# Simple Resistive Circuits 

## Assessment Problems

AP 3.1


Start from the right hand side of the circuit and make series and parallel combinations of the resistors until one equivalent resistor remains. Begin by combining the $6 \Omega$ resistor and the $10 \Omega$ resistor in series:
$6 \Omega+10 \Omega=16 \Omega$
Now combine this $16 \Omega$ resistor in parallel with the $64 \Omega$ resistor:
$16 \Omega \| 64 \Omega=\frac{(16)(64)}{16+64}=\frac{1024}{80}=12.8 \Omega$
This equivalent $12.8 \Omega$ resistor is in series with the $7.2 \Omega$ resistor:
$12.8 \Omega+7.2 \Omega=20 \Omega$
Finally, this equivalent $20 \Omega$ resistor is in parallel with the $30 \Omega$ resistor:
$20 \Omega \| 30 \Omega=\frac{(20)(30)}{20+30}=\frac{600}{50}=12 \Omega$
Thus, the simplified circuit is as shown:

[a] With the simplified circuit we can use Ohm's law to find the voltage across both the current source and the $12 \Omega$ equivalent resistor:

$$
v=(12 \Omega)(5 \mathrm{~A})=60 \mathrm{~V}
$$

[b] Now that we know the value of the voltage drop across the current source, we can use the formula $p=-v i$ to find the power associated with the source:
$p=-(60 \mathrm{~V})(5 \mathrm{~A})=-300 \mathrm{~W}$
Thus, the source delivers 300 W of power to the circuit.
[c] We now can return to the original circuit, shown in the first figure. In this circuit, $v=60 \mathrm{~V}$, as calculated in part (a). This is also the voltage drop across the $30 \Omega$ resistor, so we can use Ohm's law to calculate the current through this resistor:
$i_{A}=\frac{60 \mathrm{~V}}{30 \Omega}=2 \mathrm{~A}$
Now write a KCL equation at the upper left node to find the current $i_{B}$ :
$-5 \mathrm{~A}+i_{A}+i_{B}=0 \quad$ so $\quad i_{B}=5 \mathrm{~A}-i_{A}=5 \mathrm{~A}-2 \mathrm{~A}=3 \mathrm{~A}$
Next, write a KVL equation around the outer loop of the circuit, using Ohm's law to express the voltage drop across the resistors in terms of the current through the resistors:
$-v+7.2 i_{B}+6 i_{C}+10 i_{C}=0$
So $\quad 16 i_{C}=v-7.2 i_{B}=60 \mathrm{~V}-(7.2)(3)=38.4 \mathrm{~V}$
Thus $\quad i_{C}=\frac{38.4}{16}=2.4 \mathrm{~A}$
Now that we have the current through the $10 \Omega$ resistor we can use the formula $p=R i^{2}$ to find the power:

$$
p_{10 \Omega}=(10)(2.4)^{2}=57.6 \mathrm{~W}
$$

## AP 3.2


[a] We can use voltage division to calculate the voltage $v_{o}$ across the $75 \mathrm{k} \Omega$ resistor:

$$
v_{o}(\text { no load })=\frac{75,000}{75,000+25,000}(200 \mathrm{~V})=150 \mathrm{~V}
$$

[b] When we have a load resistance of $150 \mathrm{k} \Omega$ then the voltage $v_{o}$ is across the parallel combination of the $75 \mathrm{k} \Omega$ resistor and the $150 \mathrm{k} \Omega$ resistor. First, calculate the equivalent resistance of the parallel combination:
$75 \mathrm{k} \Omega \| 150 \mathrm{k} \Omega=\frac{(75,000)(150,000)}{75,000+150,000}=50,000 \Omega=50 \mathrm{k} \Omega$
Now use voltage division to find $v_{o}$ across this equivalent resistance:
$v_{o}=\frac{50,000}{50,000+25,000}(200 \mathrm{~V})=133.3 \mathrm{~V}$
[c] If the load terminals are short-circuited, the $75 \mathrm{k} \Omega$ resistor is effectively removed from the circuit, leaving only the voltage source and the $25 \mathrm{k} \Omega$ resistor. We can calculate the current in the resistor using Ohm's law:
$i=\frac{200 \mathrm{~V}}{25 \mathrm{k} \Omega}=8 \mathrm{~mA}$
Now we can use the formula $p=R i^{2}$ to find the power dissipated in the $25 \mathrm{k} \Omega$ resistor:
$p_{25 k}=(25,000)(0.008)^{2}=1.6 \mathrm{~W}$
[d] The power dissipated in the $75 \mathrm{k} \Omega$ resistor will be maximum at no load since $v_{o}$ is maximum. In part (a) we determined that the no-load voltage is 150 V , so be can use the formula $p=v^{2} / R$ to calculate the power:
$p_{75 k}(\max )=\frac{(150)^{2}}{75,000}=0.3 \mathrm{~W}$
AP 3.3

[a] We will write a current division equation for the current throught the $80 \Omega$ resistor and use this equation to solve for $R$ :

$$
\begin{aligned}
& i_{80 \Omega}=\frac{R}{R+40 \Omega+80 \Omega}(20 \mathrm{~A})=4 \mathrm{~A} \quad \text { so } \quad 20 R=4(R+120) \\
& \text { Thus } \quad 16 R=480 \quad \text { and } \quad R=\frac{480}{16}=30 \Omega
\end{aligned}
$$

[b] With $R=30 \Omega$ we can calculate the current through R using current division, and then use this current to find the power dissipated by $R$, using the formula $p=R i^{2}$ : $i_{R}=\frac{40+80}{40+80+30}(20 \mathrm{~A})=16 \mathrm{~A} \quad$ so $\quad p_{R}=(30)(16)^{2}=7680 \mathrm{~W}$
[c] Write a KVL equation around the outer loop to solve for the voltage $v$, and then use the formula $p=-v i$ to calculate the power delivered by the current source:
$-v+(60 \Omega)(20 \mathrm{~A})+(30 \Omega)(16 \mathrm{~A})=0 \quad$ so $\quad v=1200+480=1680 \mathrm{~V}$
Thus, $\quad p_{\text {source }}=-(1680 \mathrm{~V})(20 \mathrm{~A})=-33,600 \mathrm{~W}$
Thus, the current source generates $33,600 \mathrm{~W}$ of power.
AP 3.4

[a] First we need to determine the equivalent resistance to the right of the $40 \Omega$ and $70 \Omega$ resistors:

$$
R_{\mathrm{eq}}=20 \Omega\|30 \Omega\|(50 \Omega+10 \Omega) \quad \text { so } \quad \frac{1}{R_{\mathrm{eq}}}=\frac{1}{20 \Omega}+\frac{1}{30 \Omega}+\frac{1}{60 \Omega}=\frac{1}{10 \Omega}
$$

Thus, $\quad R_{\text {eq }}=10 \Omega$
Now we can use voltage division to find the voltage $v_{o}$ :

$$
v_{o}=\frac{40}{40+10+70}(60 \mathrm{~V})=20 \mathrm{~V}
$$

[b] The current through the $40 \Omega$ resistor can be found using Ohm's law:
$i_{40 \Omega}=\frac{v_{o}}{40}=\frac{20 \mathrm{~V}}{40 \Omega}=0.5 \mathrm{~A}$
This current flows from left to right through the $40 \Omega$ resistor. To use current division, we need to find the equivalent resistance of the two parallel branches containing the $20 \Omega$ resistor and the $50 \Omega$ and $10 \Omega$ resistors:

$$
20 \Omega \|(50 \Omega+10 \Omega)=\frac{(20)(60)}{20+60}=15 \Omega
$$

Now we use current division to find the current in the $30 \Omega$ branch:

$$
i_{30 \Omega}=\frac{15}{15+30}(0.5 \mathrm{~A})=0.16667 \mathrm{~A}=166.67 \mathrm{~mA}
$$

[c] We can find the power dissipated by the $50 \Omega$ resistor if we can find the current in this resistor. We can use current division to find this current from the current in the $40 \Omega$ resistor, but first we need to calculate the equivalent resistance of the $20 \Omega$ branch and the $30 \Omega$ branch:
$20 \Omega \| 30 \Omega=\frac{(20)(30)}{20+30}=12 \Omega$
Current division gives:
$i_{50 \Omega}=\frac{12}{12+50+10}(0.5 \mathrm{~A})=0.08333 \mathrm{~A}$
Thus, $\quad p_{50 \Omega}=(50)(0.08333)^{2}=0.34722 \mathrm{~W}=347.22 \mathrm{~mW}$

## AP 3.5 [a]



We can find the current $i$ using Ohm's law:
$i=\frac{1 \mathrm{~V}}{100 \Omega}=0.01 \mathrm{~A}=10 \mathrm{~mA}$
[b]

$R_{m}=50 \Omega \| 5.555 \Omega=5 \Omega$
We can use the meter resistance to find the current using Ohm's law:

$$
i_{\mathrm{meas}}=\frac{1 \mathrm{~V}}{100 \Omega+5 \Omega}=0.009524=9.524 \mathrm{~mA}
$$

AP 3.6 [a]


Use voltage division to find the voltage $v$ :

$$
v=\frac{75,000}{75,000+15,000}(60 \mathrm{~V})=50 \mathrm{~V}
$$

[b]


The meter resistance is a series combination of resistances:
$R_{m}=149,950+50=150,000 \Omega$
We can use voltage division to find $v$, but first we must calculate the equivalent resistance of the parallel combination of the $75 \mathrm{k} \Omega$ resistor and the voltmeter:
$75,000 \Omega \| 150,000 \Omega=\frac{(75,000)(150,000)}{75,000+150,000}=50 \mathrm{k} \Omega$
Thus, $\quad v_{\text {meas }}=\frac{50,000}{50,000+15,000}(60 \mathrm{~V})=46.15 \mathrm{~V}$
AP 3.7 [a] Using the condition for a balanced bridge, the products of the opposite resistors must be equal. Therefore,

$$
100 R_{x}=(1000)(150) \quad \text { so } \quad R_{x}=\frac{(1000)(150)}{100}=1500 \Omega=1.5 \mathrm{k} \Omega
$$

[b] When the bridge is balanced, there is no current flowing through the meter, so the meter acts like an open circuit. This places the following branches in parallel: The branch with the voltage source, the branch with the series combination $R_{1}$ and $R_{3}$ and the branch with the series combination of $R_{2}$ and $R_{x}$. We can find the current in the latter two branches using Ohm's law:
$i_{R_{1}, R_{3}}=\frac{5 \mathrm{~V}}{100 \Omega+150 \Omega}=20 \mathrm{~mA} ; \quad i_{R_{2}, R_{x}}=\frac{5 \mathrm{~V}}{1000+1500}=2 \mathrm{~mA}$
We can calculate the power dissipated by each resistor using the formula $p=R i^{2}$ :
$p_{100 \Omega}=(100 \Omega)(0.02 \mathrm{~A})^{2}=40 \mathrm{~mW}$
$p_{150 \Omega}=(150 \Omega)(0.02 \mathrm{~A})^{2}=60 \mathrm{~mW}$
$p_{1000 \Omega}=(1000 \Omega)(0.002 \mathrm{~A})^{2}=4 \mathrm{~mW}$
$p_{1500 \Omega}=(1500 \Omega)(0.002 \mathrm{~A})^{2}=6 \mathrm{~mW}$
Since none of the power dissipation values exceeds 250 mW , the bridge can be left in the balanced state without exceeding the power-dissipating capacity of the resistors.

AP 3.8 Convert the three Y-connected resistors, $20 \Omega, 10 \Omega$, and $5 \Omega$ to three $\Delta$-connected resistors $R_{\mathrm{a}}, R_{\mathrm{b}}$, and $R_{\mathrm{c}}$. To assist you the figure below has both the Y-connected resistors and the $\Delta$-connected resistors


$$
\begin{aligned}
& R_{\mathrm{a}}=\frac{(5)(10)+(5)(20)+(10)(20)}{20}=17.5 \Omega \\
& R_{\mathrm{b}}=\frac{(5)(10)+(5)(20)+(10)(20)}{10}=35 \Omega \\
& R_{\mathrm{c}}=\frac{(5)(10)+(5)(20)+(10)(20)}{5}=70 \Omega
\end{aligned}
$$

The circuit with these new $\Delta$-connected resistors is shown below:


From this circuit we see that the $70 \Omega$ resistor is parallel to the $28 \Omega$ resistor:
$70 \Omega \| 28 \Omega=\frac{(70)(28)}{70+28}=20 \Omega$
Also, the $17.5 \Omega$ resistor is parallel to the $105 \Omega$ resistor:
$17.5 \Omega \| 105 \Omega=\frac{(17.5)(105)}{17.5+105}=15 \Omega$
Once the parallel combinations are made, we can see that the equivalent $20 \Omega$ resistor is in series with the equivalent $15 \Omega$ resistor, giving an equivalent resistance
of $20 \Omega+15 \Omega=35 \Omega$. Finally, this equivalent $35 \Omega$ resistor is in parallel with the other $35 \Omega$ resistor:
$35 \Omega \| 35 \Omega=\frac{(35)(35)}{35+35}=17.5 \Omega$
Thus, the resistance seen by the 2 A source is $17.5 \Omega$, and the voltage can be calculated using Ohm's law:
$v=(17.5 \Omega)(2 \mathrm{~A})=35 \mathrm{~V}$

## Problems

P 3.1 [a] The $6 \Omega$ and $12 \Omega$ resistors are in series, as are the $9 \Omega$ and $7 \Omega$ resistors. The simplified circuit is shown below:

[b] The $3 \mathrm{k} \Omega, 5 \mathrm{k} \Omega$, and $7 \mathrm{k} \Omega$ resistors are in series. The simplified circuit is shown below:

[c] The $300 \Omega, 400 \Omega$, and $500 \Omega$ resistors are in series. The simplified circuit is shown below:


P 3.2 [a] The $10 \Omega$ and $40 \Omega$ resistors are in parallel, as are the $100 \Omega$ and $25 \Omega$ resistors. The simplified circuit is shown below:

[b] The $9 \mathrm{k} \Omega, 18 \mathrm{k} \Omega$, and $6 \mathrm{k} \Omega$ resistors are in parallel. The simplified circuit is shown below:

[c] The $600 \Omega, 200 \Omega$, and $300 \Omega$ resistors are in series. The simplified circuit is shown below:


P 3.5 Always work from the side of the circuit furthest from the source. Remember that the current in all series-connected circuits is the same, and that the voltage drop across all parallel-connected resistors is the same.
[a] $R_{\text {eq }}=6+12+[4 \|(9+7)]=18+(4 \| 16)=18+3.2=21.2 \Omega$
[b] $R_{\text {eq }}=4 \mathrm{k}+[10 \mathrm{k} \|(3 \mathrm{k}+5 \mathrm{k}+7 \mathrm{k})]=4 \mathrm{k}+(10 \mathrm{k} \| 15 \mathrm{k})=4 \mathrm{k}+6 \mathrm{k}=10 \mathrm{k} \Omega$
[c] $R_{\text {eq }}=(300+400+500)+(600 \| 1200)=1200+400=1600 \Omega$

P 3.6 Always work from the side of the circuit furthest from the source. Remember that the current in all series-connected circuits is the same, and that the voltage drop across all parallel-connected resistors is the same.
[a] $R_{\text {eq }}=18+(100\|25\|(22+(10 \| 40)))=18+(20 \|(22+8)=18+12=30 \Omega$
[b] $R_{\text {eq }}=10 \mathrm{k}\|(5 \mathrm{k}+2 \mathrm{k}+(9 \mathrm{k}\|18 \mathrm{k}\| 6 \mathrm{k}))=10 \mathrm{k}\|(7 \mathrm{k}+3 \mathrm{k})=10 \mathrm{k} \| 10 \mathrm{k}=5 \mathrm{k} \Omega$
[c] $R_{\text {eq }}=600\|200\| 300\|(250+150)=600\| 200\|300\| 400=80 \Omega$
P 3.13 [a] We can calculate the no-load voltage using voltage division to determine the voltage drop across the $500 \Omega$ resistor:

$$
v_{o}=\frac{500}{(2000+500)}(75 \mathrm{~V})=15 \mathrm{~V}
$$

[b] We can calculate the power if we know the current in each of the resistors. Under no-load conditions, the resistors are in series, so we can use Ohm's law to calculate the current they share:
$i=\frac{75 \mathrm{~V}}{2000 \Omega+500 \Omega}=0.03 \mathrm{~A}=30 \mathrm{~mA}$
Now use the formula $p=R i^{2}$ to calculate the power dissipated by each resistor:

$$
\begin{aligned}
& P_{R_{1}}=(2000)(0.03)^{2}=1.8 \mathrm{~W}=1800 \mathrm{~mW} \\
& P_{R_{2}}=(500)(0.03)^{2}=0.45 \mathrm{~W}=450 \mathrm{~mW}
\end{aligned}
$$

[c] Since $R_{1}$ and $R_{2}$ carry the same current and $R_{1}>R_{2}$ to satisfy the no-load voltage requirement, first pick $R_{1}$ to meet the 1 W specification
$i_{R_{1}}=\frac{75-15}{R_{1}}, \quad$ Therefore, $\left(\frac{60}{R_{1}}\right)^{2} R_{1} \leq 1$
Thus, $R_{1} \geq \frac{60^{2}}{1} \quad$ or $\quad R_{1} \geq 3600 \Omega$
Now use the voltage specification:

$$
\frac{R_{2}}{R_{2}+3600}(75)=15
$$

Thus, $R_{2}=900 \Omega$
$R_{1}=1600 \Omega$ and $R_{2}=400 \Omega$ are the smallest values of resistors that satisfy the 1 W specification.

P 3.14 Use voltage division to determine $R_{2}$ from the no-load voltage specification:
$6 \mathrm{~V}=\frac{R_{2}}{\left(R_{2}+40\right)}(18 \mathrm{~V}) ; \quad$ so $\quad 18 R_{2}=6\left(R_{2}+40\right)$
Thus, $\quad 12 R_{2}=240 \quad$ so $\quad R_{2}=\frac{240}{12}=20 \Omega$

Now use voltage division again, this time to determine the value of $R_{\mathrm{e}}$, the parallel combination of $R_{2}$ and $R_{L}$. We use the loaded voltage specification:
$4 \mathrm{~V}=\frac{R_{\mathrm{e}}}{\left(40+R_{\mathrm{e}}\right)}(18 \mathrm{~V}) \quad$ so $\quad 18 R_{\mathrm{e}}=4\left(40+R_{\mathrm{e}}\right)$
Thus, $\quad 14 R_{\mathrm{e}}=160 \quad$ so $\quad R_{\mathrm{e}}=\frac{160}{14}=11.43 \Omega$
Now use the definition $R_{\mathrm{e}}$ to calculate the value of $R_{L}$ given that $R_{2}=20 \Omega$ :
$R_{\mathrm{e}}=\frac{20 R_{L}}{20+R_{L}}=11.43 \quad$ so $\quad 20 R_{L}=11.43\left(R_{L}+20\right)$
Therefore, $\quad 8.57 R_{L}=228.6 \quad$ and $\quad R_{L}=\frac{226.8}{8.57}=26.67 \Omega$
P 3.21 Begin by using the relationships among the branch currents to express all branch currents in terms of $i_{4}$ :
$i_{1}=2 i_{2}=2\left(10 i_{3}\right)=20 i_{4}$
$i_{2}=10 i_{3}=10 i_{4}$
$i_{3}=i_{4}$

Now use KCL at the top node to relate the branch currents to the current supplied by the source.
$i_{1}+i_{2}+i_{3}+i_{4}=8 \mathrm{~mA}$
Express the branch currents in terms of $i_{4}$ and solve for $i_{4}$ :
$8 \mathrm{~mA}=20 i_{4}+10 i_{4}+i_{4}+i_{4}=32 i_{4} \quad$ so $\quad i_{4}=\frac{0.008}{32}=0.00025=0.25 \mathrm{~mA}$

Since the resistors are in parallel, the same voltage, 4 V appears across each of them. We know the current and the voltage for $R_{4}$ so we can use Ohm's law to calculate $R_{4}$ :
$R_{4}=\frac{v_{g}}{i_{4}}=\frac{4 \mathrm{~V}}{0.25 \mathrm{~mA}}=16 \mathrm{k} \Omega$
Calculate $i_{3}$ from $i_{4}$ and use Ohm's law as above to find $R_{3}$ :
$i_{3}=i_{4}=0.25 \mathrm{~mA} \quad \therefore \quad R_{3}=\frac{v_{g}}{i_{3}}=\frac{4 \mathrm{~V}}{0.25 \mathrm{~mA}}=16 \mathrm{k} \Omega$
Calculate $i_{2}$ from $i_{4}$ and use Ohm's law as above to find $R_{2}$ :
$i_{2}=10 i_{4}=10(0.25 \mathrm{~mA})=2.5 \mathrm{~mA} \quad \therefore \quad R_{2}=\frac{v_{g}}{i_{2}}=\frac{4 \mathrm{~V}}{2.5 \mathrm{~mA}}=1.6 \mathrm{k} \Omega$
Calculate $i_{1}$ from $i_{4}$ and use Ohm's law as above to find $R_{1}$ :
$i_{1}=20 i_{4}=20(0.25 \mathrm{~mA})=5 \mathrm{~mA} \quad \therefore \quad R_{1}=\frac{v_{g}}{i_{1}}=\frac{4 \mathrm{~V}}{5 \mathrm{~mA}}=800 \Omega$
The resulting circuit is shown below:


P 3.22 [a]


Using voltage division,

$$
v_{18 \Omega}=\frac{18}{18+30}(40)=15 \mathrm{~V} \text { positive at the top }
$$

[b]


Using current division,
$i_{30 \Omega}=\frac{24}{24+30+18}\left(60 \times 10^{-3}\right)=20 \mathrm{~mA}$ flowing from right to left
[c]


The 9 mA current in the $1.2 \mathrm{k} \Omega$ resistor is also the current in the $2 \mathrm{k} \Omega$ resistor. It then divides among the $4 \mathrm{k} \Omega, 30 \mathrm{k} \Omega$, and $60 \mathrm{k} \Omega$ resistors.
$4 \mathrm{k} \Omega \| 60 \mathrm{k} \Omega=3.75 \mathrm{k} \Omega$
Using current division,
$i_{30 \mathrm{k} \Omega}=\frac{3.75 \mathrm{k}}{30 \mathrm{k}+3.75 \mathrm{k}}\left(9 \times 10^{-3}\right)=1 \mathrm{~m} \mathrm{~A}, \quad$ flowing bottom to top
[d]


The voltage drop across the $4 \mathrm{k} \Omega$ resistor is the same as the voltage drop across the series combination of the $1.2 \mathrm{k} \Omega$, the $(7.2 \mathrm{k} \| 2.4 \mathrm{k}) \Omega$ combined resistor, and the $2 \mathrm{k} \Omega$ resistor. Note that
$7.2 \mathrm{k} \| 2.4 \mathrm{k}=\frac{(7200)(2400)}{9600}=1.8 \mathrm{k} \Omega$
Using voltage division,
$v_{o}=\frac{1800}{1200+1800+2000}(50)=18 \mathrm{~V}$ positive at the top

P 3.23 [a]


First, note the following: $18\|9=6 \Omega ; 20\| 5=4 \Omega$; and the voltage drop across the $18 \Omega$ resistor is the same as the voltage drop across the parallel combination of the $18 \Omega$ and $9 \Omega$ resistors. Using voltage division, $v_{o}=\frac{6}{6+4+10}(0.1 \mathrm{~V})=30 \mathrm{mV}$ positive at the left
[b]


The equivalent resistance of the $5 \Omega, 15 \Omega$, and $60 \Omega$ resistors is $R_{e}=(5+15) \| 60=15 \Omega$
Using voltage division to find the voltage across the equivalent resistance,
$v_{R_{e}}=\frac{15}{15+10}(10)=6 \mathrm{~V}$
Using voltage division again,
$v_{o}=\frac{15}{5+15}(6)=4.5 \mathrm{~V}$ positive at the top
[c]


Find equivalent resistance on the right side

$$
R_{r}=5.2+\frac{(12)(5+3)}{(12+3+5)}=10 \Omega
$$

Find voltage bottom to top across $R_{r}$
$(10)(3)=30 \mathrm{~V}$
Find the equivalent resistance on the left side

$$
R_{l}=6+\frac{(40)(45+15)}{(40+45+15)}=30 \Omega
$$

The current in the $6 \Omega$ is
$i_{6 \Omega}=\frac{30}{30}=1 \mathrm{~A} \quad$ left to right
Use current division to find $i_{o}$

$$
i_{o}=(1)\left(\frac{40}{40+15+45}\right)=0.4 \mathrm{~A} \quad \text { bottom to top }
$$

P 3.31 [a] The model of the ammeter is an ideal ammeter in parallel with a resistor whose resistance is given by
$R_{s}=\frac{100 \mu \mathrm{~V}}{10 \mu \mathrm{~A}}=10 \Omega$.
We can calculate the current through the real meter using current division:

$$
i_{m}=\frac{(10 / 99)}{10+(10 / 99)}\left(i_{\text {meas }}\right)=\frac{10}{990+10}\left(i_{\text {meas }}\right)=\frac{1}{100} i_{\text {meas }}
$$

[b] $R_{s}=\frac{100 \mu \mathrm{~V}}{10 \mu \mathrm{~A}}=10 \Omega$.

$$
i_{m}=\frac{(100 / 999,990)}{10+(100 / 999,990)}\left(i_{\text {meas }}\right)=\frac{1}{100,000}\left(i_{\text {meas }}\right)
$$

[c] Yes
P 3.34 For all full-scale readings the total resistance is

$$
R_{V}+R_{\text {movement }}=\frac{\text { full-scale reading }}{10^{-3}}
$$

We can calculate the resistance of the movement as follows:

$$
R_{\text {movement }}=\frac{20 \mathrm{mV}}{1 \mathrm{~mA}}=20 \Omega
$$

Therefore, $\quad R_{V}=1000$ (full-scale reading) -20
[a] $R_{V}=1000(50)-20=49,980 \Omega$
[b] $R_{V}=1000(5)-20=4980 \Omega$
[c] $R_{V}=1000(0.25)-20=230 \Omega$
[d] $R_{V}=1000(0.025)-20=5 \Omega$
P 3.49 [a]


The condition for a balanced bridge is that the product of the opposite resistors must be equal:
$(200)\left(R_{x}\right)=(500)(800) \quad$ so $\quad R_{x}=\frac{(500)(800)}{200}=2000 \Omega$
[b] The source current is the sum of the two branch currents. Each branch current can be determined using Ohm's law, since the resistors in each branch are in series and the voltage drop across each branch is 6 V :
$i_{s}=\frac{6 \mathrm{~V}}{200 \Omega+800 \Omega}+\frac{6 \mathrm{~V}}{500 \Omega+2000 \Omega}=8.4 \mathrm{~mA}$
[c] We can use current division to find the current in each branch:
$i_{\text {left }}=\frac{500+2000}{500+2000+200+800}(8.4 \mathrm{~mA})=6 \mathrm{~mA}$
$i_{\text {right }}=8.4 \mathrm{~mA}-6 \mathrm{~mA}=2.4 \mathrm{~mA}$
Now we can use the formula $p=R i^{2}$ to find the power dissipated by each resistor:

$$
\begin{array}{lr}
p_{200}=(200)(0.006)^{2}=7.2 \mathrm{~mW} & p_{800}=(800)(0.006)^{2}=28.8 \mathrm{~mW} \\
p_{500}=(500)(0.0024)^{2}=2.88 \mathrm{~mW} & p_{2000}=(2000)(0.0024)^{2}=11.52 \mathrm{~mW}
\end{array}
$$

Thus, the $800 \Omega$ resistor absorbs the most power; it absorbs 28.8 mW of power.
[d] From the analysis in part (c), the $500 \Omega$ resistor absorbs the least power; it absorbs 2.88 mW of power.

P 3.53 Begin by transforming the Y-connected resistors ( $10 \Omega, 40 \Omega, 50 \Omega$ ) to $\Delta$-connected resistors. Both the Y-connected and $\Delta$-connected resistors are shown below to assist in using Eqs. $3.44-3.46$ :


Now use Eqs. $3.44-3.46$ to calculate the values of the $\Delta$-connected resistors:
$R_{1}=\frac{(40)(10)}{10+40+50}=4 \Omega ; \quad R_{2}=\frac{(50)(10)}{10+40+50}=5 \Omega ; \quad R_{3}=\frac{(40)(50)}{10+40+50}=20 \Omega$
The transformed circuit is shown below:


The equivalent resistance seen by the 24 V source can be calculated by making series and parallel combinations of the resistors to the right of the 24 V source:
$R_{\mathrm{eq}}=(15+5)\|(4+1)+20=20\| 5+20=4+20=24 \Omega$
Therefore, the current $i$ in the 24 V source is given by
$i=\frac{24 \mathrm{~V}}{24 \Omega}=1 \mathrm{~A}$

Use current division to calculate the currents $i_{1}$ and $i_{2}$. Note that the current $i_{1}$ flows in the branch containing the $15 \Omega$ and $5 \Omega$ series connected resistors, while the current $i_{2}$ flows in the parallel branch that contains the series connection of the $1 \Omega$ and $4 \Omega$ resistors:
$i_{1}=\frac{1+4}{1+4+15+5}(i)=\frac{5}{25}(1 \mathrm{~A})=0.2 \mathrm{~A}, \quad$ and $\quad i_{2}=1 \mathrm{~A}-0.2 \mathrm{~A}=0.8 \mathrm{~A}$
Now use KVL and Ohm's law to calculate $v_{1}$. Note that $v_{1}$ is the sum of the voltage drop across the $4 \Omega$ resistor, $4 i_{2}$, and the voltage drop across the $20 \Omega$ resistor, $20 i$ :
$v_{1}=4 i_{2}+20 i=4(0.8 \mathrm{~A})+20(1 \mathrm{~A})=3.2+20=23.2 \mathrm{~V}$
Finally, use KVL and Ohm's law to calculate $v_{2}$. Note that $v_{2}$ is the sum of the voltage drop across the $5 \Omega$ resistor, $5 i_{1}$, and the voltage drop across the $20 \Omega$ resistor, 20i:
$v_{2}=5 i_{1}+20 i=5(0.2 \mathrm{~A})+20(1 \mathrm{~A})=1+20=21 \mathrm{~V}$
P 3.54 [a] Calculate the values of the Y-connected resistors that are equivalent to the $10 \Omega, 40 \Omega$, and $50 \Omega \Delta$-connected resistors:

$$
\begin{aligned}
& R_{X}=\frac{(10)(50)}{10+40+50}=5 \Omega ; \quad R_{Y}=\frac{(40)(50)}{10+40+50}=20 \Omega ; \\
& R_{Z}=\frac{(10)(40)}{10+40+50}=4 \Omega
\end{aligned}
$$

Replacing the $R_{2}-R_{3}-R_{4}$ delta with its equivalent $Y$ gives


Now calculate the equivalent resistance $R_{\mathrm{ab}}$ by making series and parallel combinations of the resistors:

$$
R_{\mathrm{ab}}=13+5+[(4+8) \|(20+4)]+7=33 \Omega
$$

[b] Calculate the values of the $\Delta$-connected resistors that are equivalent to the $8 \Omega, 10 \Omega$, and $40 \Omega$ Y-connected resistors:

$$
\begin{aligned}
& R_{X}=\frac{(10)(40)+(40)(8)+(8)(10)}{8}=\frac{800}{8}=100 \Omega \\
& R_{Y}=\frac{(10)(40)+(40)(8)+(8)(10)}{10}=\frac{800}{10}=80 \Omega \\
& R_{Z}=\frac{(10)(40)+(40)(8)+(8)(10)}{40}=\frac{800}{40}=20 \Omega
\end{aligned}
$$

Replacing the $R_{2}, R_{4}, R_{5}$ wye with its equivalent $\Delta$ gives


Make series and parallel combinations of the resistors to find the equivalent resistance $R_{\mathrm{ab}}$ :
$100 \Omega\|50 \Omega=33.33 \Omega ; \quad 80 \Omega\| 4 \Omega=3.81 \Omega$

$$
\begin{aligned}
& \therefore \quad 100\|50+80\| 4=33.33+3.81=37.14 \Omega \\
& \therefore \quad 37.14 \| 20=\frac{(37.14)(20)}{57.14}=13 \Omega
\end{aligned}
$$

$$
\therefore \quad R_{\mathrm{ab}}=13+13+7=33 \Omega
$$

[c] Convert the delta connection $R_{4}-R_{5}-R_{6}$ to its equivalent wye.
Convert the wye connection $R_{3}-R_{4}-R_{6}$ to its equivalent delta.
P 3.55 Replace the upper and lower deltas with the equivalent wyes:

$$
\begin{aligned}
& R_{1 \mathrm{U}}=\frac{(50)(10)}{100}=5 \Omega ; R_{2 \mathrm{U}}=\frac{(50)(40)}{100}=20 \Omega ; R_{3 \mathrm{U}}=\frac{(40)(10)}{100}=4 \Omega \\
& R_{1 \mathrm{~L}}=\frac{(60)(10)}{100}=6 \Omega ; R_{2 \mathrm{~L}}=\frac{(60)(30)}{100}=18 \Omega ; R_{3 \mathrm{~L}}=\frac{(30)(10)}{100}=3 \Omega
\end{aligned}
$$

The resulting circuit is shown below:


Now make series and parallel combinations of the resistors:

$$
\begin{aligned}
& (4+6)\|(20+32+20+18)=10\| 90=9 \Omega \\
& R_{\mathrm{ab}}=33+5+9+3+40=90 \Omega
\end{aligned}
$$

