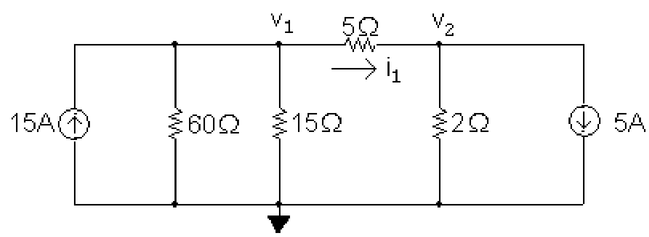
4

Techniques of Circuit Analysis

Assessment Problems

AP 4.1 **[a]** Redraw the circuit, labeling the reference node and the two node voltages:



The two node voltage equations are

$$-15 + \frac{v_1}{60} + \frac{v_1}{15} + \frac{v_1 - v_2}{5} = 0$$

$$5 + \frac{v_2}{2} + \frac{v_2 - v_1}{5} = 0$$

Place these equations in standard form:

$$v_1\left(\frac{1}{60} + \frac{1}{15} + \frac{1}{5}\right) + v_2\left(-\frac{1}{5}\right) = 15$$

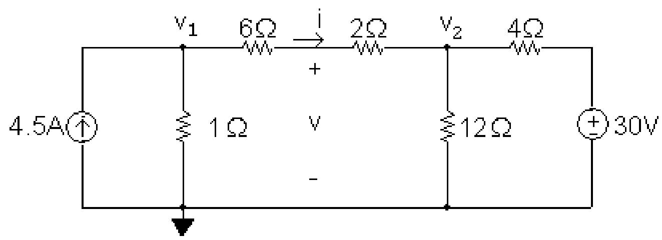
$$v_1\left(-\frac{1}{5}\right) + v_2\left(\frac{1}{2} + \frac{1}{5}\right) = -5$$

Solving, $v_1 = 60$ V and $v_2 = 10$ V;
Therefore, $i_1 = (v_1 - v_2)/5 = 10$ A

[b] $p_{15\text{A}} = -(15\text{ A})v_1 = -(15\text{ A})(60\text{ V}) = -900\text{ W} = 900\text{ W}$(delivered)

[c] $p_{5\text{A}} = (5\text{ A})v_2 = (5\text{ A})(10\text{ V}) = 50\text{ W} = -50\text{ W}$(delivered)

AP 4.2 Redraw the circuit, choosing the node voltages and reference node as shown:



The two node voltage equations are:

$$-4.5 + \frac{v_1}{1} + \frac{v_1 - v_2}{6+2} = 0$$

$$\frac{v_2}{12} + \frac{v_2 - v_1}{6+2} + \frac{v_2 - 30}{4} = 0$$

Place these equations in standard form:

$$v_1\left(1 + \frac{1}{8}\right) + v_2\left(-\frac{1}{8}\right) = 4.5$$

$$v_1\left(-\frac{1}{8}\right) + v_2\left(\frac{1}{12} + \frac{1}{8} + \frac{1}{4}\right) = 7.5$$

Solving, $v_1 = 6$ V $\qquad v_2 = 18$ V

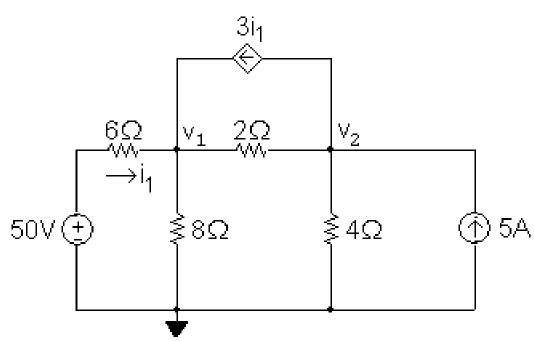
To find the voltage v, first find the current i through the series-connected $6\,\Omega$ and $2\,\Omega$ resistors:

$$i = \frac{v_1 - v_2}{6+2} = \frac{6-18}{8} = -1.5 \text{ A}$$

Using a KVL equation, calculate v:

$$v = 2i + v_2 = 2(-1.5) + 18 = 15 \text{ V}$$

AP 4.3 **[a]** Redraw the circuit, choosing the node voltages and reference node as shown:



The node voltage equations are:

$$\frac{v_1 - 50}{6} + \frac{v_1}{8} + \frac{v_1 - v_2}{2} - 3i_1 = 0$$

$$-5 + \frac{v_2}{4} + \frac{v_2 - v_1}{2} + 3i_1 = 0$$

The dependent source requires the following constraint equation:

$$i_1 = \frac{50 - v_1}{6}$$

Place these equations in standard form:

$$\begin{array}{lllllll}
v_1\left(\frac{1}{6}+\frac{1}{8}+\frac{1}{2}\right) & + & v_2\left(-\frac{1}{2}\right) & + & i_1(-3) & = & \frac{50}{6} \\
v_1\left(-\frac{1}{2}\right) & + & v_2\left(\frac{1}{4}+\frac{1}{2}\right) & + & i_1(3) & = & 5 \\
v_1\left(\frac{1}{6}\right) & + & v_2(0) & + & i_1(1) & = & \frac{50}{6}
\end{array}$$

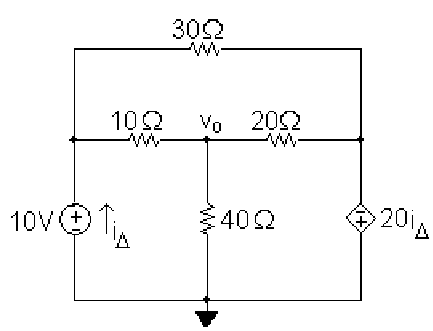
Solving, $v_1 = 32$ V; $v_2 = 16$ V; $i_1 = 3$ A

Using these values to calculate the power associated with each source:

$$\begin{aligned}
p_{50\text{V}} = -50i_1 &= -150\text{ W} \\
p_{5\text{A}} = -5(v_2) &= -80\text{ W} \\
p_{3i_1} = 3i_1(v_2 - v_1) &= -144\text{ W}
\end{aligned}$$

[b] All three sources are delivering power to the circuit because the power computed in (a) for each of the sources is negative.

AP 4.4 Redraw the circuit and label the reference node and the node at which the node voltage equation will be written:



The node voltage equation is

$$\frac{v_o}{40} + \frac{v_o - 10}{10} + \frac{v_o + 20i_\Delta}{20} = 0$$

The constraint equation required by the dependent source is

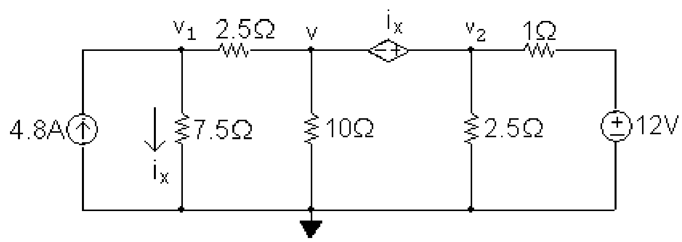
$$i_\Delta = i_{10\,\Omega} + i_{30\,\Omega} = \frac{10 - v_o}{10} + \frac{10 + 20i_\Delta}{30}$$

Place these equations in standard form:

$$v_o\left(\frac{1}{40}+\frac{1}{10}+\frac{1}{20}\right) + i_\Delta(1) = 1$$
$$v_o\left(\frac{1}{10}\right) + i_\Delta\left(1-\frac{20}{30}\right) = 1+\frac{10}{30}$$

Solving, $v_o = 24$ V $i_\Delta = -3.2$ A

AP 4.5 Redraw the circuit identifying the three node voltages and the reference node:



Note that the dependent voltage source and the node voltages v and v_2 form a supernode. The v_1 node voltage equation is

$$\frac{v_1}{7.5}+\frac{v_1-v}{2.5}-4.8=0$$

The supernode equation is

$$\frac{v-v_1}{2.5}+\frac{v}{10}+\frac{v_2}{2.5}+\frac{v_2-12}{1}=0$$

The constraint equation due to the dependent source is

$$i_x=\frac{v_1}{7.5}$$

The constraint equation due to the supernode is

$$v+i_x=v_2$$

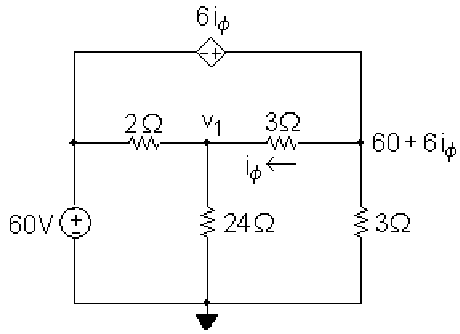
Place this set of equations in standard form:

$$v_1\left(\frac{1}{7.5}+\frac{1}{2.5}\right) + v\left(-\frac{1}{2.5}\right) + v_2(0) + i_x(0) = 4.8$$
$$v_1\left(-\frac{1}{2.5}\right) + v\left(\frac{1}{2.5}+\frac{1}{10}\right) + v_2\left(\frac{1}{2.5}+1\right) + i_x(0) = 12$$
$$v_1\left(-\frac{1}{7.5}\right) + v(0) + v_2(0) + i_x(1) = 0$$
$$v_1(0) + v(1) + v_2(-1) + i_x(1) = 0$$

Solving this set of equations for v gives $v = 8$ V

$v_1 = 15$ V, $v_2 = 10$ V, $i_x = 2$ A

AP 4.6 Redraw the circuit identifying the reference node and the two unknown node voltages. Note that the right-most node voltage is the sum of the 60 V source and the dependent source voltage.



The node voltage equation at v_1 is

$$\frac{v_1 - 60}{2} + \frac{v_1}{24} + \frac{v_1 - (60 + 6i_\phi)}{3} = 0$$

The constraint equation due to the dependent source is

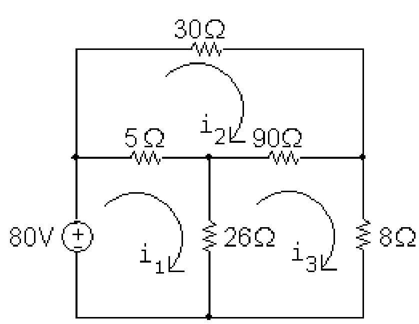
$$i_\phi = \frac{60 + 6i_\phi - v_1}{3}$$

Place these two equations in standard form:

$$\begin{aligned} v_1\left(\frac{1}{2} + \frac{1}{24} + \frac{1}{3}\right) &+ i_\phi(-2) &= 30 + 20 \\ v_1\left(\frac{1}{3}\right) &+ i_\phi(1-2) &= 20 \end{aligned}$$

Solving, $v_1 = 48$ V $\quad i_\phi = -4$ A

AP 4.7 **[a]** Redraw the circuit identifying the three mesh currents:



The mesh current equations are:

$$\begin{aligned} -80 + 5(i_1 - i_2) + 26(i_1 - i_3) &= 0 \\ 30i_2 + 90(i_2 - i_3) + 5(i_2 - i_1) &= 0 \\ 8i_3 + 26(i_3 - i_1) + 90(i_3 - i_2) &= 0 \end{aligned}$$

Place these equations in standard form:

$$\begin{aligned} 31i_1 - 5i_2 - 26i_3 &= 80 \\ -5i_1 + 125i_2 - 90i_3 &= 0 \\ -26i_1 - 90i_2 + 124i_3 &= 0 \end{aligned}$$

Solving,

$i_1 = 5$ A; $\quad i_2 = 2$ A; $\quad i_3 = 2.5$ A

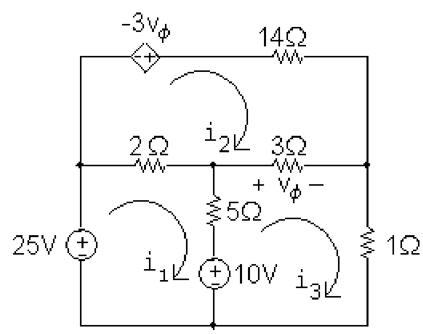
$p_{80V} = -(80)i_1 = -(80)(5) = -400$ W

Therefore the 80 V source is delivering 400 W to the circuit.

[b] $p_{8\Omega} = (8)i_3^2 = 8(2.5)^2 = 50$ W, so the $8\,\Omega$ resistor dissipates 50 W.

AP 4.8 **[a]** $b = 8, \quad n = 6, \quad b - n + 1 = 3$

[b] Redraw the circuit identifying the three mesh currents:



The three mesh-current equations are

$$\begin{aligned} -25 + 2(i_1 - i_2) + 5(i_1 - i_3) + 10 &= 0 \\ -(-3v_\phi) + 14i_2 + 3(i_2 - i_3) + 2(i_2 - i_1) &= 0 \\ 1i_3 - 10 + 5(i_3 - i_1) + 3(i_3 - i_2) &= 0 \end{aligned}$$

The dependent source constraint equation is

$v_\phi = 3(i_3 - i_2)$

Place these four equations in standard form:

$$\begin{aligned} 7i_1 - 2i_2 - 5i_3 + 0v_\phi &= 15 \\ -2i_1 + 19i_2 - 3i_3 + 3v_\phi &= 0 \\ -5i_1 - 3i_2 + 9i_3 + 0v_\phi &= 10 \\ 0i_1 + 3i_2 - 3i_3 + 1v_\phi &= 0 \end{aligned}$$

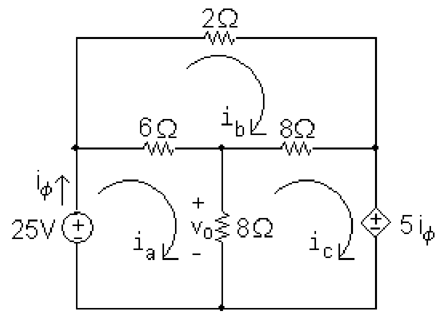
Solving

$i_1 = 4$ A; $\quad i_2 = -1$ A; $\quad i_3 = 3$ A; $\quad v_\phi = 12$ V

$$p_{\rm ds} = -(-3v_\phi)i_2 = 3(12)(-1) = -36 \text{ W}$$

Thus, the dependent source is delivering 36 W, or absorbing −36 W.

AP 4.9 Redraw the circuit identifying the three mesh currents:



The mesh current equations are:

$$\begin{aligned} -25 + 6(i_{\rm a} - i_{\rm b}) + 8(i_{\rm a} - i_{\rm c}) &= 0 \\ 2i_{\rm b} + 8(i_{\rm b} - i_{\rm c}) + 6(i_{\rm b} - i_{\rm a}) &= 0 \\ 5i_\phi + 8(i_{\rm c} - i_{\rm a}) + 8(i_{\rm c} - i_{\rm b}) &= 0 \end{aligned}$$

The dependent source constraint equation is $i_\phi = i_{\rm a}$. We can substitute this simple expression for i_ϕ into the third mesh equation and place the equations in standard form:

$$\begin{aligned} 14i_{\rm a} - 6i_{\rm b} - 8i_{\rm c} &= 25 \\ -6i_{\rm a} + 16i_{\rm b} - 8i_{\rm c} &= 0 \\ -3i_{\rm a} - 8i_{\rm b} + 16i_{\rm c} &= 0 \end{aligned}$$

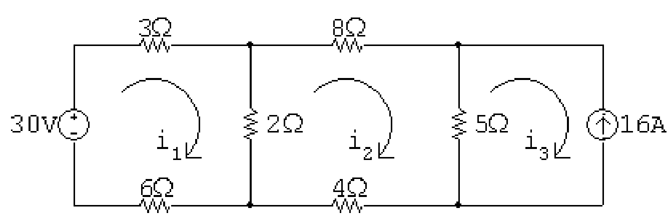
Solving,

$$i_{\rm a} = 4 \text{ A}; \qquad i_{\rm b} = 2.5 \text{ A}; \qquad i_{\rm c} = 2 \text{ A}$$

Thus,

$$v_o = 8(i_{\rm a} - i_{\rm c}) = 8(4-2) = 16 \text{ V}$$

AP 4.10 Redraw the circuit identifying the mesh currents:



Since there is a current source on the perimeter of the i_3 mesh, we know that $i_3 = -16$ A. The remaining two mesh equations are

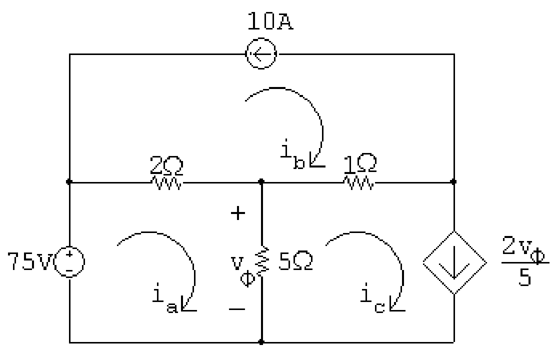
$$\begin{aligned} -30 + 3i_1 + 2(i_1 - i_2) + 6i_1 &= 0 \\ 8i_2 + 5(i_2 + 16) + 4i_2 + 2(i_2 - i_1) &= 0 \end{aligned}$$

Place these equations in standard form:

$$\begin{aligned} 11i_1 - 2i_2 &= 30 \\ -2i_1 + 19i_2 &= -80 \end{aligned}$$

Solving: $i_1 = 2$ A, $i_2 = -4$ A, $i_3 = -16$ A
The current in the $2\,\Omega$ resistor is $i_1 - i_2 = 6$ A $\therefore$ $p_{2\,\Omega} = (6)^2(2) = 72$ W
Thus, the $2\,\Omega$ resistors dissipates 72 W.

AP 4.11 Redraw the circuit and identify the mesh currents:



There are current sources on the perimeters of both the i_b mesh and the i_c mesh, so we know that

$$i_b = -10\text{ A}; \qquad i_c = \frac{2v_\phi}{5}$$

The remaining mesh current equation is

$$-75 + 2(i_a + 10) + 5(i_a - 0.4v_\phi) = 0$$

The dependent source requires the following constraint equation:

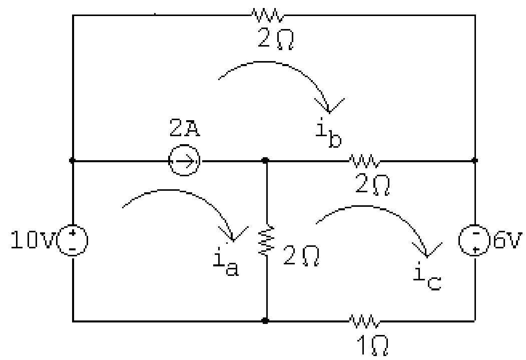
$$v_\phi = 5(i_a - i_c) = 5(i_a - 0.4v_\phi)$$

Place the mesh current equation and the dependent source equation is standard form:

$$\begin{aligned} 7i_a - 2v_\phi &= 55 \\ 5i_a - 3v_\phi &= 0 \end{aligned}$$

Solving: $i_a = 15$ A; $i_b = -10$ A; $i_c = 10$ A; $v_\phi = 25$ V
Thus, $i_a = 15$ A.

AP 4.12 Redraw the circuit and identify the mesh currents:



The 2 A current source is shared by the meshes i_a and i_b. Thus we combine these meshes to form a supermesh and write the following equation:

$$-10 + 2i_b + 2(i_b - i_c) + 2(i_a - i_c) = 0$$

The other mesh current equation is

$$-6 + 1i_c + 2(i_c - i_a) + 2(i_c - i_b) = 0$$

The supermesh constraint equation is

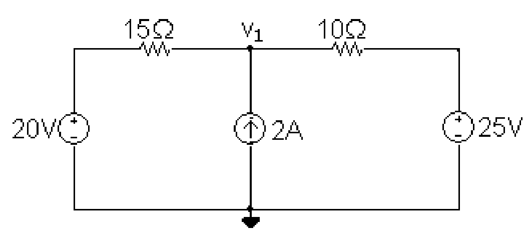
$$i_a - i_b = 2$$

Place these three equations in standard form:

$$\begin{aligned} 2i_a + 4i_b - 4i_c &= 10 \\ -2i_a - 2i_b + 5i_c &= 6 \\ i_a - i_b + 0i_c &= 2 \end{aligned}$$

Solving, $i_a = 7$ A; $i_b = 5$ A; $i_c = 6$ A
Thus, $p_{1\,\Omega} = i_c^2(1) = (6)^2(1) = 36$ W

AP 4.13 Redraw the circuit and identify the reference node and the node voltage v_1:



The node voltage equation is

$$\frac{v_1 - 20}{15} - 2 + \frac{v_1 - 25}{10} = 0$$

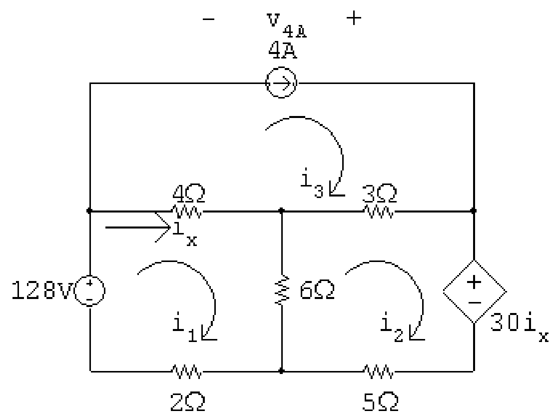
Rearranging and solving,

$$v_1\left(\frac{1}{15}+\frac{1}{10}\right) = 2+\frac{20}{15}+\frac{25}{10} \qquad \therefore \quad v_1 = 35 \text{ V}$$

$$p_{2A} = -35(2) = -70 \text{ W}$$

Thus the 2 A current source delivers 70 W.

AP 4.14 Redraw the circuit and identify the mesh currents:



There is a current source on the perimeter of the i_3 mesh, so $i_3 = 4$ A. The other two mesh current equations are

$$\begin{aligned} -128 + 4(i_1 - 4) + 6(i_1 - i_2) + 2i_1 &= 0 \\ 30i_x + 5i_2 + 6(i_2 - i_1) + 3(i_2 - 4) &= 0 \end{aligned}$$

The constraint equation due to the dependent source is

$$i_x = i_1 - i_3 = i_1 - 4$$

Substitute the constraint equation into the second mesh equation and place the resulting two mesh equations in standard form:

$$\begin{aligned} 12i_1 - 6i_2 &= 144 \\ 24i_1 + 14i_2 &= 132 \end{aligned}$$

Solving,

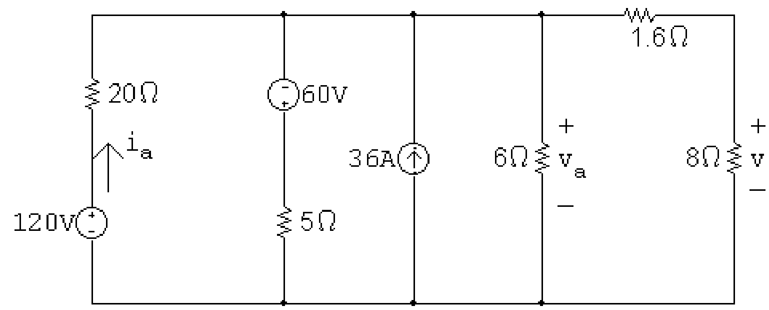
$$i_1 = 9 \text{ A}; \qquad i_2 = -6 \text{ A}; \qquad i_3 = 4 \text{ A}; \qquad i_x = 9 - 4 = 5 \text{ A}$$

$$\therefore \quad v_{4\text{A}} = 3(i_3 - i_2) - 4i_x = 10 \text{ V}$$

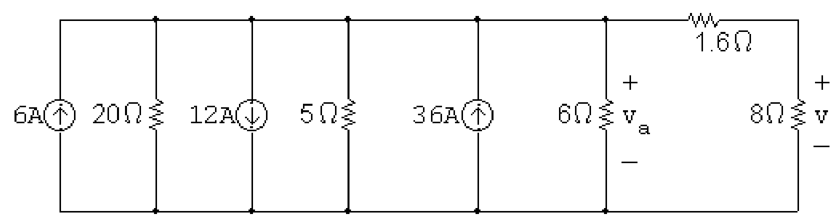
$$p_{4A} = -v_{4\text{A}}(4) = -(10)(4) = -40 \text{ W}$$

Thus, the 2 A current source delivers 40 W.

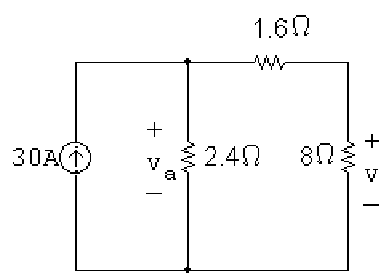
AP 4.15 **[a]** Redraw the circuit with a helpful voltage and current labeled:

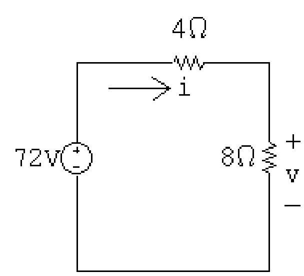


Transform the 120 V source in series with the $20\,\Omega$ resistor into a 6 A source in parallel with the $20\,\Omega$ resistor. Also transform the -60 V source in series with the $5\,\Omega$ resistor into a -12 A source in parallel with the $5\,\Omega$ resistor. The result is the following circuit:



Combine the three current sources into a single current source, using KCL, and combine the $20\,\Omega$, $5\,\Omega$, and $6\,\Omega$ resistors in parallel. The resulting circuit is shown on the left. To simplify the circuit further, transform the resulting 30 A source in parallel with the $2.4\,\Omega$ resistor into a 72 V source in series with the $2.4\,\Omega$ resistor. Combine the $2.4\,\Omega$ resistor in series with the $1.6\,\Omega$ resisor to get a very simple circuit that still maintains the voltage v. The resulting circuit is on the right.





Use voltage division in the circuit on the right to calculate v as follows:

$$v = \frac{8}{12}(72) = 48 \text{ V}$$

[b] Calculate i in the circuit on the right using Ohm's law:

$$i = \frac{v}{8} = \frac{48}{8} = 6 \text{ A}$$

Now use i to calculate v_a in the circuit on the left:

$v_\mathrm{a} = 6(1.6 + 8) = 57.6$ V

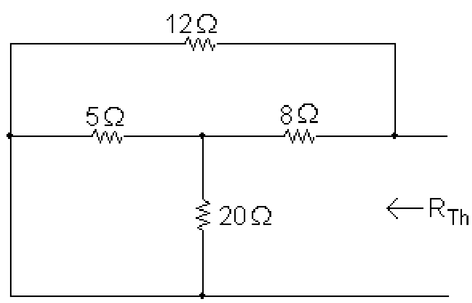
Returning back to the original circuit, note that the voltage v_a is also the voltage drop across the series combination of the 120 V source and $20\,\Omega$ resistor. Use this fact to calculate the current in the 120 V source, i_a:

$$i_\mathrm{a} = \frac{120 - v_\mathrm{a}}{20} = \frac{120 - 57.6}{20} = 3.12 \text{ A}$$

$$p_{120V} = -(120)i_\mathrm{a} = -(120)(3.12) = -374.40 \text{ W}$$

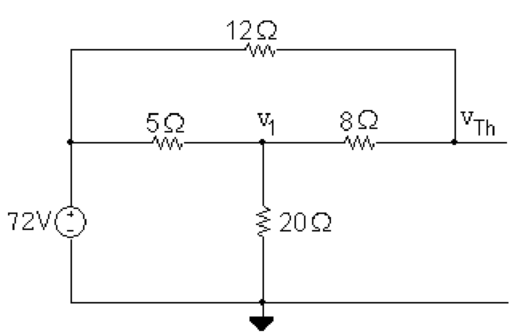
Thus, the 120 V source delivers 374.4 W.

AP 4.16 To find R_Th, replace the 72 V source with a short circuit:



Note that the $5\,\Omega$ and $20\,\Omega$ resistors are in parallel, with an equivalent resistance of $5\|20 = 4\,\Omega$. The equivalent $4\,\Omega$ resistance is in series with the $8\,\Omega$ resistor for an equivalent resistance of $4 + 8 = 12\,\Omega$. Finally, the $12\,\Omega$ equivalent resistance is in parallel with the $12\,\Omega$ resistor, so $R_\mathrm{Th} = 12\|12 = 6\,\Omega$.

Use node voltage analysis to find v_Th. Begin by redrawing the circuit and labeling the node voltages:



The node voltage equations are

$$\frac{v_1 - 72}{5} + \frac{v_1}{20} + \frac{v_1 - v_\mathrm{Th}}{8} = 0$$

$$\frac{v_\mathrm{Th} - v_1}{8} + \frac{v_\mathrm{Th} - 72}{12} = 0$$

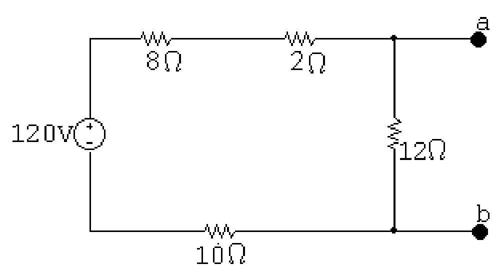
Place these equations in standard form:

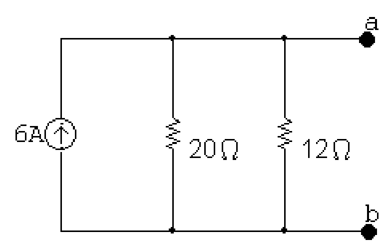
$$v_1\left(\frac{1}{5}+\frac{1}{20}+\frac{1}{8}\right) + v_{\text{Th}}\left(-\frac{1}{8}\right) = \frac{72}{5}$$

$$v_1\left(-\frac{1}{8}\right) + v_{\text{Th}}\left(\frac{1}{8}+\frac{1}{12}\right) = 6$$

Solving, $v_1 = 60$ V and $v_{\text{Th}} = 64.8$ V. Therefore, the Thévenin equivalent circuit is a 64.8 V source in series with a $6\,\Omega$ resistor.

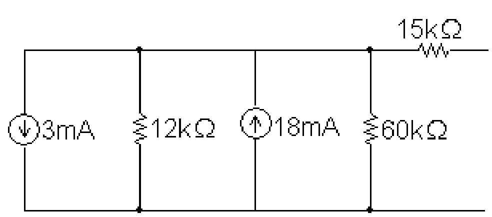
AP 4.17 We begin by performing a source transformation, turning the parallel combination of the 15 A source and $8\,\Omega$ resistor into a series combination of a 120 V source and an $8\,\Omega$ resistor, as shown in the figure on the left. Next, combine the $2\,\Omega$, $8\,\Omega$ and $10\,\Omega$ resistors in series to give an equivalent $20\,\Omega$ resistance. Then transform the series combination of the 120 V source and the $20\,\Omega$ equivalent resistance into a parallel combination of a 6 A source and a $20\,\Omega$ resistor, as shown in the figure on the right.





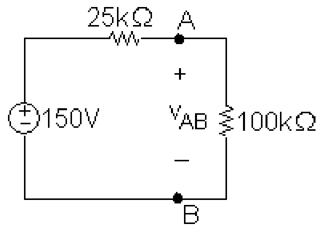
Finally, combine the $20\,\Omega$ and $12\,\Omega$ parallel resistors to give $R_{\text{N}} = 20\|12 = 7.5\,\Omega$. Thus, the Norton equivalent circuit is the parallel combination of a 6 A source and a $7.5\,\Omega$ resistor.

AP 4.18 Find the Thévenin equivalent with respect to A, B using source transformations. To begin, convert the series combination of the -36 V source and 12 kΩ resistor into a parallel combination of a -3 mA source and 12 kΩ resistor. The resulting circuit is shown below:



Now combine the two parallel current sources and the two parallel resistors to give a $-3 + 18 = 15$ mA source in parallel with a 12 k$\|60$ k$=10$ kΩ resistor. Then transform the 15 mA source in parallel with the 10 kΩ resistor into a 150 V source in series with a 10 kΩ resistor, and combine this 10 kΩ resistor in series with the 15 kΩ resistor. The Thévenin equivalent is thus a 150 V source in series with a 25 kΩ

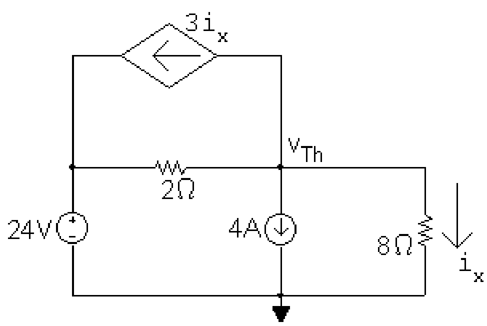
resistor, as seen to the left of the terminals A,B in the circuit below.



Now attach the voltmeter, modeled as a $100\ \text{k}\Omega$ resistor, to the Thévenin equivalent and use voltage division to calculate the meter reading v_{AB}:

$$v_{\text{AB}} = \frac{100{,}000}{125{,}000}(150) = 120\ \text{V}$$

AP 4.19 Begin by calculating the open circuit voltage, which is also v_{Th}, from the circuit below:



Summing the currents away from the node labeled v_{Th} We have

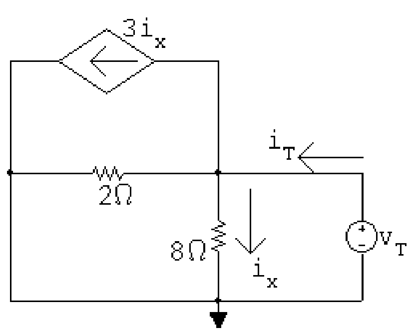
$$\frac{v_{\text{Th}}}{8} + 4 + 3i_x + \frac{v_{\text{Th}} - 24}{2} = 0$$

Also, using Ohm's law for the $8\,\Omega$ resistor,

$$i_x = \frac{v_{\text{Th}}}{8}$$

Substituting the second equation into the first and solving for v_{Th} yields $v_{\text{Th}} = 8$ V.

Now calculate R_{Th}. To do this, we use the test source method. Replace the voltage source with a short circuit, the current source with an open circuit, and apply the test voltage v_{T}, as shown in the circuit below:



Write a KCL equation at the middle node:

$$i_{\mathrm{T}} = i_x + 3i_x + v_{\mathrm{T}}/2 = 4i_x + v_{\mathrm{T}}/2$$

Use Ohm's law to determine i_x as a function of v_{T}:

$$i_x = v_{\mathrm{T}}/8$$

Substitute the second equation into the first equation:

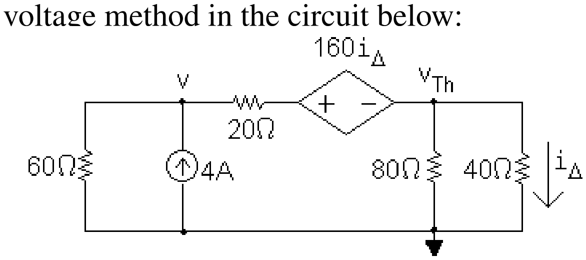
$$i_{\mathrm{T}} = 4(v_{\mathrm{T}}/8) + v_{\mathrm{T}}/2 = v_{\mathrm{T}}$$

Thus,

$$R_{\mathrm{Th}} = v_{\mathrm{T}}/i_{\mathrm{T}} = 1\,\Omega$$

The Thévenin equivalent is an 8 V source in series with a $1\,\Omega$ resistor.

AP 4.20 Begin by calculating the open circuit voltage, which is also v_{Th}, using the node voltage method in the circuit below:



The node voltage equations are

$$\begin{aligned}
\frac{v}{60} + \frac{v - (v_{\mathrm{Th}} + 160i_\Delta)}{20} - 4 &= 0, \\
\frac{v_{\mathrm{Th}}}{40} + \frac{v_{\mathrm{Th}}}{80} + \frac{v_{\mathrm{Th}} + 160i_\Delta - v}{20} &= 0
\end{aligned}$$

The dependent source constraint equation is

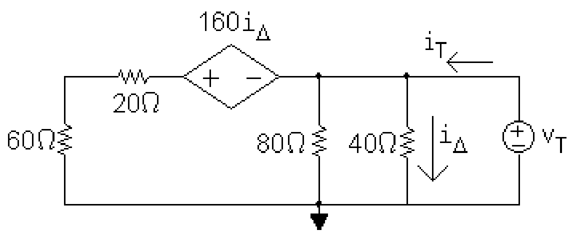
$$i_\Delta = \frac{v_{\mathrm{Th}}}{40}$$

Substitute the constraint equation into the node voltage equations and put the two equations in standard form:

$$\begin{aligned}
v\left(\frac{1}{60} + \frac{1}{20}\right) &+ v_{\mathrm{Th}}\left(-\frac{5}{20}\right) &= 4 \\
v\left(-\frac{1}{20}\right) &+ v_{\mathrm{Th}}\left(\frac{1}{40} + \frac{1}{80} + \frac{5}{20}\right) &= 0
\end{aligned}$$

Solving, $v = 172.5$ V and $v_{\mathrm{Th}} = 30$ V.

Now use the test source method to calculate the test current and thus R_{Th}. Replace the current source with a short circuit and apply the test source to get the following circuit:



Write a KCL equation at the rightmost node:

$$i_{\mathrm{T}} = \frac{v_{\mathrm{T}}}{80} + \frac{v_{\mathrm{T}}}{40} + \frac{v_{\mathrm{T}} + 160 i_{\Delta}}{80}$$

The dependent source constraint equation is

$$i_{\Delta} = \frac{v_{\mathrm{T}}}{40}$$

Substitute the constraint equation into the KCL equation and simplify the right-hand side:

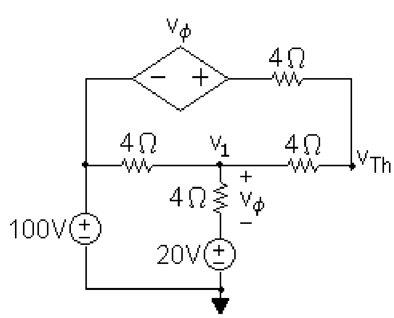
$$i_{\mathrm{T}} = \frac{v_{\mathrm{T}}}{10}$$

Therefore,

$$R_{\mathrm{Th}} = \frac{v_{\mathrm{T}}}{i_{\mathrm{T}}} = 10\,\Omega$$

Thus, the Thévenin equivalent is a 30 V source in series with a $10\,\Omega$ resistor.

AP 4.21 First find the Thévenin equivalent circuit. To find v_{Th}, create an open circuit between nodes a and b and use the node voltage method with the circuit below:



The node voltage equations are:

$$\frac{v_{\mathrm{Th}} - (100 + v_\phi)}{4} + \frac{v_{\mathrm{Th}} - v_1}{4} = 0$$
$$\frac{v_1 - 100}{4} + \frac{v_1 - 20}{4} + \frac{v_1 - v_{\mathrm{Th}}}{4} = 0$$

The dependent source constraint equation is

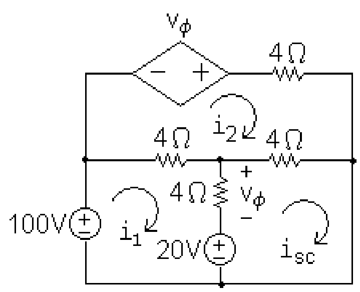
$$v_\phi = v_1 - 20$$

Place these three equations in standard form:

$$\begin{aligned}
v_{\mathrm{Th}}\left(\frac{1}{4}+\frac{1}{4}\right) &+ v_1\left(-\frac{1}{4}\right) &+ v_\phi\left(-\frac{1}{4}\right) &= 25\\
v_{\mathrm{Th}}\left(-\frac{1}{4}\right) &+ v_1\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right) &+ v_\phi\,(0) &= 30\\
v_{\mathrm{Th}}\,(0) &+ v_1\,(1) &+ v_\phi\,(-1) &= 20
\end{aligned}$$

Solving, $v_{\mathrm{Th}} = 120$ V, $v_1 = 80$ V, and $v_\phi = 60$ V.

Now create a short circuit between nodes a and b and use the mesh current method with the circuit below:



The mesh current equations are

$$\begin{aligned}
-100 + 4(i_1 - i_2) + v_\phi + 20 &= 0\\
-v_\phi + 4i_2 + 4(i_2 - i_{\mathrm{sc}}) + 4(i_2 - i_1) &= 0\\
-20 - v_\phi + 4(i_{\mathrm{sc}} - i_2) &= 0
\end{aligned}$$

The dependent source constraint equation is

$$v_\phi = 4(i_1 - i_{\mathrm{sc}})$$

Place these four equations in standard form:

$$\begin{aligned}
4i_1 - 4i_2 + 0i_{\mathrm{sc}} + v_\phi &= 80\\
-4i_1 + 12i_2 - 4i_{\mathrm{sc}} - v_\phi &= 0\\
0i_1 - 4i_2 + 4i_{\mathrm{sc}} - v_\phi &= 20\\
4i_1 + 0i_2 - 4i_{\mathrm{sc}} - v_\phi &= 0
\end{aligned}$$

Solving, $i_1 = 45$ A, $i_2 = 30$ A, $i_{sc} = 40$ A, and $v_\phi = 20$ V. Thus,

$$R_{Th} = \frac{v_{Th}}{i_{sc}} = \frac{120}{40} = 3\,\Omega$$

[a] For maximum power transfer, $R = R_{Th} = 3\,\Omega$

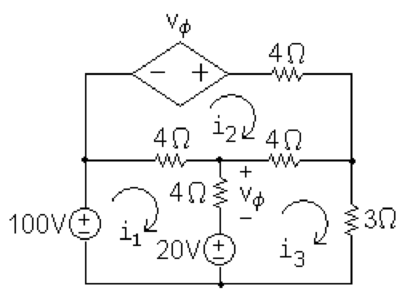
[b] The Thévenin voltage, $v_{Th} = 120$ V, splits equally between the Thévenin resistance and the load resistance, so

$$v_{load} = \frac{120}{2} = 60 \text{ V}$$

Therefore,

$$p_{max} = \frac{v_{load}^2}{R_{load}} = \frac{60^2}{3} = 1200 \text{ W}$$

AP 4.22 Sustituting the value $R = 3\,\Omega$ into the circuit and identifying three mesh currents we have the circuit below:



The mesh current equations are:

$$\begin{aligned} -100 + 4(i_1 - i_2) + v_\phi + 20 &= 0 \\ -v_\phi + 4i_2 + 4(i_2 - i_3) + 4(i_2 - i_1) &= 0 \\ -20 - v_\phi + 4(i_3 - i_2) + 3i_3 &= 0 \end{aligned}$$

The dependent source constraint equation is

$$v_\phi = 4(i_1 - i_3)$$

Place these four equations in standard form:

$$\begin{aligned} 4i_1 - 4i_2 + 0i_3 + v_\phi &= 80 \\ -4i_1 + 12i_2 - 4i_3 - v_\phi &= 0 \\ 0i_1 - 4i_2 + 7i_3 - v_\phi &= 20 \\ 4i_1 + 0i_2 - 4i_3 - v_\phi &= 0 \end{aligned}$$

Solving, $i_1 = 30$ A, $i_2 = 20$ A, $i_3 = 20$ A, and $v_\phi = 40$ V.

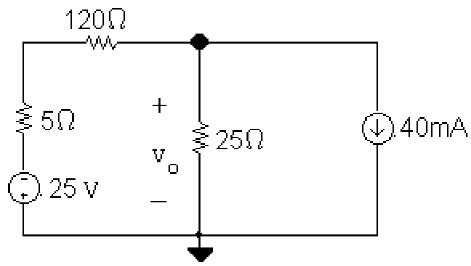
[a] $p_{100\text{V}} = -(100)i_1 = -(100)(30) = -3000$ W. Thus, the 100 V source is delivering 3000 W.

[b] $p_{\text{depsource}} = -v_\phi i_2 = -(40)(20) = -800$ W. Thus, the dependent source is delivering 800 W.

[c] From Assessment Problem 4.21(b), the power delivered to the load resistor is 1200 W, so the load power is $(1200/3800)100 = 31.58\%$ of the combined power generated by the 100 V source and the dependent source.

Problems

P 4.6



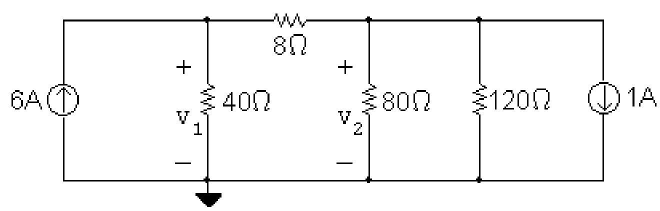
Note that we have chosen the lower node as the reference node, and that the voltage at the upper node with respect to the reference node is v_o. Write a KCL equation (node voltage equation)by summing the currents leaving the upper node:

$$\frac{v_o + 25}{120 + 5} + \frac{v_o}{25} + 0.04 = 0$$

Solve by multiplying both sides of the KCL equation by 125 and collecting the terms involving v_o on one side of the equation and the constants on the other side of the equation:

$$v_o + 25 + 5v_o + 5 = 0 \qquad \therefore \qquad 6v_o = -30 \qquad \text{so} \qquad v_o = -30/6 = -5 \text{ V}$$

P 4.9



The two node voltage equations are:

$$\begin{aligned} -6 + \frac{v_1}{40} + \frac{v_1 - v_2}{8} &= 0 \\ \frac{v_2 - v_1}{8} + \frac{v_2}{80} + \frac{v_2}{120} + 1 &= 0 \end{aligned}$$

Place these equations in standard form:

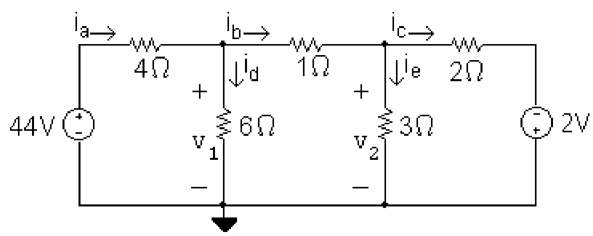
$$\begin{aligned} v_1\left(\frac{1}{40} + \frac{1}{8}\right) &+ v_2\left(-\frac{1}{8}\right) &= 6 \\ v_1\left(-\frac{1}{8}\right) &+ v_2\left(\frac{1}{8} + \frac{1}{80} + \frac{1}{120}\right) &= -1 \end{aligned}$$

Solving, $v_1 = 120$ V and $v_2 = 96$ V.

Check this result by calculating the power associated with each component:

Component	Power Delivered (W)	Power Absorbed (W)
6A	$-(6\ \text{A})(120\ \text{V}) = -720$	
$40\,\Omega$		$\dfrac{120^2}{40} = 360$
$8\,\Omega$		$\dfrac{(120-96)^2}{8} = 72$
$80\,\Omega$		$\dfrac{96^2}{80} = 115.2$
$120\,\Omega$		$\dfrac{96^2}{120} = 76.8$
1 A		$(96\ \text{V})(1\ \text{A}) = 96$
Total	-720	720

P 4.10 **[a]**



The two node voltage equations are:

$$\frac{v_1}{6} + \frac{v_1 - 44}{4} + \frac{v_1 - v_2}{1} = 0$$

$$\frac{v_2}{3} + \frac{v_2 - v_1}{1} + \frac{v_2 + 2}{2} = 0$$

Place these equations in standard form:

$$v_1\left(\frac{1}{6} + \frac{1}{4} + 1\right) + v_2(-1) = \frac{44}{4}$$

$$v_1(-1) + v_2\left(\frac{1}{3} + 1 + \frac{1}{2}\right) = -\frac{2}{2}$$

Solving, $v_1 = 12$ V; $v_2 = 6$ V

Now calculate the branch currents from the node voltage values:

$$i_\mathrm{a} = \frac{44-12}{4} = 8\text{ A}$$

$$i_\mathrm{b} = \frac{12}{6} = 2\text{ A}$$

$$i_\mathrm{c} = \frac{12-6}{1} = 6\text{ A}$$

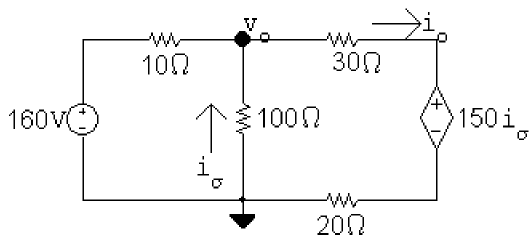
$$i_\mathrm{d} = \frac{6}{3} = 2\text{ A}$$

$$i_\mathrm{e} = \frac{6+2}{2} = 4\text{ A}$$

[b] $p_{\text{sources}} = p_{44\text{V}} + p_{2\text{V}} = -(44)i_\mathrm{a} - (2)i_\mathrm{e} = -(44)(8) - (2)(4) = -352 - 8 = -360$ W

Thus, the power developed in the circuit is 360 W. Note that the resistors cannot develop power!

P 4.19



The node voltage equation is

$$\frac{v_o - 160}{10} + \frac{v_o}{100} + \frac{v_o - 150i_\sigma}{30+20} = 0$$

The dependent source constraint equation is:

$$i_\sigma = -\frac{v_o}{100}$$

Place these equations in standard form:

$$v_o\left(\frac{1}{10} + \frac{1}{100} + \frac{1}{50}\right) + i_\sigma\left(-\frac{150}{50}\right) = \frac{160}{10}$$

$$v_o\left(\frac{1}{100}\right) + i_\sigma(1) = 0$$

Solving, $v_o = 100$ V; $i_\sigma = -1$ A

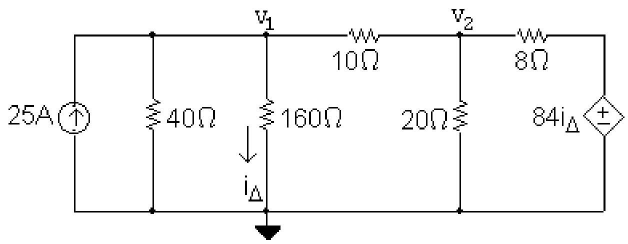
Now find the power:

$$i_o = \frac{160-100}{10} - 1 = 5\text{ A}$$

$p_{\text{ds}} = [150(-1)](5) = -750$ W.

Thus, the dependent source delivers 750 W

P 4.20 **[a]**



The node voltage equations are:

$$-25 + \frac{v_1}{40} + \frac{v_1}{160} + \frac{v_1 - v_2}{10} = 0$$

$$\frac{v_2 - v_1}{10} + \frac{v_2}{20} + \frac{v_2 - 84i_\Delta}{8} = 0$$

The dependent source constraint equation is:

$i_\Delta = v_1/160$

Place these three equations in standard form:

$$\begin{aligned}
v_1\left(\frac{1}{40} + \frac{1}{160} + \frac{1}{10}\right) &+ v_2\left(-\frac{1}{10}\right) &+ i_\Delta(0) &= 25 \\
v_1\left(-\frac{1}{10}\right) &+ v_2\left(\frac{1}{10} + \frac{1}{20} + \frac{1}{8}\right) &+ i_\Delta\left(-\frac{84}{8}\right) &= 0 \\
v_1\left(-\frac{1}{160}\right) &+ v_2(0) &+ i_\Delta(1) &= 0
\end{aligned}$$

Solving, $v_1 = 352$ V; $v_2 = 212$ V; $i_\Delta = 2.2$ A

Now calculate the power. Only the two sources can develop power, so focus on the sources:

$$p_{25\text{A}} = -(352)(25) = -8800 \text{ W}$$

$$i_{\text{dep source}} = (v_2 - 84i_\Delta)/8 = (212 - 84 \cdot 2.2)/8 = 3.4 \text{ A}$$

$$p_{\text{dep source}} = (84 \cdot 2.2)(3.4) = 628.32 \text{ W}$$

Thus, only the current source develops power, so the total power developed in the circuit is 8800 W

[b] The dependent source and all of the resistors dissipate the power developed by the current source. Check that the power developed equals the power dissipated:

$$p_{40\Omega} = (352)^2/40 = 3097.6 \text{ W}$$

$$p_{160\Omega} = (352)^2/160 = 774.4 \text{ W}$$

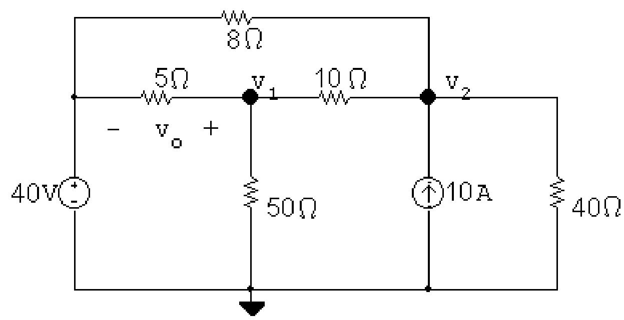
$$p_{10\Omega} = (352 - 212)^2/10 = 1960 \text{ W}$$

$$p_{20\Omega} = (212)^2/20 = 2247.2 \text{ W}$$

$$p_{8\Omega} = (212 - 84 \cdot 2.2)^2/8 = 92.48 \text{ W}$$

$\sum p_{\text{diss}} = 628.32 + 3097.6 + 774.4 + 1960 + 2247.2 + 92.48 = 8800$ W so the power balances.

P 4.21



The two node voltage equations are:

$$\frac{v_1-40}{5}+\frac{v_1}{50}+\frac{v_1-v_2}{10} = 0$$

$$\frac{v_2-v_1}{10}-10+\frac{v_2}{40}+\frac{v_2-40}{8} = 0$$

Place these equations in standard form:

$$v_1\left(\frac{1}{5}+\frac{1}{50}+\frac{1}{10}\right) + v_2\left(-\frac{1}{10}\right) = \frac{40}{5}$$

$$v_1\left(-\frac{1}{10}\right) + v_2\left(\frac{1}{10}+\frac{1}{40}+\frac{1}{8}\right) = 10+\frac{40}{8}$$

Solving, $v_1 = 50$ V; $v_2 = 80$ V.

Thus, $v_o = v_1 - 40 = 50 - 40 = 10$ V.

POWER CHECK:

$$i_g = (50-40)/5 + (80-40)/8 = 7 \text{ A}$$

$$p_{40\text{V}} = (40)(7) = 280 \text{ W} \quad \text{(abs)}$$

$$p_{5\Omega} = (50-40)^2/5 = 20 \text{ W} \quad \text{(abs)}$$

$$p_{8\Omega} = (80-40)^2/8 = 200 \text{ W} \quad \text{(abs)}$$

$$p_{10\Omega} = (80-50)^2/10 = 90 \text{ W} \quad \text{(abs)}$$

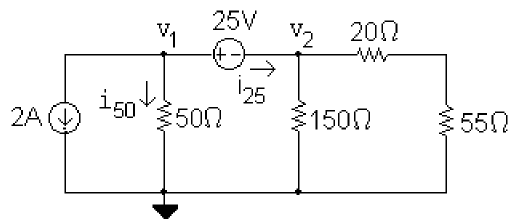
$$p_{50\Omega} = 50^2/50 = 50 \text{ W} \quad \text{(abs)}$$

$$p_{40\Omega} = 80^2/40 = 160 \text{ W} \quad \text{(abs)}$$

$$p_{10\text{A}} = -(80)(10) = -800 \text{ W} \quad \text{(del)}$$

$$\sum p_{\text{abs}} = 280+20+200+90+50+160 = 800 \text{ W} = \sum p_{\text{del}}$$

P 4.26



This circuit has a supernode includes the nodes v_1, v_2 and the 25 V source. The supernode equation is

$$2 + \frac{v_1}{50} + \frac{v_2}{150} + \frac{v_2}{20 + 55} = 0$$

The supernode constraint equation is

$$v_2 + 25 = v_1$$

Place these two equations in standard form:

$$\begin{aligned} v_1\left(\frac{1}{50}\right) &+ v_2\left(\frac{1}{150} + \frac{1}{75}\right) &= -2 \\ v_1(1) &+ v_2(-1) &= 25 \end{aligned}$$

Solving, $v_1 = -37.5$ V and $v_2 = -62.5$ V.

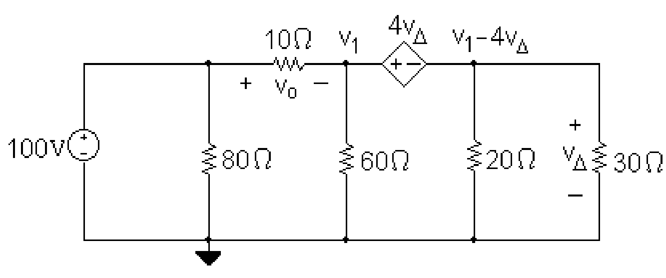
$$p_{25\text{V}} = (25)i_{25}$$

$$i_{25} = -2\text{ A} - i_{50} = -2\text{ A} - \frac{v_1}{50} = 2\text{ A} - \frac{-37.5}{50} = -2\text{ A} + 0.75\text{ A} = -1.25\text{ A}$$

Thus, $\quad p_{25\text{V}} = (25)(-1.25) = -31.25$ W

The 25 V source delivers 31.25 W.

P 4.27



The supernode equation is:

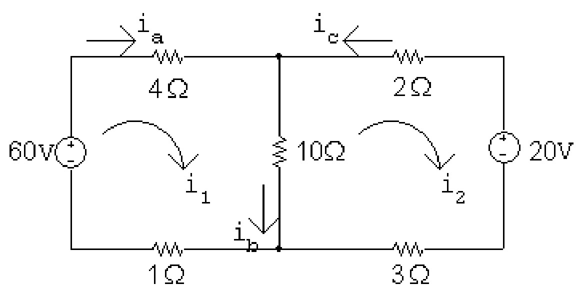
$$\frac{v_1 - 100}{10} + \frac{v_1}{60} + \frac{v_1 - 4v_\Delta}{20} + \frac{v_1 - 4v_\Delta}{30} = 0$$

The constraint equation for the dependent source is:
$4v_\Delta = v_1 - v_\Delta$
Place these equations in standard form:

$$\begin{aligned} v_1\left(\frac{1}{10}+\frac{1}{60}+\frac{1}{20}+\frac{1}{30}\right) &+ v_\Delta\left(-\frac{4}{20}-\frac{4}{30}\right) &= \frac{100}{10} \\ v_1(1) &+ v_\Delta(-5) &= 0 \end{aligned}$$

Solving, $\quad v_1 = 75$ V; $\quad v_\Delta = 15$ V
Thus, $v_o = 100 - v_1 = 25$ V

P 4.31 **[a]**



The mesh current equations are:

$$-60 + 4i_1 + 10(i_1 - i_2) + 1i_1 = 0$$
$$20 + 3i_2 + 10(i_2 - i_1) + 2i_2 = 0$$

Place the equations in standard form:

$$i_1(4+10+1) + i_2(-10) = 60$$
$$i_1(-10) + i_2(3+10+2) = -20$$

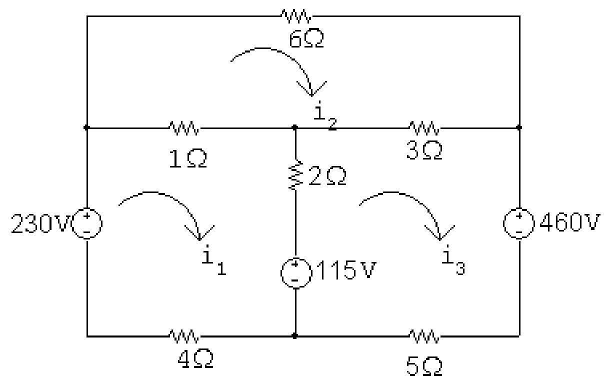
Solving, $\quad i_1 = 5.6$ A; $\quad i_2 = 2.4$ A
Now solve for the requested currents:
$i_\mathrm{a} = i_1 = 5.6$ A; $\quad i_\mathrm{b} = i_1 - i_2 = 3.2$ A; $\quad i_\mathrm{c} = -i_2 = -2.4$ A

[b] If the polarity of the 60 V source is reversed, we have the following mesh current equations in standard form:

$$i_1(4+10+1) + i_2(-10) = -60$$
$$i_1(-10) + i_2(3+10+2) = -20$$

Solving, $\quad i_1 = -8.8$ A; $\quad i_2 = -7.2$ A
Now solve for the requested currents:
$i_\mathrm{a} = i_1 = -8.8$ A; $\quad i_\mathrm{b} = i_1 - i_2 = -1.6$ A; $\quad i_\mathrm{c} = -i_2 = 7.2$ A

P 4.32 **[a]**



The mesh current equations are:

$$\begin{aligned}
-230 + 1(i_1 - i_2) + 2(i_1 - i_3) + 115 + 4i_1 &= 0\\
6i_2 + 3(i_2 - i_3) + 1(i_2 - i_1) &= 0\\
460 + 5i_3 - 115 + 2(i_3 - i_1) + 3(i_3 - i_2) &= 0
\end{aligned}$$

Place these equations in standard form:

$$\begin{aligned}
i_1(1+2+4) + i_2(-1) + i_3(-2) &= 115\\
i_1(-1) + i_2(6+3+1) + i_3(-3) &= 0\\
i_1(-2) + i_2(-3) + i_3(5+2+3) &= -345
\end{aligned}$$

Solving, $\quad i_1 = 4.4$ A; $\quad i_2 = -10.6$ A; $\quad i_3 = -36.8$ A

The only components that can develop power in the circuit are the sources:

$$\begin{aligned}
p_{230\text{V}} &= -(230)(4.4) = -1012 \text{ W}\\
p_{115\text{V}} &= -(115)(-36.8 - 4.4) = 4738 \text{ W}\\
p_{460\text{V}} &= (460)(-36.8) = -16{,}928 \text{ W}
\end{aligned}$$

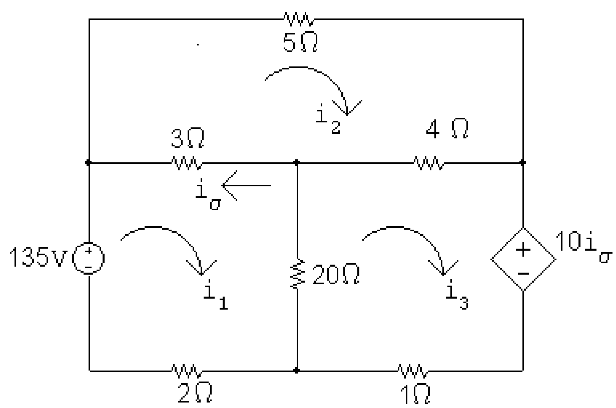
$\therefore \quad \sum p_{\text{dev}} = 1012 + 16{,}928 = 17940$ W

[b] From part (a) we know that the 115 V source is dissipating power; compute the power dissipated by the resistors:

$$\begin{aligned}
p_{1\Omega} &= (1)(4.4 + 10.6)^2 = 225 \text{ W}\\
p_{4\Omega} &= (4)(4.4)^2 = 77.44 \text{ W}\\
p_{6\Omega} &= (6)(-10.6)^2 = 674.16 \text{ W}\\
p_{2\Omega} &= (2)(4.4 + 36.8)^2 = 3394.88 \text{ W}\\
p_{3\Omega} &= (3)(-10.6 + 36.8)^2 = 2059.32 \text{ W}\\
p_{5\Omega} &= (5)(-36.8)^2 = 6771.2 \text{ W}
\end{aligned}$$

$\therefore \quad \sum p_{\text{dis}} = 4738 + 225 + 77.44 + 674.16 + 3394.88 + 2059.32 + 6771.2$
$= 17940$ W (checks!)

P 4.33



The mesh current equations are:

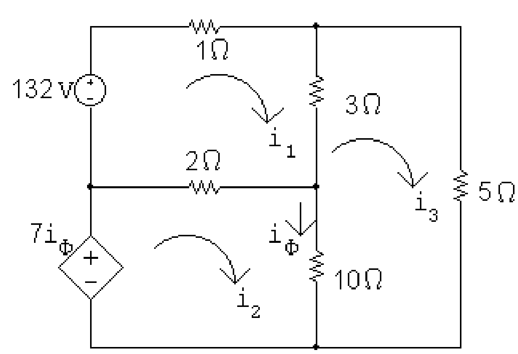
$$-135 + 3(i_1 - i_2) + 20(i_1 - i_3) + 2i_1 = 0$$
$$5i_2 + 4(i_2 - i_3) + 3(i_2 - i_1) = 0$$
$$10i_\sigma + 1i_3 + 20(i_3 - i_1) + 4(i_3 - i_2) = 0$$

The dependent source constraint equation is:
$i_\sigma = i_2 - i_1$
Place these equations in standard form:

$$i_1(3 + 20 + 2) + i_2(-3) + i_3(-20) + i_\sigma(0) = 135$$
$$i_1(-3) + i_2(5 + 4 + 3) + i_3(-4) + i_\sigma(0) = 0$$
$$i_1(-20) + i_2(-4) + i_3(1 + 20 + 4) + i_\sigma(10) = 0$$
$$i_1(1) + i_2(-1) + i_3(0) + i_\sigma(1) = 0$$

Solving, $i_1 = 64.8$ A, $i_2 = 39$ A; $i_3 = 68.4$ A; $i_\sigma = -25.8$ A
Calculate the power:
$p_{20\Omega} = 20(68.4 - 64.8)^2 = 259.2$ W
Thus the $20\,\Omega$ resistor dissipates 259.2 W.

P 4.34



The mesh current equations:

$$\begin{aligned} -132 + 1i_1 + 3(i_1 - i_3) + 2(i_1 - i_2) &= 0 \\ -7i_\phi + 2(i_2 - i_1) + 10(i_2 - i_3) &= 0 \\ 5i_3 + 10(i_3 - i_2) + 3(i_3 - i_1) &= 0 \end{aligned}$$

The dependent source constraint equation:
$i_\phi = i_2 - i_3$

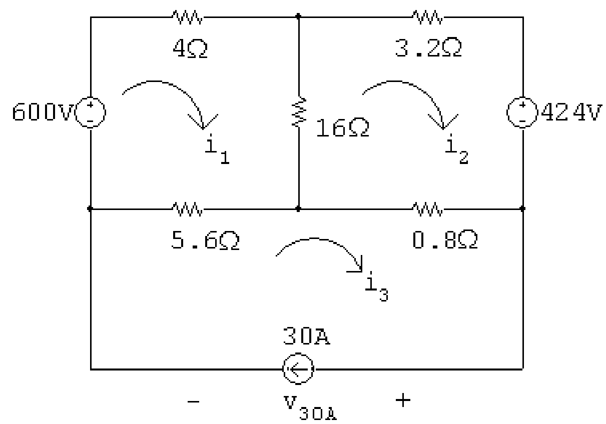
Place these equations in standard form:

$$\begin{aligned} i_1(1+3+2) + i_2(-2) + i_3(-3) + i_\phi(0) &= 132 \\ i_1(-2) + i_2(10+2) + i_3(-10) + i_\phi(-7) &= 0 \\ i_1(-3) + i_2(-10) + i_3(5+10+3) + i_\phi(0) &= 0 \\ i_1(0) + i_2(-1) + i_3(1) + i_\phi(1) &= 0 \end{aligned}$$

Solving, $i_1 = 48$ A; $i_2 = 36$ A; $i_3 = 28$ A; $i_\phi = 8$ A

Solve for the power:
$p_{\text{dep source}} = -7(i_\phi)i_2 = -7(8)(36) = -2016$ W
Thus, the dependent source is developing 2016 W.

P 4.37



$600 = 25.6i_1 - 16i_2 - 5.6i_3$

$-424 = -16i_1 + 20i_2 - 0.8i_3$

$30 = i_3$

Solving, $i_1 = 35$ A; $i_2 = 8$ A; $i_3 = 30$ A

[a] $v_{30\text{A}} = 0.8(i_2 - i_3) + 5.6(i_1 - i_3)$

$= 0.8(8 - 30) + 5.6(35 - 30) = 10.4$ V

$p_{30\text{A}} = 30v_{30\text{A}} = 30(10.4) = 312$ W (abs)

Therefore, the 30 A source delivers -312 W.

[b] $p_{600\text{V}} = -600(35) = -21{,}000$ W(del)

$p_{424\text{V}} = 424(8) = 3392$ W(abs)

Therefore, the total power delivered is $21{,}000$ W

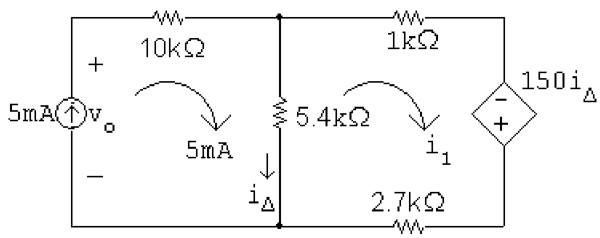
[c]

$$p_{4\Omega} = (35)^2(4) = 4900 \text{ W}$$
$$p_{3.2\Omega} = (8)^2(3.2) = 204.8 \text{ W}$$
$$p_{16\Omega} = (35-8)^2(16) = 11{,}664 \text{ W}$$
$$p_{5.6\Omega} = (35-30)^2(5.6) = 140 \text{ W}$$
$$p_{0.8\Omega} = (-30+8)^2(0.8) = 387.2 \text{ W}$$

$\sum p_{\text{resistors}} = 17{,}296$ W

$\sum p_{\text{abs}} = 17{,}296 + 312 + 3392 = 21{,}000$ W (CHECKS)

P 4.38 **[a]**

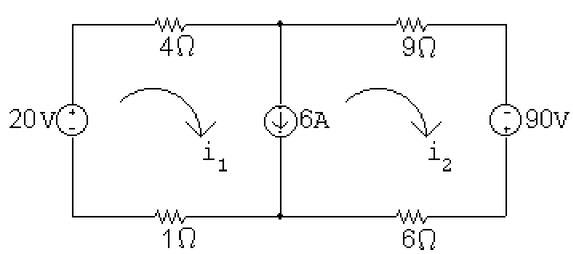


The mesh current equation for the right mesh is:
$5400(i_1 - 0.005) + 3700i_1 - 150(0.005 - i_1) = 0$
Solving, $\quad 9250i_1 = 27.75 \quad \therefore \; i_1 = 3$ mA
Then, $\quad i_\Delta = 0.005 - i_1 = 0.005 - 0.003 = 0.002 = 2$ mA

[b] $v_o = (0.005)(10{,}000) + (0.002)(5400) = 60.8$ V
$p_{5\text{mA}} = -(60.8)(0.005) = -304$ mW
Thus, the 5 mA source delivers 304 mW

[c] $150i_\Delta = 150(0.002) = 0.3$ V
$p_{\text{dep source}} = 150i_\Delta i_1 = -(0.3)(0.003) = -0.9$ mW
The dependent source delivers 0.9 mW.

P 4.43



The supermesh equation is:
$-20 + 4i_1 + 9i_2 - 90 + 6i_2 + 1i_1 = 0$
The supermesh constraint equation is :
$i_1 - i_2 = 6$
Place these equations in standard form:

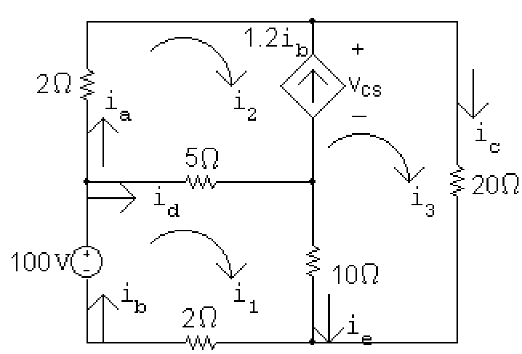
$$\begin{aligned} i_1(4+1) + i_2(9+6) &= 20 + 90 \\ i_1(1) + i_2(-1) &= 6 \end{aligned}$$

Solving, $\quad i_1 = 10$ A; $\quad i_2 = 4$ A
Now find the power:

$$\begin{aligned} p_{4\Omega} &= 10^2(4) = 400 \text{ W} \\ p_{1\Omega} &= 10^2(1) = 100 \text{ W} \\ p_{9\Omega} &= 4^2(9) = 144 \text{ W} \\ p_{6\Omega} &= 4^2(6) = 96 \text{ W} \\ p_{20\text{V}} &= -(20)(10) = -200 \text{ W} \\ v_{6\text{A}} &= 9i_2 - 90 + 6i_2 = (9)(4) - 90 + (6)(4) = -30 \text{ V} \\ p_{6\text{A}} &= (-30)(6) = -180 \text{ W} \\ p_{90\text{V}} &= -(90)(4) = -360 \text{ W} \end{aligned}$$

In summary:
$\sum p_{\text{dev}} = 200 + 180 + 360 = 740$ W
$\sum p_{\text{diss}} = 400 + 100 + 144 + 96 = 740$ W
Thus the power dissipated in the circuit is 740 W

P 4.46 **[a]**



The i_1 mesh current equation:
$-100 + 5(i_1 - i_2) + 10(i_1 - i_3) + 2i_1 = 0$
The i_2 — i_3 supermesh equationa:
$2i_2 + 20i_3 + 10(i_3 - i_1) + 5(i_2 - i_1) = 0$
The supermesh constraint:
$i_3 - i_2 = 1.2i_\mathrm{b} = 1.2i_1$

Place these equations in standard form:

$$\begin{aligned} i_1(5 + 10 + 2) + i_2(-5) + i_3(-10) &= 100 \\ i_1(-10 - 5) + i_2(2 + 5) + i_3(20 + 10) &= 0 \\ i_1(1.2) + i_2(1) + i_3(-1) &= 0 \end{aligned}$$

Solving, $\quad i_1 = 7.4$ A; $\quad i_2 = -4.2$ A; $\quad i_3 = 4.68$ A

Solve for the requested currents:

$$\begin{aligned} i_a &= i_2 = -4.2 \text{ A} \\ i_b &= i_1 = 7.4 \text{ A} \\ i_c &= i_3 = 4.68 \text{ A} \\ i_d &= i_1 - i_2 = 11.6 \text{ A} \\ i_e &= i_1 - i_3 = 2.72 \text{ A} \end{aligned}$$

[b] Find v_cs:
$2i_2 + v_\mathrm{cs} + 5(i_2 - i_1) = 0 \quad \therefore \quad v_\mathrm{cs} = -2(-4.2) - 5(-4.2 - 7.4) = 66.4$ V

Calculate the power:

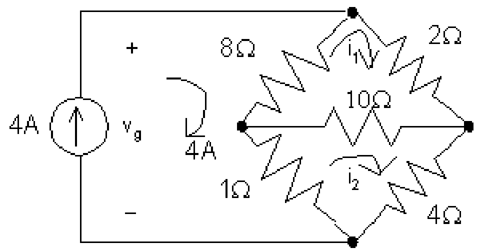
$$\begin{aligned} p_{100\mathrm{V}} &= -(100)(7.4) = -740 \text{ W} \\ p_{\text{dep source}} &= -(66.4)[1.2(7.4)] = -589.632 \text{ W} \\ p_{2\Omega} &= 2(-4.2)^2 = 35.28 \text{ W} \\ p_{5\Omega} &= 5(7.4 + 4.2)^2 = 672.8 \text{ W} \\ p_{2\Omega} &= 2(7.4)^2 = 109.52 \text{ W} \\ p_{10\Omega} &= 10(7.4 - 4.68)^2 = 73.984 \text{ W} \\ p_{20\Omega} &= 20(4.68)^2 = 438.048 \text{ W} \end{aligned}$$

$$\sum p_\mathrm{dev} = 740 + 589.632 = 1329.632 \text{ W}$$

$$\sum p_\mathrm{dis} = 35.28 + 672.8 + 109.52 + 73.984 + 438.048 = 1329.632 \text{ W}$$

P 4.52 **[a]** There are three unknown node voltages and only two unknown mesh currents. Use the mesh current method to minimize the number of simultaneous equations.

[b]



The mesh current equations:

$$2i_1 + 10(i_1 - i_2) + 8(i_1 - 4) = 0$$

$$4i_2 + 1(i_2 - 4) + 10(i_2 - i_1) = 0$$

Place the equations in standard form:

$$i_1(2 + 10 + 8) + i_2(-10) = 32$$

$$i_1(-10) + i_2(4 + 1 + 10) = 4$$

Solving, $i_1 = 2.6$ A; $\quad i_2 = 2$ A

Find the power in the $10\,\Omega$ resistor:

$i_{10\Omega} = i_1 - i_2 = 0.6$ A

$p_{10\Omega} = (0.6)^2(10) = 3.6$ W

[c] No, the voltage across the 4 A current source is readily available from the mesh currents, and solving two simultaneous mesh-current equations is less work than solving three node voltage equations.

[d] $v_g = 2i_1 + 4i_2 = 2(2.6) + 4(2) = 13.2$ V

$p_{4\text{A}} = -(13.2)(4) = -52.8$ W

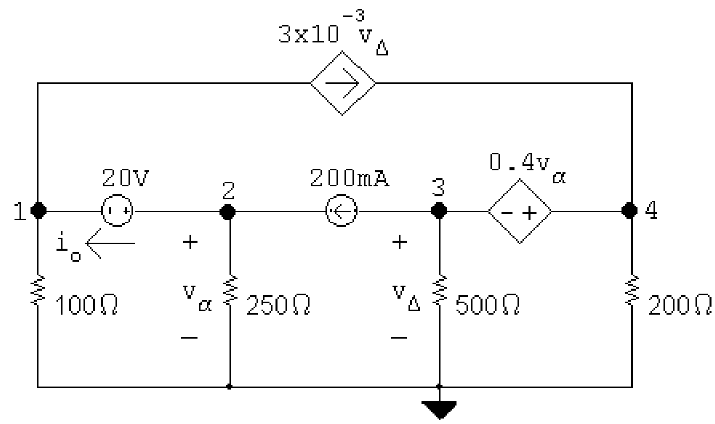
Thus the 4 A source develops 52.8 W.

P 4.54 **[a]** The node voltage method requires summing the currents at two supernodes in terms of four node voltages and using two constraint equations to reduce the system of equations to two unknowns. If the connection at the bottom of the circuit is used as the reference node, then the voltages controlling the dependent sources are node voltages. This makes it easy to formulate the constraint equations. The current in the 20 V source is obtained by summing the currents at either terminal of the source.

The mesh current method requires summing the voltages around the two meshes not containing current sources in terms of four mesh currents. In addition the voltages controlling the dependent sources must be expressed in terms of the mesh currents. Thus the constraint equations are more complicated, and the reduction to two equations and two unknowns involves more algebraic manipulation. The current in the 20 V source is found by subtracting two mesh currents.

Because the constraint equations are easier to formulate in the node voltage method, it is the preferred approach.

[b]



Node voltage equations:

$$\frac{v_1}{100} + \frac{v_2}{250} - 0.2 + 3 \times 10^{-3} v_3 = 0$$

$$\frac{v_3}{500} + \frac{v_4}{200} - 3 \times 10^{-3} v_3 + 0.2 = 0$$

Constraints:

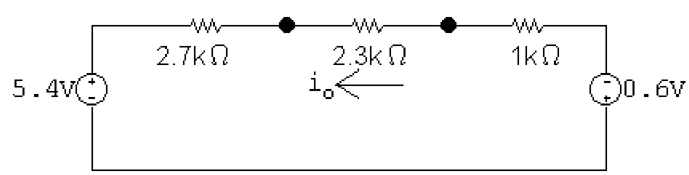
$v_2 - v_1 = 20; \qquad v_4 - v_3 = 0.4v_\alpha; v_\alpha = v_2$

Solving, $v_2 = 44$ V

$i_o = 0.2 - 44/250 = 24$ mA

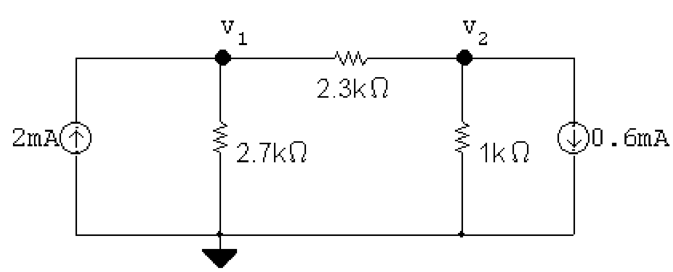
$p_{20\text{V}} = 20i_o = 480$ mW (abs)

P 4.55 **[a]** Apply source transformations to both current sources to get



$$i_o = \frac{-(5.4 + 0.6)}{2700 + 2300 + 1000} = -1 \text{ mA}$$

[b]



The node voltage equations:

$$-2\times 10^{-3}+\frac{v_1}{2700}+\frac{v_1-v_2}{2300} = 0$$

$$\frac{v_2}{1000}+\frac{v_2-v_1}{2300}+0.6\times 10^{-3} = 0$$

Place these equations in standard form:

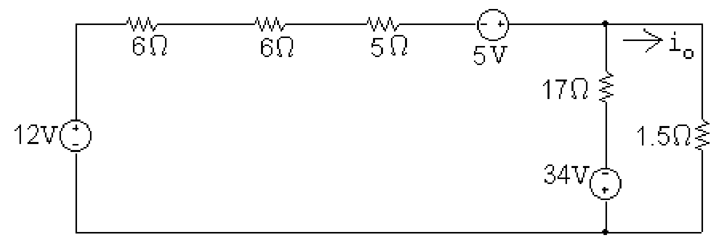
$$v_1\left(\frac{1}{2700}+\frac{1}{2300}\right)+v_2\left(-\frac{1}{2300}\right) = 2\times 10^{-3}$$

$$v_1\left(-\frac{1}{2300}\right)+v_2\left(\frac{1}{1000}+\frac{1}{2300}\right) = -0.6\times 10^{-3}$$

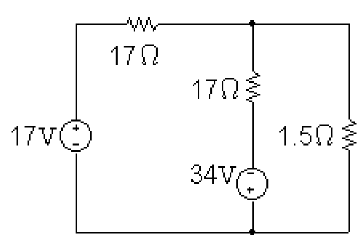
Solving, $v_1 = 2.7$ V; $\qquad v_2 = 0.4$ V

$$\therefore\quad i_o = \frac{v_2-v_1}{2300} = -1 \text{ mA}$$

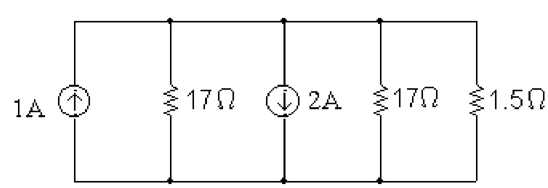
P 4.58 **[a]** Applying a source transformation to each current source yields



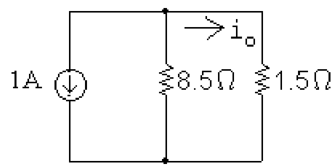
Now combine the 12 V and 5 V sources into a single voltage source and the $6\,\Omega$, $6\,\Omega$ and $5\,\Omega$ resistors into a single resistor to get



Now use a source transformation on each voltage source, thus

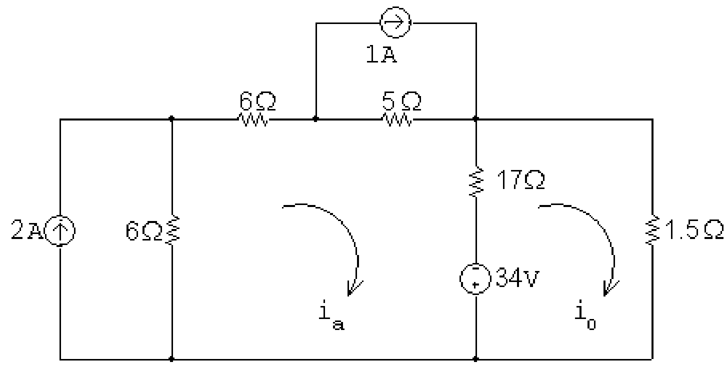


which can be reduced to



$$\therefore \quad i_o = \frac{8.5}{10}(-1) = -0.85 \text{ A}$$

[b]



The mesh current equations are:

$$6(i_\mathrm{a} - 2) + 6i_\mathrm{a} + 5(i_\mathrm{a} - 1) + 17(i_\mathrm{a} - i_o) - 34 = 0$$

$$1.5i_o + 34 + 17(i_o - i_\mathrm{a}) = 0$$

Put these equations in standard form:

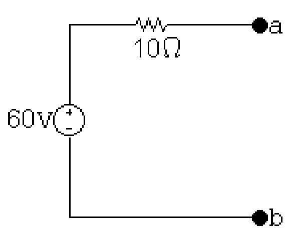
$$i_\mathrm{a}(6 + 6 + 5 + 17) + i_o(-17) = 12 + 5 + 34$$

$$i_\mathrm{a}(-17) + i_o(1.5 + 17) = -34$$

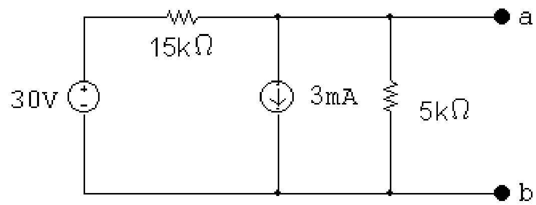
Solving, $\quad i_\mathrm{a} = 1.075$ A; $\quad i_o = -0.85$ A

P 4.59 $V_{\mathrm{Th}} = \dfrac{30}{30 + 10}(80) = 60$ V

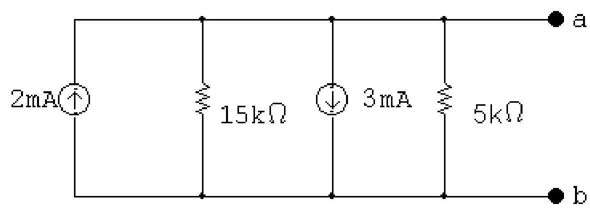
$R_{\mathrm{Th}} = 10\|30 + 2.5 = 10\,\Omega$



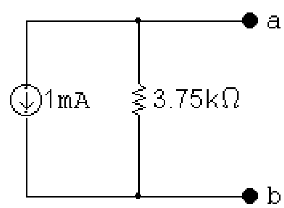
P 4.62 First we make the observation that the 10 mA current source and the 10 kΩ resistor will have no influence on the behavior of the circuit with respect to the terminals a,b. This follows because they are in parallel with an ideal voltage source. Hence our circuit can be simplified to



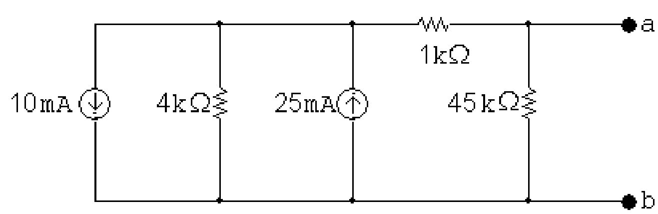
or

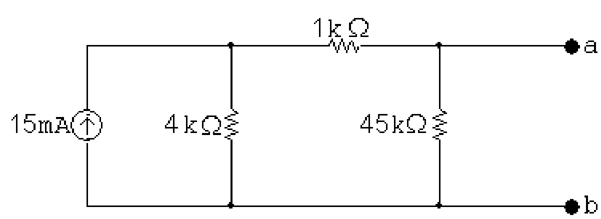


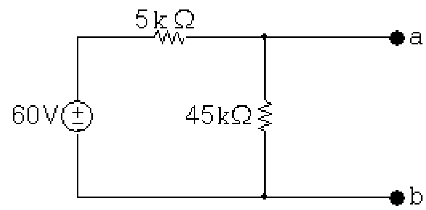
Therefore the Norton equivalent is determined by adding the current sources and combining the resistors in parallel:

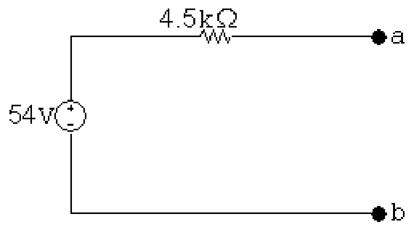


P 4.63 **[a]** First, find the Thévenin equivalent with respect to a,b using a succession of source transformations.

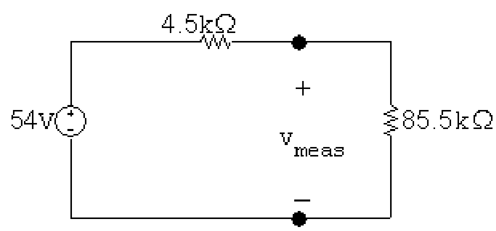








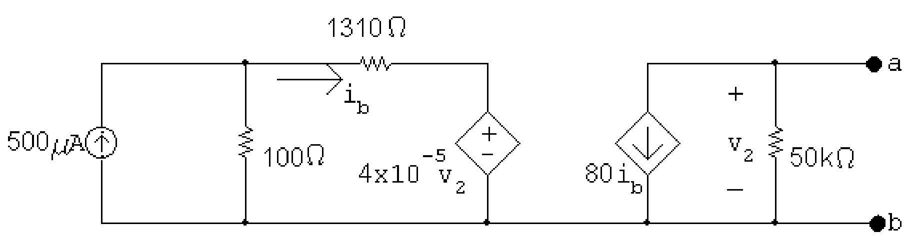
$\therefore \quad V_{\text{Th}} = 54 \text{ V} \qquad R_{\text{Th}} = 4.5 \text{ k}\Omega$



$$v_{\text{meas}} = \frac{85.5}{90}(54) = 51.3 \text{ V}$$

[b] $\%\text{error} = \left(\frac{51.3 - 54}{54}\right) \times 100 = -5\%$

P 4.65



OPEN CIRCUIT

Use Ohm's law to solve for v_2 on the right hand side of the circuit:

$v_2 = -80i_{\text{b}}(50{,}000) = -40 \times 10^5 i_{\text{b}}$

Use this value of v_2 to express the value of the dependent voltage source in terms of i_{b}:

$4 \times 10^{-5} v_2 = 4 \times 10^{-5}(-40 \times 10^5 i_{\text{b}}) = -160 i_{\text{b}}$

Write the mesh current equation for the i_{b} mesh:

$1310 i_{\text{b}} - 160 i_{\text{b}} + 100(i_{\text{b}} - 500 \times 10^{-6}) = 0$

Solving,
$1250i_{\rm b} = 0.05 \quad \therefore \quad i_{\rm b} = 0.05/1250 = 40\,\mu$ A
Thus,
$V_{\rm Th} = v_2 = -40 \times 10^5 i_{\rm b} = -40 \times 10^5(40 \times 10^{-6}) = -160$ V
SHORT CIRCUIT

$v_2 = 0; \qquad i_{\rm sc} = -80i_b$

Calculate $i_{\rm b}$ using current division on the left hand side of the circuit:

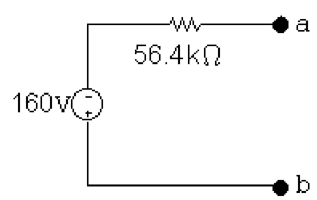
$$i_b = \frac{100}{100 + 1310} 500 \times 10^{-6} = 35.461\,\mu \text{ A}$$

Calculate the short circuit current from the right hand side of the circuit:

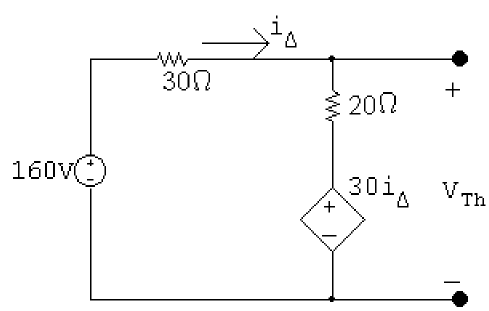
$$i_{\rm sc} = -80(35.461 \times 10^{-6}) = -2.8369 \times 10^{-3}\,\text{mA}$$

Calculate $R_{\rm Th}$ from the short circuit current and open circuit voltage:

$$R_{\rm Th} = \frac{-160}{-2.8369 \times 10^{-3}} = 56.4\ \text{k}\Omega$$



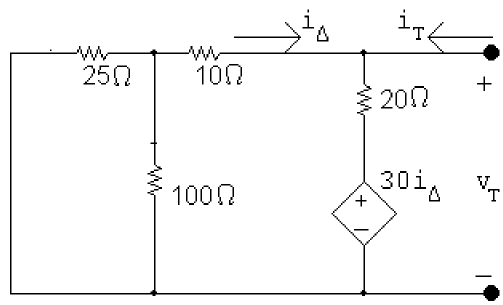
P 4.75 We begin by finding the Thévenin equivalent with respect to R_o. After making a couple of source transformations the circuit simplifies to



$$i_\Delta = \frac{160 - 30i_\Delta}{50}; \qquad i_\Delta = 2 \text{ A}$$

$$V_{\rm Th} = 20i_\Delta + 30i_\Delta = 50i_\Delta = 100 \text{ V}$$

Using the test-source method to find the Thévenin resistance gives

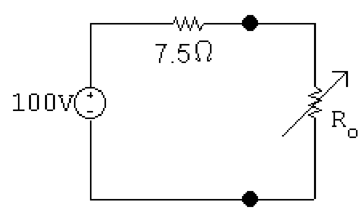


$$i_{\mathrm{T}} = \frac{v_{\mathrm{T}}}{30} + \frac{v_{\mathrm{T}} - 30(-v_{\mathrm{T}}/30)}{20}$$

$$\frac{i_{\mathrm{T}}}{v_{\mathrm{T}}} = \frac{1}{30} + \frac{1}{10} = \frac{4}{30} = \frac{2}{15}$$

$$R_{\mathrm{Th}} = \frac{v_{\mathrm{T}}}{i_{\mathrm{T}}} = \frac{15}{2} = 7.5\,\Omega$$

Thus our problem is reduced to analyzing the circuit shown below.



$$p = \left(\frac{100}{7.5 + R_o}\right)^2 R_o = 250$$

$$\frac{10^4}{R_o^2 + 15R_o + 56.25} R_o = 250$$

$$\frac{10^4 R_o}{250} = R_o^2 + 15R_o + 56.25$$

$$40R_o = R_o^2 + 15R_o + 56.25$$

$$R_o^2 - 25R_o + 56.25 = 0$$

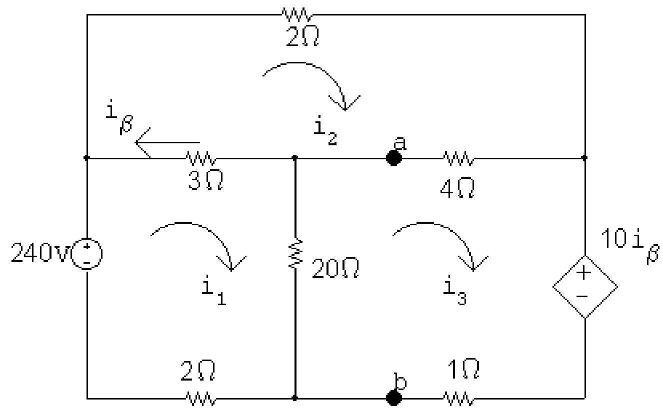
$$R_o = 12.5 \pm \sqrt{156.25 - 56.25} = 12.5 \pm 10$$

$$R_o = 22.5\,\Omega$$

$$R_o = 2.5\,\Omega$$

P 4.76 **[a]** Find the Thévenin equivalent with respect to the terminals of R_L.

Open circuit voltage:



The mesh current equations are:

$$
\begin{aligned}
-240 + 3(i_1 - i_2) + 20(i_1 - i_3) + 2i_1 &= 0 \\
2i_2 + 4(i_2 - i_3) + 3(i_2 - i_1) &= 0 \\
10i_\beta + 1i_3 + 20(i_3 - i_1) + 4(i_3 - i_2) &= 0
\end{aligned}
$$

The dependent source constraint equation is:

$i_\beta = i_2 - i_1$

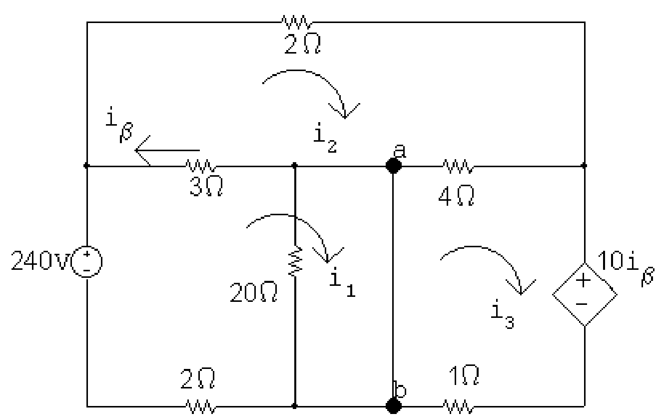
Place these equations in standard form:

$$
\begin{aligned}
i_1(3 + 20 + 2) + i_2(-3) + i_3(-20) + i_\beta(0) &= 240 \\
i_1(-3) + i_2(2 + 4 + 3) + i_3(-4) + i_\beta(0) &= 0 \\
i_1(-20) + i_2(-4) + i_3(4 + 1 + 20) + i_\beta(10) &= 0 \\
i_1(1) + i_2(-1) + i_3(0) + i_\beta(1) &= 0
\end{aligned}
$$

Solving, $i_1 = 99.6$ A; $i_2 = 78$ A; $i_3 = 100.8$ A; $i_\beta = -21.6$ A

$V_{\rm Th} = 20(i_1 - i_3) = -24$ V

Short-circuit current:



The mesh current equations are:

$$-240 + 3(i_1 - i_2) + 2i_1 = 0$$
$$2i_2 + 4(i_2 - i_3) + 3(i_2 - i_1) = 0$$
$$10i_\beta + 1i_3 + 4(i_3 - i_2) = 0$$

The dependent source constraint equation is:

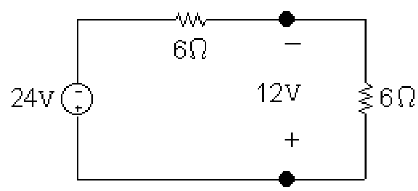
$i_\beta = i_2 - i_1$

Place these equations in standard form:

$$i_1(3+2) + i_2(-3) + i_3(0) + i_\beta(0) = 240$$
$$i_1(-3) + i_2(2+4+3) + i_3(-4) + i_\beta(0) = 0$$
$$i_1(0) + i_2(-4) + i_3(4+1) + i_\beta(10) = 0$$
$$i_1(1) + i_2(-1) + i_3(0) + i_\beta(1) = 0$$

Solving, $i_1 = 92$ A; $i_2 = 73.33$ A; $i_3 = 96$ A; $i_\beta = -18.67$ A

$i_{sc} = i_1 - i_3 = -4$ A; $\quad R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{-24}{-4} = 6\,\Omega$



$R_L = R_{Th} = 6\,\Omega$

[b] $p_{max} = \frac{12^2}{6} = 24$ W