6

Inductance, Capacitance, and Mutual Inductance

Assessment Problems

AP 6.1 **[a]** $i_g = 8e^{-300t} - 8e^{-1200t}\,\text{A}$

$$v = L\frac{di_g}{dt} = -9.6e^{-300t} + 38.4e^{-1200t}\,\text{V}, \qquad t > 0^+$$

$$v(0^+) = -9.6 + 38.4 = 28.8\,\text{V}$$

[b] $v = 0$ when $38.4e^{-1200t} = 9.6e^{-300t}$ or $t = (\ln 4)/900 = 1.54\,\text{ms}$

[c] $p = vi = 384e^{-1500t} - 76.8e^{-600t} - 307.2e^{-2400t}\,\text{W}$

[d] $\dfrac{dp}{dt} = 0$ when $e^{1800t} - 12.5e^{900t} + 16 = 0$

Let $x = e^{900t}$ and solve the quadratic $x^2 - 12.5x + 16 = 0$

$$x = 1.45, \qquad t = \frac{\ln 1.45}{900} = 411.05\,\mu\text{s}$$

$$x = 11.05, \qquad t = \frac{\ln 11.05}{900} = 2.67\,\text{ms}$$

p is maximum at $t = 411.05\,\mu\text{s}$

[e] $p_{\max} = 384e^{-1.5(0.41105)} - 76.8e^{-0.6(0.41105)} - 307.2e^{-2.4(0.41105)} = 32.72\,\text{W}$

[f] $i_{\max} = 8[e^{-0.3(1.54)} - e^{-1.2(1.54)}] = 3.78\,\text{A}$

$$w_{\max} = (1/2)(4 \times 10^{-3})(3.78)^2 = 28.6\,\text{mJ}$$

[g] W is max when i is max, i is max when di/dt is zero.

When $di/dt = 0$, $v = 0$, therefore $t = 1.54\,\text{ms}$.

AP 6.2 **[a]** $i = C\frac{dv}{dt} = 24 \times 10^{-6}\frac{d}{dt}[e^{-15,000t}\sin 30{,}000t]$

$$= [0.72\cos 30{,}000t - 0.36\sin 30{,}000t]e^{-15,000t}\,\text{A}, \qquad i(0^+) = 0.72\,\text{A}$$

[b] $i\left(\frac{\pi}{80}\,\text{ms}\right) = -31.66\,\text{mA}, \quad v\left(\frac{\pi}{80}\,\text{ms}\right) = 20.505\,\text{V},$

$p = vi = -649.23\,\text{mW}$

[c] $w = \left(\frac{1}{2}\right)Cv^2 = 126.13\,\mu\text{J}$

AP 6.3 **[a]** $v = \left(\frac{1}{C}\right)\int_{0^-}^{t} i\,dx + v(0^-)$

$$= \frac{1}{0.6 \times 10^{-6}}\int_{0^-}^{t} 3\cos 50{,}000x\,dx = 100\sin 50{,}000t\,\text{V}$$

[b] $p(t) = vi = [300\cos 50{,}000t]\sin 50{,}000t$

$$= 150\sin 100{,}000t\,\text{W}, \qquad p_{(\text{max})} = 150\,\text{W}$$

[c] $w_{(\text{max})} = \left(\frac{1}{2}\right)Cv_{\text{max}}^2 = 0.30(100)^2 = 3000\,\mu\text{J} = 3\,\text{mJ}$

AP 6.4 **[a]** $L_{\text{eq}} = \frac{60(240)}{300} = 48\,\text{mH}$

[b] $i(0^+) = 3 + -5 = -2\,\text{A}$

[c] $i = \frac{125}{6}\int_{0^+}^{t}(-0.03e^{-5x})\,dx - 2 = 0.125e^{-5t} - 2.125\,\text{A}$

[d] $i_1 = \frac{50}{3}\int_{0^+}^{t}(-0.03e^{-5x})\,dx + 3 = 0.1e^{-5t} + 2.9\,\text{A}$

$i_2 = \frac{25}{6}\int_{0^+}^{t}(-0.03e^{-5x})\,dx - 5 = 0.025e^{-5t} - 5.025\,\text{A}$

$i_1 + i_2 = i$

AP 6.5 $v_1 = 0.5 \times 10^6\int_{0^+}^{t} 240 \times 10^{-6}e^{-10x}\,dx - 10 = -12e^{-10t} + 2\,\text{V}$

$v_2 = 0.125 \times 10^6\int_{0^+}^{t} 240 \times 10^{-6}e^{-10x}\,dx - 5 = -3e^{-10t} - 2\,\text{V}$

$v_1(\infty) = 2\,\text{V}, \qquad v_2(\infty) = -2\,\text{V}$

$W = \left[\frac{1}{2}(2)(4) + \frac{1}{2}(8)(4)\right] \times 10^{-6} = 20\,\mu\text{J}$

AP 6.6 **[a]** Summing the voltages around mesh 1 yields

$$4\frac{di_1}{dt} + 8\frac{d(i_2 + i_g)}{dt} + 20(i_1 - i_2) + 5(i_1 + i_g) = 0$$

or

$$4\frac{di_1}{dt} + 25i_1 + 8\frac{di_2}{dt} - 20i_2 = -\left(5i_g + 8\frac{di_g}{dt}\right)$$

Summing the voltages around mesh 2 yields

$$16\frac{d(i_2 + i_g)}{dt} + 8\frac{di_1}{dt} + 20(i_2 - i_1) + 780i_2 = 0$$

or

$$8\frac{di_1}{dt} - 20i_1 + 16\frac{di_2}{dt} + 800i_2 = -16\frac{di_g}{dt}$$

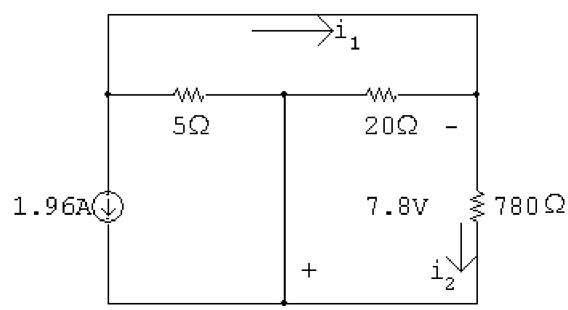
[b] From the solutions given in part (b)

$$i_1(0) = -0.4 - 11.6 + 12 = 0; \qquad i_2(0) = -0.01 - 0.99 + 1 = 0$$

These values agree with zero initial energy in the circuit. At infinity,

$$i_1(\infty) = -0.4\text{A}; \qquad i_2(\infty) = -0.01\text{A}$$

When $t = \infty$ the circuit reduces to



$$\therefore \quad i_1(\infty) = -\left(\frac{7.8}{20} + \frac{7.8}{780}\right) = -0.4\text{A}; \quad i_2(\infty) = -\frac{7.8}{780} = -0.01\text{A}$$

From the solutions for i_1 and i_2 we have

$$\frac{di_1}{dt} = 46.40e^{-4t} - 60e^{-5t}$$

$$\frac{di_2}{dt} = 3.96e^{-4t} - 5e^{-5t}$$

Also, $\displaystyle\frac{di_g}{dt} = 7.84e^{-4t}$

Thus

$$4\frac{di_1}{dt} = 185.60e^{-4t} - 240e^{-5t}$$

$$25i_1 = -10 - 290e^{-4t} + 300e^{-5t}$$

$$8\frac{di_2}{dt} = 31.68e^{-4t} - 40e^{-5t}$$

$$20i_2 = -0.20 - 19.80e^{-4t} + 20e^{-5t}$$

$$5i_g = 9.8 - 9.8e^{-4t}$$

$$8\frac{di_g}{dt} = 62.72e^{-4t}$$

Test:

$$185.60e^{-4t} - 240e^{-5t} - 10 - 290e^{-4t} + 300e^{-5t} + 31.68e^{-4t} - 40e^{-5t}$$

$$+0.20 + 19.80e^{-4t} - 20e^{-5t} \stackrel{?}{=} -[9.8 - 9.8e^{-4t} + 62.72e^{-4t}]$$

$$-9.8 + (300 - 240 - 40 - 20)e^{-5t}$$

$$+(185.60 - 290 + 31.68 + 19.80)e^{-4t} \stackrel{?}{=} -(9.8 + 52.92e^{-4t})$$

$$-9.8 + 0e^{-5t} + (237.08 - 290)e^{-4t} \stackrel{?}{=} -9.8 - 52.92e^{-4t}$$

$$-9.8 - 52.92e^{-4t} = -9.8 - 52.92e^{-4t} \quad \text{(OK)}$$

Also,

$$8\frac{di_1}{dt} = 371.20e^{-4t} - 480e^{-5t}$$

$$20i_1 = -8 - 232e^{-4t} + 240e^{-5t}$$

$$16\frac{di_2}{dt} = 63.36e^{-4t} - 80e^{-5t}$$

$$800i_2 = -8 - 792e^{-4t} + 800e^{-5t}$$

$$16\frac{di_g}{dt} = 125.44e^{-4t}$$

Test:

$$371.20e^{-4t} - 480e^{-5t} + 8 + 232e^{-4t} - 240e^{-5t} + 63.36e^{-4t} - 80e^{-5t}$$

$$-8 - 792e^{-4t} + 800e^{-5t} \stackrel{?}{=} -125.44e^{-4t}$$

$$(8 - 8) + (800 - 480 - 240 - 80)e^{-5t}$$

$$+(371.20 + 232 + 63.36 - 792)e^{-4t} \stackrel{?}{=} -125.44e^{-4t}$$

$$(800 - 800)e^{-5t} + (666.56 - 792)e^{-4t} \stackrel{?}{=} -125.44e^{-4t}$$

$$-125.44e^{-4t} = -125.44e^{-4t} \quad \text{(OK)}$$

Problems

P 6.1 **[a]**

$$\begin{aligned} i &= 0 & t &< 0 \\ i &= 50t\,\text{A} & 0 &\le t \le 5\,\text{ms} \\ i &= 0.5 - 50t\,\text{A} & 5 &\le t \le 10\,\text{ms} \\ i &= 0 & 10\,\text{ms} &< t \end{aligned}$$

[b] $v = L\dfrac{di}{dt} = 20 \times 10^{-3}(50) = 1\,\text{V} \qquad 0 \le t \le 5\,\text{ms}$

$v = 20 \times 10^{-3}(-50) = -1\,\text{V} \qquad 5 \le t \le 10\,\text{ms}$

$$\begin{aligned} v &= 0 & t &< 0 \\ v &= 1\,\text{V} & 0 &< t < 5\,\text{ms} \\ v &= -1\,\text{V} & 5 &< t < 10\,\text{ms} \\ v &= 0 & 10\,\text{ms} &< t \end{aligned}$$

$p = vi$

$$\begin{aligned} p &= 0 & t &< 0 \\ p &= (50t)(1) = 50t\,\text{W} & 0 &< t < 5\,\text{ms} \\ p &= (0.5 - 50t)(-1) = 50t - 0.5\,\text{W} & 5 &< t < 10\,\text{ms} \\ p &= 0 & 10\,\text{ms} &< t \end{aligned}$$

$$\begin{aligned} w &= 0 & t &< 0 \\ w &= \int_0^t (50x)\,dx = 50\frac{x^2}{2}\Big|_0^t = 25t^2\,\text{J} & 0 &< t < 5\,\text{ms} \\ w &= \int_{0.005}^t (50x - 0.5)\,dx + 0.625 \times 10^{-3} \\ &= 25x^2 - 0.5x\Big|_{0.005}^t + 0.625 \times 10^{-3} \\ &= 25t^2 - 0.5t + 2.5 \times 10^{-3}\,\text{J} & 5 &< t < 10\,\text{ms} \\ w &= 0 & 10\,\text{ms} &< t \end{aligned}$$

P 6.14 $i_C = C(dv/dt)$

$0 < t < 0.5:$

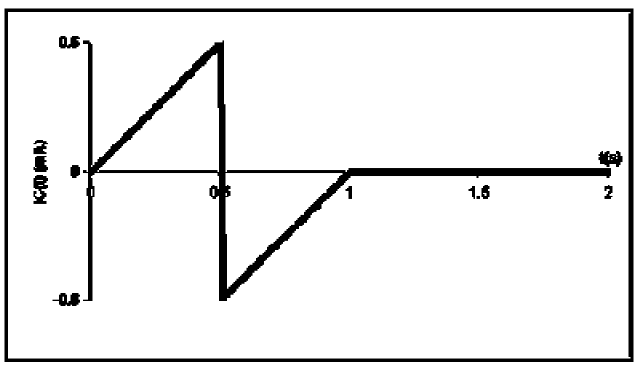
$$v_c = 30t^2\,\text{V}$$

$$i_C = 20\times10^{-6}(60)t = 1.2t\,\text{mA}$$

$0.5 < t < 1:$

$$v_c = 30(t-1)^2\,\text{V}$$

$$i_C = 20\times10^{-6}(60)(t-1) = 1.2(t-1)\,\text{mA}$$



P 6.15 **[a]** $0 \le t \le 5\,\mu\text{s}$

$$C = 5\,\mu\text{F} \qquad \frac{1}{C} = 2\times10^5$$

$$v = 2\times10^5\int_0^t 4\,dx + 12$$

$$v = 8\times10^5 t + 12\,\text{V} \qquad 0 \le t \le 5\,\mu\text{s}$$

$$v(5\,\mu\text{s}) = 4 + 12 = 16\,\text{V}$$

[b] $5\,\mu\text{s} \le t \le 20\,\mu\text{s}$

$$v = 2\times10^5\int_{5\times10^{-6}}^t -2\,dx + 16 = -4\times10^5 t + 2 + 16$$

$$v = -4\times10^5 t + 18\text{V} \qquad 5 \le t \le 20\,\mu\text{s}$$

$$v(20\,\mu\text{s}) = -4\times10^5(20\times10^{-6}) + 18 = 10\,\text{V}$$

[c] $20\,\mu\text{s} \le t \le 25\,\mu\text{s}$

$$v = 2\times 10^5 \int_{20\times 10^{-6}}^{t} 6\,dx + 10 = 12\times 10^5 t - 24 + 10$$

$$v = 12\times 10^5 t - 14\,\text{V}, \qquad 20\,\mu\text{s} \le t \le 25\,\mu\text{s}$$

$$v(25\,\mu\text{s}) = 12\times 10^5(25\times 10^{-6}) - 14 = 16\,\text{V}$$

[d] $25\,\mu\text{s} \le t \le 35\,\mu\text{s}$

$$v = 2\times 10^5 \int_{25\times 10^{-6}}^{t} 4\,dx + 16 = 8\times 10^5 t - 20 + 16$$

$$v = 8\times 10^5 t - 4\,\text{V}, \qquad 25\,\mu\text{s} \le t \le 35\,\mu\text{s}$$

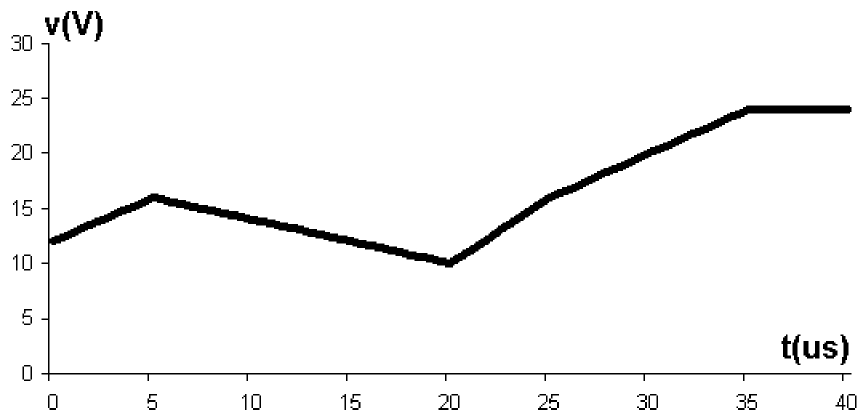
$$v(35\,\mu\text{s}) = 8\times 10^5(35\times 10^{-6}) - 4 = 24\,\text{V}$$

[e] $35\,\mu\text{s} \le t < \infty$

$$v = 2\times 10^5 \int_{35\times 10^{-6}}^{t} 0\,dx + 24 = 24$$

$$v = 24\,\text{V}, \qquad 35\,\mu\text{s} \le t < \infty$$

[f]



P 6.21 $30\|20 = 12\,\text{H}$

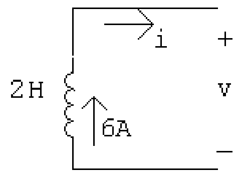
$80\|(8+12) = 16\,\text{H}$

$60\|(14+16) = 20\,\text{H}$

$15\|(20+10) = 20\,\text{H}$

$L_{\text{ab}} = 5 + 10 = 15\,\text{H}$

P 6.22 **[a]**



$$
\begin{aligned}
i(t) &= -\frac{1}{2}\int_0^t 12e^{-x}\,dx + 6 \\
&= 6e^{-x}\Big|_0^t + 6 \\
&= 6e^{-t} - 6 + 6 \\
i(t) &= 6e^{-t}\,\text{A}, \qquad t \geq 0
\end{aligned}
$$

[b]

$$
\begin{aligned}
i_1(t) &= -\frac{1}{3}\int_0^t 12e^{-x}\,dx + 2 \\
&= 4e^{-x}\Big|_0^t + 2 \\
&= 4(e^{-t} - 1) + 2 \\
i_1(t) &= 4e^{-t} - 2\,\text{A}, \qquad t \geq 0
\end{aligned}
$$

[c]

$$
\begin{aligned}
i_2(t) &= -\frac{1}{6}\int_0^t 12e^{-x}\,dx + 4 \\
&= 2e^{-x}\Big|_0^t + 4 \\
&= 2(e^{-t} - 1) + 4 \\
i_2(t) &= 2e^{-t} + 2\,\text{A}, \qquad t \geq 0
\end{aligned}
$$

[d] $p = vi = (12e^{-t})(6e^{-t}) = 72e^{-2t}\,\text{W}$

$$
\begin{aligned}
w &= \int_0^\infty p\,dt = \int_0^\infty 72e^{-2t}\,dt \\
&= 72\frac{e^{-2t}}{-2}\Big|_0^\infty \\
&= 36\,\text{J}
\end{aligned}
$$

[e] $w = \frac{1}{2}(3)(2)^2 + \frac{1}{2}(6)(4)^2 = 54\,\text{J}$

[f] $w_{\text{trapped}} = \frac{1}{2}(3)(-2)^2 + \frac{1}{2}(6)(2)^2 = 18\,\text{J}$

$w_{\text{trapped}} = 54 - 36 = 18\,\text{J}$ checks

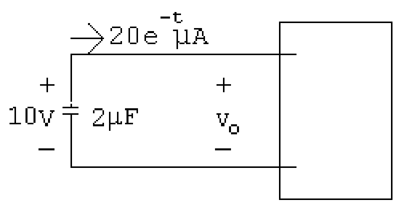
[g] Yes, they agree.

P 6.26 Work from the right hand side of the circuit, simplifying step by step:

1. $48\,\mu$F in series with $16\,\mu$F : $1/C = 1/16\,\mu + 1/48\,\mu \quad \therefore \quad C = 12\,\mu$F
   The voltages add in series, so the $12\,\mu$F capacitor has a voltage of 20 V, negative at the top.
2. Previous $12\,\mu$F in parallel with $3\,\mu$F : $C = 12\,\mu + 3\,\mu = 15\,\mu$F
   The voltage is 20 V, negative at the top.
3. Previous $15\,\mu$F in series with $30\,\mu$F :
   $1/C = 1/15\,\mu + 1/30\,\mu \quad \therefore \quad C = 10\,\mu$F
   The voltages add in series, so the $10\,\mu$F capacitor has a voltage of 10 V, positive at the right.
4. Previous $10\,\mu$F in parallel with $10\,\mu$F : $C = 10\,\mu + 10\,\mu = 20\,\mu$F
   The voltage is 10 V, negative at the top.
5. Previous $20\,\mu$F in series with $5\,\mu$F and $4\,\mu$F :
   $1/C = 1/20\,\mu + 1/5\,\mu + 1/4\,\mu \quad \therefore \quad C = 2\,\mu$F
   The voltages in series add: $5\text{V} - 10V + 30\text{V} = 25\text{V}$ positive at the top.

The equivalent capacitance is $2\,\mu$F with a voltage of 25 V, positive at the top.

P 6.27 **[a]**



$$v_o = -\frac{1}{2\times10^{-6}}\int_0^t 20\times10^{-6}e^{-x}\,dx + 10$$
$$= 10e^{-x}\Big|_0^t + 10$$
$$= 10e^{-t}\,\text{V}, \qquad t \geq 0$$

[b] $$v_1 = -\frac{1}{3\times10^{-6}}(20\times10^{-6})e^{-x}\Big|_0^t + 4$$
$$= 6.67e^{-t} - 2.67\,\text{V}, \qquad t \geq 0$$

[c] $$v_2 = -\frac{1}{6\times10^{-6}}(20\times10^{-6})e^{-x}\Big|_0^t + 6$$
$$= 3.33e^{-t} + 2.67\,\text{V}, \qquad t \geq 0$$

[d] $$p = vi = (10e^{-t})(20\times10^{-6})e^{-t}$$
$$= 200\times10^{-6}e^{-2t}$$

$$w = \int_0^\infty 200 \times 10^{-6} e^{-2t}\,dt$$

$$= 200 \times 10^{-6} \frac{e^{-2t}}{-2} \Big|_0^\infty$$

$$= -100 \times 10^{-6}(0-1) = 100\,\mu\text{J}$$

[e] $w = \frac{1}{2}(3 \times 10^{-6})(4)^2 + \frac{1}{2}(6 \times 10^{-9})(6)^2$

$= 132\,\mu\text{J}$

[f] $w_{\text{trapped}} = \frac{1}{2}(3 \times 10^{-6})(8/3)^2 + \frac{1}{2}(6 \times 10^{-6})(8/3)^2$

$= 32\,\mu\text{J}$

CHECK: $100 + 32 = 132\,\mu\text{J}$

[g] Yes, they agree.

P 6.34 **[a]** $-2\frac{di_g}{dt} + 16\frac{di_2}{dt} + 32i_2 = 0$

$$16\frac{di_2}{dt} + 32i_2 = 2\frac{di_g}{dt}$$

[b] $i_2 = e^{-t} - e^{-2t}\,\text{A}$

$$\frac{di_2}{dt} = -e^{-t} + 2e^{-2t}\,\text{A/s}$$

$$i_g = 8 - 8e^{-t}\,\text{A}$$

$$\frac{di_g}{dt} = 8e^{-t}\,\text{A/s}$$

$$\therefore \quad -16e^{-t} + 32e^{-2t} + 32e^{-t} - 32e^{-2t} = 16e^{-t}$$

[c] $v_1 = 4\frac{di_g}{dt} - 2\frac{di_2}{dt}$

$= 4(8e^{-t}) - 2(-e^{-t} + 2e^{-2t})$

$= 34e^{-t} - 4e^{-2t}\,\text{V}, \qquad t > 0$

[d] $v_1(0) = 34 - 4 = 30\,\text{V};$ Also

$$v_1(0) = 4\frac{di_g}{dt}(0) - 2\frac{di_2}{dt}(0)$$

$$= 4(8) - 2(-1+2) = 32 - 2 = 30\,\text{V}$$

Yes, the initial value of v_1 is consistent with known circuit behavior.