5

The Operational Amplifier

Assessment Problems

AP 5.1 **[a]** This is an inverting amplifier, so

$$v_o = (-R_f/R_i)v_s = (-80/16)v_s, \qquad \text{so} \qquad v_o = -5v_s$$

v_s(V)	0.4	2.0	3.5	−0.6	−1.6	−2.4
v_o(V)	−2.0	−10.0	−15.0	3.0	8.0	10.0

Two of the v_s values, 3.5 V and −2.4 V, cause the op amp to saturate.

[b] Use the negative power supply value to determine the largest input voltage:

$$-15 = -5v_s, \quad v_s = 3 \text{ V}$$

Use the positive power supply value to determine the smallest input voltage:

$$10 = -5v_s, \qquad v_s = -2 \text{ V}$$

Therefore $\quad -2 \leq v_s \leq 3 \text{ V}$

AP 5.2 From Assessment Problem 5.1

$$\begin{aligned} v_o &= (-R_f/R_i)v_s = (-R_x/16{,}000)v_s \\ &= (-R_x/16{,}000)(-0.640) = 0.64R_x/16{,}000 = 4\times10^{-5}R_x \end{aligned}$$

Use the negative power supply value to determine one limit on the value of R_x:

$$4\times10^{-5}R_x = -15 \qquad \text{so} \qquad R_x = -15/4\times10^{-5} = -375\,\text{k}\Omega$$

Since we cannot have negative resistor values, the lower limit for R_x is 0. Now use the positive power supply value to determine the upper limit on the value of R_x:

$$4\times10^{-5}R_x = 10 \qquad \text{so} \qquad R_x = 10/4\times10^{-5} = 250\,\text{k}\Omega$$

Therefore,

$$0 \leq R_x \leq 250\,\text{k}\Omega$$

AP 5.3 **[a]** This is an inverting summing amplifier so

$$v_o = (-R_f/R_\mathrm{a})v_\mathrm{a} + (-R_f/R_\mathrm{b})v_\mathrm{b} = -(250/5)v_\mathrm{a} - (250/25)v_\mathrm{b} = -50v_\mathrm{a} - 10v_\mathrm{b}$$

Substituting the values for v_a and v_b:

$$v_o = -50(0.1) - 10(0.25) = -5 - 2.5 = -7.5\ \mathrm{V}$$

[b] Substitute the value for v_b into the equation for v_o from part (a) and use the negative power supply value:

$$v_o = -50v_\mathrm{a} - 10(0.25) = -50v_\mathrm{a} - 2.5 = -10\ \mathrm{V}$$

$$\text{Therefore}\quad 50v_\mathrm{a} = 7.5, \quad \text{so} \quad v_\mathrm{a} = 0.15\ \mathrm{V}$$

[c] Substitute the value for v_a into the equation for v_o from part (a) and use the negative power supply value:

$$v_o = -50(0.10) - 10v_\mathrm{b} = -5 - 10v_\mathrm{b} = -10\ \mathrm{V};$$

$$\text{Therefore}\quad 10v_\mathrm{b} = 5, \quad \text{so} \quad v_\mathrm{b} = 0.5\ \mathrm{V}$$

[d] The effect of reversing polarity is to change the sign on the v_b term in each equation from negative to positive.
Repeat part (a):

$$v_o = -50v_\mathrm{a} + 10v_\mathrm{b} = -5 + 2.5 = -2.5\ \mathrm{V}$$

Repeat part (b):

$$v_o = -50v_\mathrm{a} + 2.5 = -10\ \mathrm{V}; \qquad 50v_\mathrm{a} = 12.5, \quad v_\mathrm{a} = 0.25\ \mathrm{V}$$

Repeat part (c):

$$v_o = -5 + 10v_\mathrm{b} = 15\ \mathrm{V}; \qquad 10v_\mathrm{b} = 20; \qquad v_\mathrm{b} = 2.0\ \mathrm{V}$$

AP 5.4 **[a]** Write a node voltage equation at v_n; remember that for an ideal op amp, the current into the op amp at the inputs is zero:

$$\frac{v_n}{4500} + \frac{v_n - v_o}{63{,}000} = 0$$

Solve for v_o in terms of v_n by multiplying both sides by $63{,}000$ and collecting terms:

$$14v_n + v_n - v_o = 0 \qquad \text{so} \qquad v_o = 15v_n$$

Now use voltage division to calculate v_p. We can use voltage division because the op amp is ideal, so no current flows into the non-inverting input terminal and the 400 mV divides between the 15 kΩ resistor and the R_x resistor:

$$v_p = \frac{R_x}{15{,}000 + R_x}(0.400)$$

Now substitute the value $R_x = 60\ \text{k}\Omega$:

$$v_p = \frac{60{,}000}{15{,}000 + 60{,}000}(0.400) = 0.32\ \text{V}$$

Finally, remember that for an ideal op amp, $v_n = v_p$, so substitute the value of v_p into the equation for v_0

$$v_o = 15v_n = 15v_p = 15(0.32) = 4.8\ \text{V}$$

[b] Substitute the expression for v_p into the equation for v_o and set the resulting equation equal to the positive power supply value:

$$v_o = 15\left(\frac{0.4R_x}{15{,}000 + R_x}\right) = 5$$

$$15(0.4R_x) = 5(15{,}000 + R_x) \quad \text{so} \quad R_x = 75\,\text{k}\Omega$$

AP 5.5 **[a]** Since this is a difference amplifier, we can use the expression for the output voltage in terms of the input voltages and the resistor values given in Eq. 5.22:

$$v_o = \frac{20(60)}{10(24)}v_\text{b} - \frac{50}{10}v_\text{a}$$

Simplify this expression and substitute in the value for v_b:

$$v_o = 5(v_\text{b} - v_\text{a}) = 20 - 5v_\text{a}$$

Set this expression for v_o to the positive power supply value:

$$20 - 5v_\text{a} = 10\ \text{V} \quad \text{so} \quad v_\text{a} = 2\ \text{V}$$

Now set the expression for v_o to the negative power supply value:

$$20 - 5v_\text{a} = -10\ \text{V} \quad \text{so} \quad v_\text{a} = 6\ \text{V}$$

Therefore $\quad 2 \le v_\text{a} \le 6\ \text{V}$

[b] Begin as before by substituting the appropriate values into Eq. 5.22:

$$v_o = \frac{8(60)}{10(12)}v_\text{b} - 5v_\text{a} = 4v_\text{b} - 5v_\text{a}$$

Now substitute the value for v_b:

$$v_o = 4(4) - 5v_\text{a} = 16 - 5v_\text{a}$$

Set this expression for v_o to the positive power supply value:

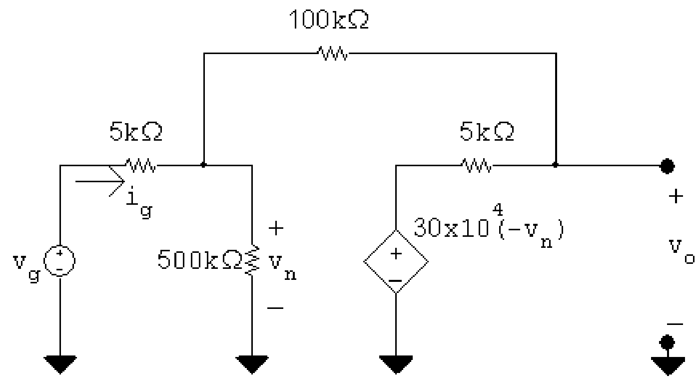
$$16 - 5v_\text{a} = 10\ \text{V} \quad \text{so} \quad v_\text{a} = 1.2\ \text{V}$$

Now set the expression for v_o to the negative power supply value:

$$16 - 5v_\text{a} = -10\ \text{V} \quad \text{so} \quad v_\text{a} = 5.2\ \text{V}$$

Therefore $\quad 1.2 \le v_\text{a} \le 5.2\ \text{V}$

AP 5.6 **[a]** Replace the op amp with the more realistic model of the op amp from Fig. 5.15:



Write the node voltage equation at the left hand node:

$$\frac{v_n}{500{,}000} + \frac{v_n - v_g}{5000} + \frac{v_n - v_o}{100{,}000} = 0$$

Multiply both sides by 500,000 and simplify:

$$v_n + 100v_n - 100v_g + 5v_n - 5v_0 = 0 \quad \text{so} \quad 21.2v_n - v_o = 20v_g$$

Write the node voltage equation at the right hand node:

$$\frac{v_o - 300{,}000(-v_n)}{5000} + \frac{v_o - v_n}{100{,}000} = 0$$

Multiply through by 100,000 and simplify:

$$20v_o + 6\times10^6 v_n + v_o - v_n = 0 \quad \text{so} \quad 6\times10^6 v_n + 21v_o = 0$$

Use Cramer's method to solve for v_o:

$$\Delta = \begin{vmatrix} 21.2 & -1 \\ 6\times10^6 & 21 \end{vmatrix} = 6{,}000{,}445.2$$

$$N_o = \begin{vmatrix} 21.2 & 20v_g \\ 6\times10^6 & 0 \end{vmatrix} = -120\times10^6 v_g$$

$$v_o = \frac{N_o}{\Delta} = -19.9985v_g; \qquad \text{so } \frac{v_o}{v_g} = -19.9985$$

[b] Use Cramer's method again to solve for v_n:

$$N_1 = \begin{vmatrix} 20v_g & -1 \\ 0 & 21 \end{vmatrix} = 420v_g$$

$$v_n = \frac{N_1}{\Delta} = 6.9995\times10^{-5} v_g$$

$$v_g = 1\text{ V}, \qquad v_n = 69.995\,\mu\text{ V}$$

[c] The resistance seen at the input to the op amp is the ratio of the input voltage to the input current, so calculate the input current as a function of the input voltage:

$$i_g = \frac{v_g - v_n}{5000} = \frac{v_g - 6.9995 \times 10^{-5} v_g}{5000}$$

Solve for the ratio of v_g to i_g to get the input resistance:

$$R_g = \frac{v_g}{i_g} = \frac{5000}{1 - 6.9995 \times 10^{-5}} = 5000.35\,\Omega$$

[d] This is a simple inverting amplifier configuration, so the voltage gain is the ratio of the feedback resistance to the input resistance:

$$\frac{v_o}{v_g} = -\frac{100{,}000}{5000} = -20$$

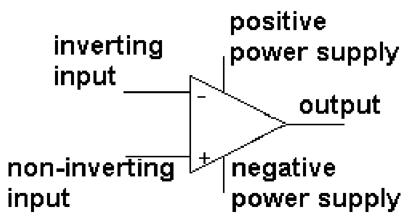
Since this is now an ideal op amp, the voltage difference between the two input terminals is zero; since $v_p = 0$, $v_n = 0$

Since there is no current into the inputs of an ideal op amp, the resistance seen by the input voltage source is the input resistance:

$$R_g = 5000\,\Omega$$

Problems

P 5.1 **[a]** The five terminals of the op amp are identified as follows:



[b] The input resistance of an ideal op amp is infinite, which constrains the value of the input currents to 0. Thus, $i_n = 0$ A.

[c] The open-loop voltage gain of an ideal op amp is infinite, which constrains the difference between the voltage at the two input terminals to 0. Thus, $(v_p - v_n) = 0$.

[d] Write a node voltage equation at v_n:

$$\frac{v_n - 2.5}{10{,}000} + \frac{v_n - v_o}{40{,}000} = 0$$

But $v_p = 0$ and $v_n = v_p = 0$. Thus,

$$\frac{-2.5}{10{,}000} - \frac{v_o}{40{,}000} = 0 \quad \text{so} \quad v_o = -10 \text{ V}$$

P 5.2 $$\frac{v_\text{b} - v_\text{a}}{20} + \frac{v_\text{b} - v_o}{100} = 0, \qquad \text{therefore} \quad v_o = 6v_\text{b} - 5v_\text{a}$$

[a] $v_\text{a} = 4$ V, $v_\text{b} = 0$ V, $v_o = -15$ V (sat)

[b] $v_\text{a} = 2$ V, $v_\text{b} = 0$ V, $v_o = -10$ V

[c] $v_\text{a} = 2$ V, $v_\text{b} = 1$ V, $v_o = -4$ V

[d] $v_\text{a} = 1$ V, $v_\text{b} = 2$ V, $v_o = 7$ V

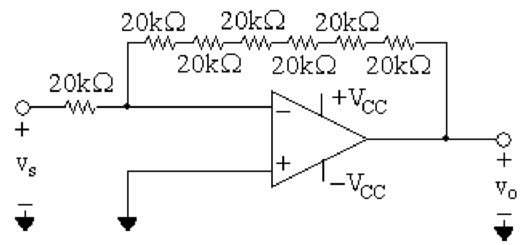
[e] If $v_\text{b} = 1.6$ V, $v_o = 9.6 - 5v_\text{a} = \pm 15$

$$\therefore \quad -1.08 \le v_\text{a} \le 4.92 \text{ V}$$

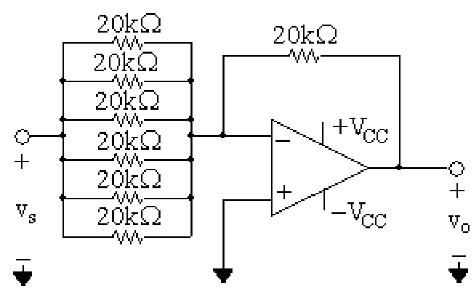
P 5.3 $$v_o = -(0.5 \times 10^{-3})(10 \times 10^3) = -5 \text{ V}$$

$$\therefore \quad i_o = \frac{-5}{5000} = -1 \,\text{mA}$$

P 5.6 **[a]** The gain of an inverting amplifier is the ratio of the feedback resistor to the input resistor. If the gain of the inverting amplifier is to be 6, the feedback resistor must be 6 times as large as the input resistor. There are many possible designs that use only 20 kΩ resistors. We present two here. Use a single 20 kΩ resistor as the input resistor, and use six 20 kΩ resistors in series as the feedback resistor to give a total of 120 kΩ.



Alternately, Use a single 20 kΩ resistor as the feedback resistor and use six 20 kΩ resistors in parallel as the input resistor to give a total of 3.33 kΩ.



[b] To amplify a 3 V signal without saturating the op amp, the power supply voltages must be greater than or equal to the product of the input voltage and the amplifier gain. Thus, the power supplies should have a magnitude of $(3)(6) = 18$ V.

P 5.7 **[a]** The circuit shown is a non-inverting amplifier.

[b] We assume the op amp to be ideal, so $v_n = v_p = 3$V. Write a KCL equation at v_n:

$$\frac{3}{40{,}000} + \frac{3 - v_o}{80{,}000} = 0$$

Solving,

$v_o = 9$ V.

P 5.16 **[a]** This circuit is an example of an inverting summing amplifier.

[b] $v_o = -\frac{220}{33}v_\text{a} - \frac{220}{22}v_\text{b} - \frac{220}{80}v_\text{c} = -8 + 15 - 11 = -4\ \text{V}$

[c] $v_o = -19 - 10v_\text{b} = \pm 6$

$$\therefore\quad v_\text{b} = -1.3\ \text{V} \quad \text{when} \quad v_o = -6\ \text{V};$$

$$v_\text{b} = -2.5\ \text{V} \quad \text{when} \quad v_o = 6\ \text{V}$$

$$\therefore\quad -2.5\ \text{V} \le v_\text{b} \le -1.3\ \text{V}$$

P 5.17 **[a]** Write a KCL equation at the inverting input to the op amp:

$$\frac{v_\text{d} - v_\text{a}}{40{,}000} + \frac{v_\text{d} - v_\text{b}}{22{,}000} + \frac{v_\text{d} - v_\text{c}}{100{,}000} + \frac{v_\text{d}}{352{,}000} + \frac{v_\text{d} - v_o}{220{,}000} = 0$$

Multiply through by $220{,}000$, plug in the values of input voltages, and rearrange to solve for v_o:

$$v_o = 220{,}000\left(\frac{4}{40{,}000} + \frac{-1}{22{,}000} + \frac{-5}{100{,}000} + \frac{8}{352{,}000} + \frac{8}{220{,}000}\right) = 14\ \text{V}$$

[b] Write a KCL equation at the inverting input to the op amp. Use the given values of input voltages in the equation:

$$\frac{8 - v_\text{a}}{40{,}000} + \frac{8 - 9}{22{,}000} + \frac{8 - 13}{100{,}000} + \frac{8}{352{,}000} + \frac{8 - v_o}{220{,}000} = 0$$

Simplify and solve for v_o:

$$44 - 5.5v_\text{a} - 10 - 11 + 5 + 8 - v_o = 0 \quad \text{so} \quad v_o = 36 - 5.5v_\text{a}$$

Set v_o to the positive power supply voltage and solve for v_a:

$$36 - 5.5v_\text{a} = 15 \qquad \therefore \qquad v_\text{a} = 3.818\ \text{V}$$

Set v_o to the negative power supply voltage and solve for v_a:

$$36 - 5.5v_\text{a} = -15 \qquad \therefore \qquad v_\text{a} = 9.273\ \text{V}$$

Therefore,

$$3.818\ \text{V} \le v_\text{a} \le 9.273\ \text{V}$$

P 5.19 We want the following expression for the output voltage:

$$v_o = -(2v_a + 4v_b + 6v_c + 8v_d)$$

This is an inverting summing amplifier, so each input voltage is amplified by a gain that is the ratio of the feedback resistance to the resistance in the forward path for the input voltage:

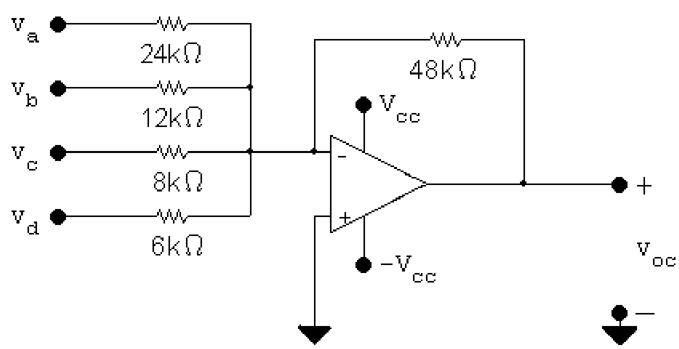
$$v_o = -\left[\frac{48}{R_\text{a}}v_\text{a} + \frac{48}{R_\text{b}}v_\text{b} + \frac{48}{R_\text{c}}v_\text{c} + \frac{48}{R_\text{d}}v_\text{d}\right]$$

Solve for each input resistance value to yield the desired gain:

$$\therefore \quad R_\text{a} = 48{,}000/2 = 24\,\text{k}\Omega \qquad R_\text{c} = 48{,}000/6 = 8\,\text{k}\Omega$$

$$R_\text{b} = 48{,}000/4 = 12\,\text{k}\Omega \qquad R_\text{d} = 48{,}000/8 = 6\,\text{k}\Omega$$

The final circuit is shown here:



P 5.22 **[a]** This circuit is an example of the non-inverting amplifier.

[b] Use voltage division to calculate v_p:

$$v_p = \frac{10{,}000}{10{,}000 + 30{,}000}v_s = \frac{v_s}{4}$$

Write a KCL equation at $v_n = v_p = v_s/4$:

$$\frac{v_s/4}{4000} + \frac{v_s/4 - v_o}{28{,}000} = 0$$

Solving,

$$v_o = 7v_s/4 + v_s/4 = 2v_s$$

[c] $2v_s = 8 \qquad \text{so} \qquad v_s = 4\ \text{V}$

$2v_s = -12 \qquad \text{so} \qquad v_s = -6\ \text{V}$

Thus, $-6\ \text{V} \leq v_s \leq 4\ \text{V}$.

P 5.23 **[a]** $v_p = v_n = \dfrac{68{,}000}{80{,}000} v_g = 0.85 v_g$

$$\therefore \quad \frac{0.85v_g}{30{,}000} + \frac{0.85v_g - v_o}{63{,}000} = 0;$$

$$\therefore \quad v_o = 2.635 v_g = 2.635(4), \qquad v_o = 10.54 \text{ V}$$

[b] $v_o = 2.635 v_g = \pm 12$

$$v_g = \pm 4.554 \text{ V}, \qquad -4.554 \leq v_g \leq 4.554 \text{ V}$$

[c] $\dfrac{0.85v_g}{30{,}000} + \dfrac{0.85v_g - v_o}{R_\mathrm{f}} = 0$

$$\left(\frac{0.85R_\mathrm{f}}{30{,}000} + 0.85\right) v_g = v_o = \pm 12$$

$$\therefore \quad 1.7\times10^{-3} R_\mathrm{f} + 51 = \pm 360; \quad 1.7\times10^{-3} R_\mathrm{f} = 360 - 51; \quad R_\mathrm{f} = 181.76\,\mathrm{k}\Omega$$

P 5.29 Use voltage division to find v_p:

$$v_p = \frac{2000}{2000 + 8000}(5) = 1 \text{ V}$$

Write a KCL equation at v_n and solve it for v_o:

$$\frac{v_n - v_a}{5000} + \frac{v_n - v_o}{R_f} = 0 \qquad \text{so} \qquad \left(\frac{R_f}{5000} + 1\right) v_n - \frac{R_f}{5000} v_a = v_o$$

Since the op amp is ideal, $v_n = v_p = 1$V, so

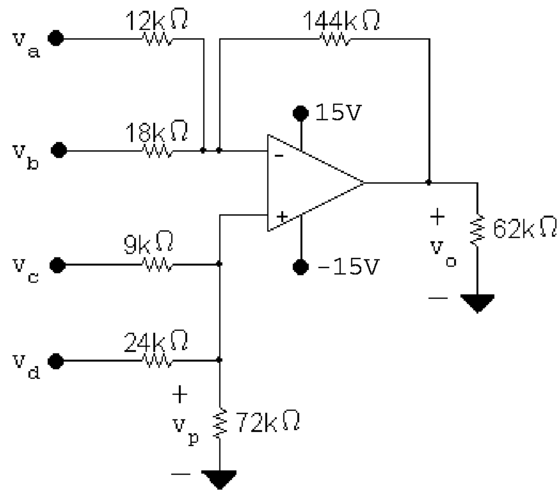
$$v_o = \left(\frac{R_f}{5000} + 1\right) - \frac{R_f}{5000} v_a$$

To satisfy the equation,

$$\left(\frac{R_f}{5000} + 1\right) = 5 \qquad \text{and} \qquad \frac{R_f}{5000} = 4$$

Thus, $R_f = 20$ kΩ.

P 5.30 **[a]**



$$\frac{v_p}{72{,}000} + \frac{v_p - v_c}{9{,}000} + \frac{v_p - v_d}{24{,}000} = 0$$

$$\therefore \quad v_p = (2/3)v_c + 0.25v_d = v_n$$

$$\frac{v_n - v_a}{12{,}000} + \frac{v_n - v_b}{18{,}000} + \frac{v_n - v_o}{144{,}000} = 0$$

$$\begin{aligned} \therefore \quad v_o &= 21v_n - 12v_a - 8v_b \\ &= 21[(2/3)v_c + 0.25v_d] - 12v_a - 8v_b \\ &= 21(0.4 + 0.2) - 12(0.5) - 8(0.3) = 4.2 \text{ V} \end{aligned}$$

[b] $v_o = 14v_c + 4.2 - 6 - 2.4$

$$\pm 15 = 14v_c - 4.2$$

$$\therefore \quad 14v_c = \pm 15 + 4.2$$

$$\therefore \quad v_c = 1.371 \text{ V} \qquad \text{and} \qquad v_c = -0.771 \text{ V}$$

$$\therefore \quad -771 \leq v_c \leq 1371 \,\text{mV}$$

P 5.31 **[a]** $v_o = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)}v_b - \frac{R_b}{R_a}v_a = \frac{47(110)}{10(80)}(0.80) - 10(0.67)$

$$v_o = 5.17 - 6.70 = -1.53 \text{ V}$$

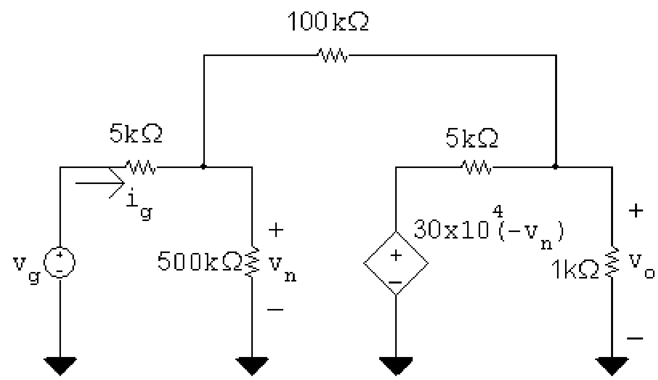
[b] $v_n = v_p = \frac{(800)(47)}{80} = 470\,\text{mV}$

$$i_a = \frac{(670 - 470)10^{-3}}{10 \times 10^3} = 20\,\mu\text{A}$$

$$R_a = \frac{v_a}{i_a} = \frac{670 \times 10^{-3}}{20 \times 10^{-6}} = 33.5\,\text{k}\Omega$$

[c] $R_{\text{in b}} = R_c + R_d = 80\,\text{k}\Omega$

P 5.39 **[a]** Replace the op amp with the model from Fig. 5.15:



Write two node voltage equations, one at the left node, the other at the right node:

$$\frac{v_n - v_g}{5000} + \frac{v_n - v_o}{100{,}000} + \frac{v_n}{500{,}000} = 0$$

$$\frac{v_o + 3 \times 10^5 v_n}{5000} + \frac{v_o - v_n}{100{,}000} + \frac{v_o}{1000} = 0$$

Simplify and place in standard form:

$$106v_n - 5v_o = 100v_g$$

$$(6{\times}10^6 - 1)v_n + 121v_o = 0$$

Let $v_g = 1$ V and solve the two simultaneous equations:

$$v_o = -19.9915\ \text{V}; \qquad v_n = 403.2\,\mu\text{V}$$

[b] From the solution in part (a), $v_n = 403.2\,\mu$V.

[c] $i_g = \dfrac{v_g - v_n}{5000} = \dfrac{v_g - 403.2 \times 10^{-6} v_g}{5000}$

$$R_g = \frac{v_g}{i_g} = \frac{5000}{1 - 403.2 \times 10^{-6}} = 5002.02\,\Omega$$

[d] For an ideal op amp, the voltage gain is the ratio between the feedback resistor and the input resistor:

$$\frac{v_o}{v_g} = -\frac{100{,}000}{5000} = -20$$

For an ideal op amp, the difference between the voltages at the input terminals is zero, and the input resistance of the op amp is infinite. Therefore,

$$v_n = v_p = 0\ \text{V}; \qquad R_g = 5000\,\Omega$$