v + 7.2i_B + 6i_C + 10i_C &= 0\\ \text{So} \qquad 16i_C &= v - 7.2i_B = 60 \text{ V} - (7.2)(3) = 38.4 \text{ V}\\ \text{Thus} \qquad i_C &= \frac{38.4}{16} = 2.4 \text{ A} \end{aligned}$$

Now that we have the current through the 10Ω resistor we can use the formula $p = Ri^2$ to find the power:

$$p_{10\,\Omega} = (10)(2.4)^2 = 57.6 \,\mathrm{W}$$

AP 3.2



[a] We can use voltage division to calculate the voltage v_o across the 75 k Ω resistor:

$$v_o(\text{no load}) = \frac{75,000}{75,000 + 25,000} (200 \text{ V}) = 150 \text{ V}$$

[b] When we have a load resistance of $150 \text{ k}\Omega$ then the voltage v_o is across the parallel combination of the $75 \text{ k}\Omega$ resistor and the $150 \text{ k}\Omega$ resistor. First, calculate the equivalent resistance of the parallel combination:

75 k
$$\Omega \| 150 \text{ k}\Omega = \frac{(75,000)(150,000)}{75,000 + 150,000} = 50,000 \,\Omega = 50 \,\text{k}\Omega$$

Now use voltage division to find v_o across this equivalent resistance:

$$v_o = \frac{50,000}{50,000 + 25,000} (200 \text{ V}) = 133.3 \text{ V}$$

[c] If the load terminals are short-circuited, the 75 k Ω resistor is effectively removed from the circuit, leaving only the voltage source and the 25 k Ω resistor. We can calculate the current in the resistor using Ohm's law:

$$i = \frac{200 \text{ V}}{25 \text{ k}\Omega} = 8 \text{ mA}$$

Now we can use the formula $p = Ri^2$ to find the power dissipated in the $25 \text{ k}\Omega$ resistor:

$$p_{25k} = (25,000)(0.008)^2 = 1.6 \text{ W}$$

[d] The power dissipated in the 75 k Ω resistor will be maximum at no load since v_o is maximum. In part (a) we determined that the no-load voltage is 150 V, so be can use the formula $p = v^2/R$ to calculate the power:

$$p_{75k}(\max) = \frac{(150)^2}{75,000} = 0.3 \text{ W}$$

AP 3.3



[a] We will write a current division equation for the current throught the 80Ω resistor and use this equation to solve for R:

$$i_{80\Omega} = \frac{R}{R + 40\,\Omega + 80\,\Omega} (20\text{ A}) = 4\text{ A} \quad \text{so} \quad 20R = 4(R + 120)$$

Thus $16R = 480$ and $R = \frac{480}{16} = 30\,\Omega$

[b] With $R = 30 \Omega$ we can calculate the current through R using current division, and then use this current to find the power dissipated by R, using the formula $p = Ri^2$:

$$i_R = \frac{40 + 80}{40 + 80 + 30} (20 \text{ A}) = 16 \text{ A}$$
 so $p_R = (30)(16)^2 = 7680 \text{ W}$

[c] Write a KVL equation around the outer loop to solve for the voltage v, and then use the formula p = -vi to calculate the power delivered by the current source:

$$-v + (60 \Omega)(20 \text{ A}) + (30 \Omega)(16 \text{ A}) = 0 \qquad \text{so} \qquad v = 1200 + 480 = 1680 \text{ V}$$

Thus, $p_{\text{source}} = -(1680 \text{ V})(20 \text{ A}) = -33,600 \text{ W}$

Thus, the current source generates 33,600 W of power.



[a] First we need to determine the equivalent resistance to the right of the 40Ω and 70Ω resistors:

$$R_{\rm eq} = 20\,\Omega \|30\,\Omega\| (50\,\Omega + 10\,\Omega) \qquad \text{so} \qquad \frac{1}{R_{\rm eq}} = \frac{1}{20\,\Omega} + \frac{1}{30\,\Omega} + \frac{1}{60\,\Omega} = \frac{1}{10\,\Omega}$$

Thus, $R_{\rm eq} = 10 \,\Omega$

AP 3.4

Now we can use voltage division to find the voltage v_o :

$$v_o = \frac{40}{40 + 10 + 70} (60 \text{ V}) = 20 \text{ V}$$

[b] The current through the 40Ω resistor can be found using Ohm's law:

$$i_{40\Omega} = \frac{v_o}{40} = \frac{20 \text{ V}}{40 \Omega} = 0.5 \text{ A}$$

This current flows from left to right through the 40 Ω resistor. To use current division, we need to find the equivalent resistance of the two parallel branches containing the 20 Ω resistor and the 50 Ω and 10 Ω resistors:

$$20\,\Omega \| (50\,\Omega + 10\,\Omega) = \frac{(20)(60)}{20 + 60} = 15\,\Omega$$

Now we use current division to find the current in the 30Ω branch:

$$i_{30\Omega} = \frac{15}{15+30}(0.5 \text{ A}) = 0.16667 \text{ A} = 166.67 \text{ mA}$$

[c] We can find the power dissipated by the 50Ω resistor if we can find the current in this resistor. We can use current division to find this current from the current in the 40Ω resistor, but first we need to calculate the equivalent resistance of the 20Ω branch and the 30Ω branch:

$$20\,\Omega \| 30\,\Omega = \frac{(20)(30)}{20+30} = 12\,\Omega$$

Current division gives:

$$i_{50\Omega} = \frac{12}{12 + 50 + 10} (0.5 \text{ A}) = 0.08333 \text{ A}$$

Thus,
$$p_{50\Omega} = (50)(0.08333)^2 = 0.34722 \text{ W} = 347.22 \text{ mW}$$

AP 3.5 [a]



We can find the current i using Ohm's law:

$$i = \frac{1 \text{ V}}{100 \Omega} = 0.01 \text{ A} = 10 \text{ mA}$$

[b]



 $R_m = 50\,\Omega \| 5.555\,\Omega = 5\,\Omega$

We can use the meter resistance to find the current using Ohm's law:

$$i_{\text{meas}} = \frac{1 \text{ V}}{100 \,\Omega + 5 \,\Omega} = 0.009524 = 9.524 \text{ mA}$$



Use voltage division to find the voltage v:



The meter resistance is a series combination of resistances:

 $R_m = 149,950 + 50 = 150,000\,\Omega$

We can use voltage division to find v, but first we must calculate the equivalent resistance of the parallel combination of the 75 k Ω resistor and the voltmeter:

$$75,000 \,\Omega \| 150,000 \,\Omega = \frac{(75,000)(150,000)}{75,000 + 150,000} = 50 \,\mathrm{k\Omega}$$

Thus, $v_{\mathrm{meas}} = \frac{50,000}{50,000 + 15,000} (60 \,\mathrm{V}) = 46.15 \,\mathrm{V}$

AP 3.7 [a] Using the condition for a balanced bridge, the products of the opposite resistors must be equal. Therefore,

$$100R_x = (1000)(150)$$
 so $R_x = \frac{(1000)(150)}{100} = 1500 \,\Omega = 1.5 \,\mathrm{k\Omega}$

[b] When the bridge is balanced, there is no current flowing through the meter, so the meter acts like an open circuit. This places the following branches in parallel: The branch with the voltage source, the branch with the series combination R_1 and R_3 and the branch with the series combination of R_2 and R_x . We can find the current in the latter two branches using Ohm's law:

$$i_{R_1,R_3} = \frac{5 \text{ V}}{100 \Omega + 150 \Omega} = 20 \text{ mA};$$
 $i_{R_2,R_x} = \frac{5 \text{ V}}{1000 + 1500} = 2 \text{ mA}$

We can calculate the power dissipated by each resistor using the formula $p = Ri^2$:

$$p_{100\Omega} = (100 \ \Omega)(0.02 \ A)^2 = 40 \ mW$$

$$p_{150\Omega} = (150 \ \Omega)(0.02 \ A)^2 = 60 \ mW$$

$$p_{1000\Omega} = (1000 \ \Omega)(0.002 \ A)^2 = 4 \ mW$$

$$p_{1500\Omega} = (1500 \ \Omega)(0.002 \ A)^2 = 6 \ mW$$

Since none of the power dissipation values exceeds 250 mW, the bridge can be left in the balanced state without exceeding the power-dissipating capacity of the resistors.

AP 3.8 Convert the three Y-connected resistors, 20Ω , 10Ω , and 5Ω to three Δ -connected resistors $R_{\rm a}$, $R_{\rm b}$, and $R_{\rm c}$. To assist you the figure below has both the Y-connected resistors and the Δ -connected resistors



$$R_{\rm a} = \frac{(5)(10) + (5)(20) + (10)(20)}{20} = 17.5\,\Omega$$
$$R_{\rm b} = \frac{(5)(10) + (5)(20) + (10)(20)}{10} = 35\,\Omega$$
$$R_{\rm c} = \frac{(5)(10) + (5)(20) + (10)(20)}{5} = 70\,\Omega$$

The circuit with these new Δ -connected resistors is shown below:



From this circuit we see that the 70 Ω resistor is parallel to the 28 Ω resistor:

$$70\,\Omega \|28\,\Omega = \frac{(70)(28)}{70+28} = 20\,\Omega$$

Also, the 17.5Ω resistor is parallel to the 105Ω resistor:

$$17.5\,\Omega \| 105\,\Omega = \frac{(17.5)(105)}{17.5+105} = 15\,\Omega$$

Once the parallel combinations are made, we can see that the equivalent 20Ω resistor is in series with the equivalent 15Ω resistor, giving an equivalent resistance

of $20 \Omega + 15 \Omega = 35 \Omega$. Finally, this equivalent 35Ω resistor is in parallel with the other 35Ω resistor:

$$35\,\Omega \| 35\,\Omega = \frac{(35)(35)}{35+35} = 17.5\,\Omega$$

Thus, the resistance seen by the 2 A source is 17.5Ω , and the voltage can be calculated using Ohm's law:

 $v = (17.5 \,\Omega)(2 \,\mathrm{A}) = 35 \,\mathrm{V}$

Problems

P 3.1 [a] The 6Ω and 12Ω resistors are in series, as are the 9Ω and 7Ω resistors. The simplified circuit is shown below:



[b] The $3 \text{ k}\Omega$, $5 \text{ k}\Omega$, and $7 \text{ k}\Omega$ resistors are in series. The simplified circuit is shown below:



[c] The 300Ω , 400Ω , and 500Ω resistors are in series. The simplified circuit is shown below:



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P 3.2 [a] The 10Ω and 40Ω resistors are in parallel, as are the 100Ω and 25Ω resistors. The simplified circuit is shown below:



[b] The 9 k Ω , 18 k Ω , and 6 k Ω resistors are in parallel. The simplified circuit is shown below:



[c] The 600Ω , 200Ω , and 300Ω resistors are in series. The simplified circuit is shown below:



- P 3.5 Always work from the side of the circuit furthest from the source. Remember that the current in all series-connected circuits is the same, and that the voltage drop across all parallel-connected resistors is the same.
 - **[a]** $R_{\rm eq} = 6 + 12 + [4||(9+7)] = 18 + (4||16) = 18 + 3.2 = 21.2 \,\Omega$
 - **[b]** $R_{eq} = 4 \text{ k} + [10 \text{ k} || (3 \text{ k} + 5 \text{ k} + 7 \text{ k})] = 4 \text{ k} + (10 \text{ k} || 15 \text{ k}) = 4 \text{ k} + 6 \text{ k} = 10 \text{ k}\Omega$
 - [c] $R_{\rm eq} = (300 + 400 + 500) + (600||1200) = 1200 + 400 = 1600 \,\Omega$

- P 3.6 Always work from the side of the circuit furthest from the source. Remember that the current in all series-connected circuits is the same, and that the voltage drop across all parallel-connected resistors is the same.
 - **[a]** $R_{\text{eq}} = 18 + (100\|25\|(22 + (10\|40))) = 18 + (20\|(22 + 8) = 18 + 12 = 30\,\Omega$
 - **[b]** $R_{\rm eq} = 10 \text{ k} \| (5 \text{ k} + 2 \text{ k} + (9 \text{ k} \| 18 \text{ k} \| 6 \text{ k})) = 10 \text{ k} \| (7 \text{ k} + 3 \text{ k}) = 10 \text{ k} \| 10 \text{ k} = 5 \text{ k} \Omega$

[c]
$$R_{eq} = 600 \|200\| 300 \| (250 + 150) = 600 \|200\| 300 \|400 = 80 \Omega$$

P 3.13 [a] We can calculate the no-load voltage using voltage division to determine the voltage drop across the 500Ω resistor:

$$v_o = \frac{500}{(2000 + 500)}(75 \text{ V}) = 15 \text{ V}$$

[b] We can calculate the power if we know the current in each of the resistors. Under no-load conditions, the resistors are in series, so we can use Ohm's law to calculate the current they share:

$$i = \frac{75 \text{ V}}{2000 \Omega + 500 \Omega} = 0.03 \text{ A} = 30 \text{ mA}$$

Now use the formula $p = Ri^2$ to calculate the power dissipated by each resistor:

$$P_{R_1} = (2000)(0.03)^2 = 1.8 \text{ W} = 1800 \text{ mW}$$

$$P_{R_2} = (500)(0.03)^2 = 0.45 \text{ W} = 450 \text{ mW}$$

[c] Since R_1 and R_2 carry the same current and $R_1 > R_2$ to satisfy the no-load voltage requirement, first pick R_1 to meet the 1 W specification

$$i_{R_1} = \frac{75 - 15}{R_1}$$
, Therefore, $\left(\frac{60}{R_1}\right)^2 R_1 \le 1$

Thus, $R_1 \ge \frac{60}{1}$ or $R_1 \ge 3600 \,\Omega$

Now use the voltage specification:

$$\frac{R_2}{R_2 + 3600}(75) = 15$$

Thus, $R_2 = 900 \,\Omega$

 $R_1 = 1600 \Omega$ and $R_2 = 400 \Omega$ are the smallest values of resistors that satisfy the 1 W specification.

P 3.14 Use voltage division to determine R_2 from the no-load voltage specification:

$$6 \text{ V} = \frac{R_2}{(R_2 + 40)} (18 \text{ V});$$
 so $18R_2 = 6(R_2 + 40)$

Thus,
$$12R_2 = 240$$
 so $R_2 = \frac{240}{12} = 20 \,\Omega$

Now use voltage division again, this time to determine the value of R_e , the parallel combination of R_2 and R_L . We use the loaded voltage specification:

$$4 \text{ V} = \frac{R_{\text{e}}}{(40 + R_{\text{e}})}(18 \text{ V})$$
 so $18R_{\text{e}} = 4(40 + R_{\text{e}})$

Thus,
$$14R_{\rm e} = 160$$
 so $R_{\rm e} = \frac{160}{14} = 11.43 \,\Omega$

Now use the definition R_e to calculate the value of R_L given that $R_2 = 20 \Omega$:

$$R_{\rm e} = \frac{20R_L}{20 + R_L} = 11.43$$
 so $20R_L = 11.43(R_L + 20)$

Therefore, $8.57R_L = 228.6$ and $R_L = \frac{226.8}{8.57} = 26.67 \,\Omega$

P 3.21 Begin by using the relationships among the branch currents to express all branch currents in terms of i_4 :

$$i_1 = 2i_2 = 2(10i_3) = 20i_4$$

 $i_2 = 10i_3 = 10i_4$
 $i_3 = i_4$

Now use KCL at the top node to relate the branch currents to the current supplied by the source.

 $i_1 + i_2 + i_3 + i_4 = 8 \text{ mA}$

Express the branch currents in terms of i_4 and solve for i_4 :

$$8 \text{ mA} = 20i_4 + 10i_4 + i_4 + i_4 = 32i_4$$
 so $i_4 = \frac{0.008}{32} = 0.00025 = 0.25 \text{ mA}$

Since the resistors are in parallel, the same voltage, 4 V appears across each of them. We know the current and the voltage for R_4 so we can use Ohm's law to calculate R_4 :

$$R_4 = \frac{v_g}{i_4} = \frac{4 \text{ V}}{0.25 \text{ mA}} = 16 \text{ k}\Omega$$

Calculate i_3 from i_4 and use Ohm's law as above to find R_3 :

$$i_3 = i_4 = 0.25 \text{ mA}$$
 \therefore $R_3 = \frac{v_g}{i_3} = \frac{4 \text{ V}}{0.25 \text{ mA}} = 16 \text{ k}\Omega$

Calculate i_2 from i_4 and use Ohm's law as above to find R_2 :

$$i_2 = 10i_4 = 10(0.25 \text{ mA}) = 2.5 \text{ mA}$$
 \therefore $R_2 = \frac{v_g}{i_2} = \frac{4 \text{ V}}{2.5 \text{ mA}} = 1.6 \text{ k}\Omega$

Calculate i_1 from i_4 and use Ohm's law as above to find R_1 :

$$i_1 = 20i_4 = 20(0.25 \text{ mA}) = 5 \text{ mA}$$
 \therefore $R_1 = \frac{v_g}{i_1} = \frac{4 \text{ V}}{5 \text{ mA}} = 800 \Omega$

The resulting circuit is shown below:



Using voltage division,

$$v_{18\Omega} = \frac{18}{18 + 30} (40) = 15 \text{ V positive at the top}$$
[b]

$$a \bullet \underbrace{12\Omega}_{\text{W}} \underbrace{30\Omega}_{\text{W}}_{\text{I}_{30}} \\ \underbrace{24\Omega}_{\text{I}_{30}} \underbrace{18\Omega}_{\text{I}_{30}} \\ b \bullet \underbrace{60\text{mA}}_{10\Omega} \\ b \bullet \underbrace{10\Omega}_{\text{I}_{30}} \\ b \bullet \underbrace{10\Omega}_{\text{I}_{30}} \underbrace{18\Omega}_{\text{I}_{30}} \\ b \bullet \underbrace{18\Omega}_{\text{I}_{30}} \underbrace{18\Omega}_{\text{I}_{30}} \underbrace{18\Omega}_{\text{I}_{30}} \\ b \bullet \underbrace{18\Omega}_{\text{I}_{30}} \underbrace{18\Omega}_{\text{I}_{30$$

Using current division,

 $i_{30\Omega} = \frac{24}{24 + 30 + 18} (60 \times 10^{-3}) = 20$ mA flowing from right to left

[c]



The 9 mA current in the 1.2 k Ω resistor is also the current in the 2 k Ω resistor. It then divides among the 4 k Ω , 30 k Ω , and 60 k Ω resistors.

4 k $\Omega \| 60 \ k\Omega = 3.75 \ k\Omega$

Using current division,

 $i_{30 \text{ k}\Omega} = \frac{3.75 \text{ k}}{30 \text{ k} + 3.75 \text{ k}} (9 \times 10^{-3}) = 1 \text{ m A}, \text{ flowing bottom to top}$

[d]

The voltage drop across the 4 k Ω resistor is the same as the voltage drop across the series combination of the 1.2 k Ω , the $(7.2 \text{ k} \| 2.4 \text{ k})\Omega$ combined resistor, and the 2 k Ω resistor. Note that

$$7.2 \text{ k} \| 2.4 \text{ k} = \frac{(7200)(2400)}{9600} = 1.8 \text{ k}\Omega$$

Using voltage division,

 $v_o = \frac{1800}{1200 + 1800 + 2000}(50) = 18$ V positive at the top





First, note the following: $18||9 = 6 \Omega$; $20||5 = 4 \Omega$; and the voltage drop across the 18Ω resistor is the same as the voltage drop across the parallel combination of the 18Ω and 9Ω resistors. Using voltage division,

$$v_o = \frac{6}{6+4+10}(0.1 \text{ V}) = 30 \text{ mV}$$
 positive at the left

[b]



The equivalent resistance of the $5\,\Omega,\,15\,\Omega,$ and $60\,\Omega$ resistors is

$$R_e = (5+15) \| 60 = 15 \,\Omega$$

Using voltage division to find the voltage across the equivalent resistance,

$$v_{R_e} = \frac{15}{15+10}(10) = 6 \text{ V}$$

Using voltage division again,

$$v_o = \frac{15}{5+15}(6) = 4.5$$
 V positive at the top

[c]



Find equivalent resistance on the right side

$$R_r = 5.2 + \frac{(12)(5+3)}{(12+3+5)} = 10 \ \Omega$$

Find voltage bottom to top across R_r

$$(10)(3) = 30$$
 V

Find the equivalent resistance on the left side

$$R_l = 6 + \frac{(40)(45+15)}{(40+45+15)} = 30 \ \Omega$$

The current in the 6Ω is

$$i_{6 \Omega} = \frac{30}{30} = 1 \text{ A}$$
 left to right

Use current division to find i_o

$$i_o = (1) \left(\frac{40}{40 + 15 + 45} \right) = 0.4 \text{ A}$$
 bottom to top

P 3.31 [a] The model of the ammeter is an ideal ammeter in parallel with a resistor whose resistance is given by

$$R_s = \frac{100\,\mu\mathrm{V}}{10\,\mu\mathrm{A}} = 10\,\Omega.$$

We can calculate the current through the real meter using current division:

$$i_m = \frac{(10/99)}{10 + (10/99)}(i_{\text{meas}}) = \frac{10}{990 + 10}(i_{\text{meas}}) = \frac{1}{100}i_{\text{meas}}$$

$$[\mathbf{b}] \ R_s = \frac{100\,\mu\text{V}}{10\,\mu\text{A}} = 10\,\Omega.$$

$$i_m = \frac{(100/999,990)}{10 + (100/999,990)}(i_{\text{meas}}) = \frac{1}{100,000}(i_{\text{meas}})$$

[c] Yes

P 3.34 For all full-scale readings the total resistance is

$$R_V + R_{\text{movement}} = \frac{\text{full-scale reading}}{10^{-3}}$$

We can calculate the resistance of the movement as follows:

$$R_{\rm movement} = \frac{20 \,\,{\rm mV}}{1 \,\,{\rm mA}} = 20 \,\Omega$$

Therefore, $R_V = 1000$ (full-scale reading) -20

[a] $R_V = 1000(50) - 20 = 49,980 \,\Omega$

[b]
$$R_V = 1000(5) - 20 = 4980 \,\Omega$$

[c] $R_V = 1000(0.25) - 20 = 230 \,\Omega$
[d] $R_V = 1000(0.025) - 20 = 5 \,\Omega$





The condition for a balanced bridge is that the product of the opposite resistors must be equal:

$$(200)(R_x) = (500)(800)$$
 so $R_x = \frac{(500)(800)}{200} = 2000 \,\Omega$

[b] The source current is the sum of the two branch currents. Each branch current can be determined using Ohm's law, since the resistors in each branch are in series and the voltage drop across each branch is 6 V:

$$i_s = \frac{6 \text{ V}}{200 \Omega + 800 \Omega} + \frac{6 \text{ V}}{500 \Omega + 2000 \Omega} = 8.4 \text{ mA}$$

[c] We can use current division to find the current in each branch:

$$i_{\text{left}} = \frac{500 + 2000}{500 + 2000 + 200 + 800} (8.4 \text{ mA}) = 6 \text{ mA}$$

 $i_{\text{right}} = 8.4 \text{ mA} - 6 \text{ mA} = 2.4 \text{ mA}$

Now we can use the formula $p = Ri^2$ to find the power dissipated by each resistor:

$$p_{200} = (200)(0.006)^2 = 7.2 \text{ mW}$$
 $p_{800} = (800)(0.006)^2 = 28.8 \text{ mW}$
 $p_{500} = (500)(0.0024)^2 = 2.88 \text{ mW}$ $p_{2000} = (2000)(0.0024)^2 = 11.52 \text{ mW}$

Thus, the 800Ω resistor absorbs the most power; it absorbs 28.8 mW of power.

[d] From the analysis in part (c), the 500Ω resistor absorbs the least power; it absorbs 2.88 mW of power.

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P 3.53 Begin by transforming the Y-connected resistors $(10 \Omega, 40 \Omega, 50 \Omega)$ to Δ -connected resistors. Both the Y-connected and Δ -connected resistors are shown below to assist in using Eqs. 3.44 – 3.46:



Now use Eqs. 3.44 - 3.46 to calculate the values of the Δ -connected resistors:

$$R_1 = \frac{(40)(10)}{10 + 40 + 50} = 4\,\Omega; \quad R_2 = \frac{(50)(10)}{10 + 40 + 50} = 5\,\Omega; \quad R_3 = \frac{(40)(50)}{10 + 40 + 50} = 20\,\Omega$$

The transformed circuit is shown below:



The equivalent resistance seen by the 24 V source can be calculated by making series and parallel combinations of the resistors to the right of the 24 V source:

$$R_{\rm eq} = (15+5) \| (4+1) + 20 = 20 \| 5 + 20 = 4 + 20 = 24 \,\Omega$$

Therefore, the current i in the 24 V source is given by

$$i = \frac{24 \text{ V}}{24 \Omega} = 1 \text{ A}$$

Use current division to calculate the currents i_1 and i_2 . Note that the current i_1 flows in the branch containing the 15Ω and 5Ω series connected resistors, while the current i_2 flows in the parallel branch that contains the series connection of the 1Ω and 4Ω resistors:

$$i_1 = \frac{1+4}{1+4+15+5}$$
 $(i) = \frac{5}{25}$ $(1 A) = 0.2 A$, and $i_2 = 1 A - 0.2 A = 0.8 A$

Now use KVL and Ohm's law to calculate v_1 . Note that v_1 is the sum of the voltage drop across the 4Ω resistor, $4i_2$, and the voltage drop across the 20Ω resistor, 20i:

$$v_1 = 4i_2 + 20i = 4(0.8 \text{ A}) + 20(1 \text{ A}) = 3.2 + 20 = 23.2 \text{ V}$$

Finally, use KVL and Ohm's law to calculate v_2 . Note that v_2 is the sum of the voltage drop across the 5Ω resistor, $5i_1$, and the voltage drop across the 20Ω resistor, 20i:

$$v_2 = 5i_1 + 20i = 5(0.2 \text{ A}) + 20(1 \text{ A}) = 1 + 20 = 21 \text{ V}$$

P 3.54 [a] Calculate the values of the Y-connected resistors that are equivalent to the 10Ω , 40Ω , and $50\Omega \Delta$ -connected resistors:

$$R_X = \frac{(10)(50)}{10+40+50} = 5\,\Omega; \qquad R_Y = \frac{(40)(50)}{10+40+50} = 20\,\Omega;$$
$$R_Z = \frac{(10)(40)}{10+40+50} = 4\,\Omega$$

Replacing the R_2 — R_3 — R_4 delta with its equivalent Y gives



Now calculate the equivalent resistance R_{ab} by making series and parallel combinations of the resistors:

$$R_{\rm ab} = 13 + 5 + \left[(4+8) \| (20+4) \right] + 7 = 33\,\Omega$$

[b] Calculate the values of the Δ -connected resistors that are equivalent to the 8Ω , 10Ω , and 40Ω Y-connected resistors:

$$R_X = \frac{(10)(40) + (40)(8) + (8)(10)}{8} = \frac{800}{8} = 100 \Omega$$

$$R_Y = \frac{(10)(40) + (40)(8) + (8)(10)}{10} = \frac{800}{10} = 80 \Omega$$

$$R_Z = \frac{(10)(40) + (40)(8) + (8)(10)}{40} = \frac{800}{40} = 20 \Omega$$

Replacing the R_2 , R_4 , R_5 wye with its equivalent Δ gives



Make series and parallel combinations of the resistors to find the equivalent resistance R_{ab} :

- $100 \Omega \| 50 \Omega = 33.33 \Omega;$ $80 \Omega \| 4 \Omega = 3.81 \Omega$
- $\therefore 100\|50 + 80\|4 = 33.33 + 3.81 = 37.14\,\Omega$

$$\therefore \quad 37.14 \| 20 = \frac{(37.14)(20)}{57.14} = 13\,\Omega$$

- : $R_{\rm ab} = 13 + 13 + 7 = 33 \,\Omega$
- [c] Convert the delta connection $R_4 R_5 R_6$ to its equivalent wye. Convert the wye connection $R_3 - R_4 - R_6$ to its equivalent delta.
- P 3.55 Replace the upper and lower deltas with the equivalent wyes:

$$R_{1U} = \frac{(50)(10)}{100} = 5\,\Omega; R_{2U} = \frac{(50)(40)}{100} = 20\,\Omega; R_{3U} = \frac{(40)(10)}{100} = 4\,\Omega$$

$$R_{1L} = \frac{(60)(10)}{100} = 6\,\Omega; R_{2L} = \frac{(60)(30)}{100} = 18\,\Omega; R_{3L} = \frac{(30)(10)}{100} = 3\,\Omega$$

The resulting circuit is shown below:



Now make series and parallel combinations of the resistors:

 $(4+6) \| (20+32+20+18) = 10 \| 90 = 9 \,\Omega$

 $R_{\rm ab} = 33 + 5 + 9 + 3 + 40 = 90\,\Omega$