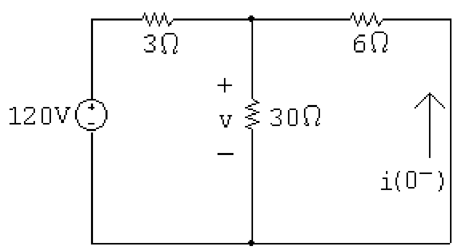
7

Response of First-Order RL and RC Circuits

Assessment Problems

AP 7.1 **[a]** The circuit for $t < 0$ is shown below. Note that the inductor behaves like a short circuit, effectively eliminating the $2\,\Omega$ resistor from the circuit.



First combine the $30\,\Omega$ and $6\,\Omega$ resistors in parallel:
$30\|6 = 5\,\Omega$
Use voltage division to find the voltage drop across the parallel resistors:
$$v = \frac{5}{5+3}(120) = 75\,\text{V}$$
Now find the current using Ohm's law:
$$i(0^-) = -\frac{v}{6} = -\frac{75}{6} = -12.5\,\text{A}$$

[b] $w(0) = \frac{1}{2}Li^2(0) = \frac{1}{2}(8 \times 10^{-3})(12.5)^2 = 625\,\text{mJ}$

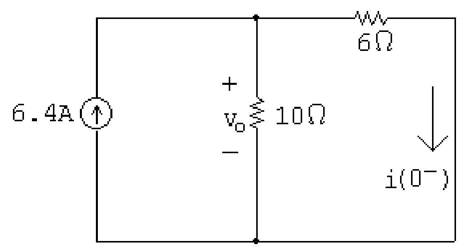
[c] To find the time constant, we need to find the equivalent resistance seen by the inductor for $t > 0$. When the switch opens, only the $2\,\Omega$ resistor remains connected to the inductor. Thus,
$$\tau = \frac{L}{R} = \frac{8 \times 10^{-3}}{2} = 4\,\text{ms}$$

[d] $i(t) = i(0^-)e^{t/\tau} = -12.5e^{-t/0.004} = -12.5e^{-250t}\,\text{A}, \qquad t \geq 0$

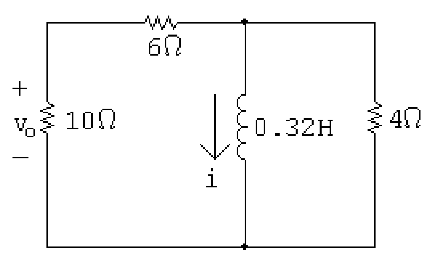
[e] $i(5\,\text{ms}) = -12.5e^{-250(0.005)} = -12.5e^{-1.25} = -3.58\,\text{A}$

So $w\,(5\,\text{ms}) = \frac{1}{2}Li^2(5\,\text{ms}) = \frac{1}{2}(8)\times 10^{-3}(3.58)^2 = 51.3\,\text{mJ}$
$w\,(\text{dis}) = 625 - 51.3 = 573.7\,\text{mJ}$
$\%\text{ dissipated} = \left(\frac{573.7}{625}\right)100 = 91.8\%$

AP 7.2 **[a]** First, use the circuit for $t < 0$ to find the initial current in the inductor:



Using current division,
$$i(0^-) = \frac{10}{10+6}(6.4) = 4\,\text{A}$$
Now use the circuit for $t > 0$ to find the equivalent resistance seen by the inductor, and use this value to find the time constant:



$$R_{\text{eq}} = 4\|(6+10) = 3.2\,\Omega, \quad \therefore \quad \tau = \frac{L}{R_{\text{eq}}} = \frac{0.32}{3.2} = 0.1\,\text{s}$$
Use the initial inductor current and the time constant to find the current in the inductor:
$i(t) = i(0^-)e^{-t/\tau} = 4e^{-t/0.1} = 4e^{-10t}\,\text{A}, \quad t \geq 0$
Use current division to find the current in the $10\,\Omega$ resistor:
$$i_o(t) = \frac{4}{4+10+6}(-i) = \frac{4}{20}(-4e^{-10t}) = -0.8e^{-10t}\,\text{A}, \quad t \geq 0^+$$
Finally, use Ohm's law to find the voltage drop across the $10\,\Omega$ resistor:
$v_o(t) = 10i_o = 10(-0.8e^{-10t}) = -8e^{-10t}\,\text{V}, \quad t \geq 0^+$

[b] The initial energy stored in the inductor is
$$w(0) = \frac{1}{2}Li^2(0^-) = \frac{1}{2}(0.32)(4)^2 = 2.56\,\text{J}$$
Find the energy dissipated in the $4\,\Omega$ resistor by integrating the power over all time:
$$v_{4\Omega}(t) = L\frac{di}{dt} = 0.32(-10)(4e^{-10t}) = -12.8e^{-10t}\,\text{V}, \qquad t \geq 0^+$$

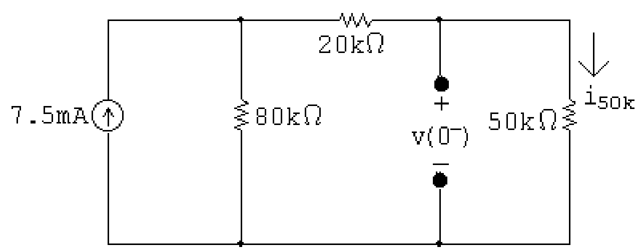
$$p_{4\Omega}(t) = \frac{v_{4\Omega}^2}{4} = 40.96e^{-20t}\,\text{W}, \qquad t \geq 0^+$$

$$w_{4\Omega}(t) = \int_0^\infty 40.96e^{-20t}dt = 2.048\,\text{J}$$

Find the percentage of the initial energy in the inductor dissipated in the $4\,\Omega$ resistor:

$$\%\text{ dissipated} = \left(\frac{2.048}{2.56}\right)100 = 80\%$$

AP 7.3 **[a]** The circuit for $t < 0$ is shown below. Note that the capacitor behaves like an open circuit.



Find the voltage drop across the open circuit by finding the voltage drop across the $50\,\text{k}\Omega$ resistor. First use current division to find the current through the $50\,\text{k}\Omega$ resistor:

$$i_{50\text{k}} = \frac{80 \times 10^3}{80 \times 10^3 + 20 \times 10^3 + 50 \times 10^3}(7.5 \times 10^{-3}) = 4\,\text{mA}$$

Use Ohm's law to find the voltage drop:
$v(0^-) = (50 \times 10^3)i_{50\text{k}} = (50 \times 10^3)(0.004) = 200\,\text{V}$

[b] To find the time constant, we need to find the equivalent resistance seen by the capacitor for $t > 0$. When the switch opens, only the $50\,\text{k}\Omega$ resistor remains connected to the capacitor. Thus,
$\tau = RC = (50 \times 10^3)(0.4 \times 10^{-6}) = 20\,\text{ms}$

[c] $v(t) = v(0^-)e^{-t/\tau} = 200e^{-t/0.02} = 200e^{-50t}\,\text{V}, \quad t \geq 0$

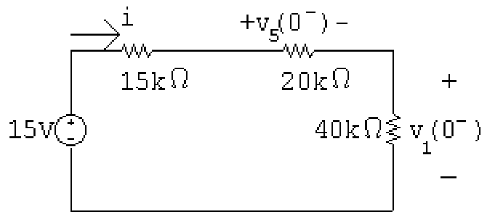
[d] $w(0) = \frac{1}{2}Cv^2 = \frac{1}{2}(0.4 \times 10^{-6})(200)^2 = 8\,\text{mJ}$

[e] $w(t) = \frac{1}{2}Cv^2(t) = \frac{1}{2}(0.4 \times 10^{-6})(200e^{-50t})^2 = 8e^{-100t}\,\text{mJ}$

The initial energy is 8 mJ, so when 75% is dissipated, 2 mJ remains:

$$8 \times 10^{-3}e^{-100t} = 2 \times 10^{-3}, \qquad e^{100t} = 4, \qquad t = (\ln 4)/100 = 13.86\,\text{ms}$$

AP 7.4 **[a]** This circuit is actually two RC circuits in series, and the requested voltage, v_o, is the sum of the voltage drops for the two RC circuits. The circuit for $t < 0$ is shown below:



Find the current in the loop and use it to find the initial voltage drops across the two RC circuits:

$$i = \frac{15}{75{,}000} = 0.2\,\text{mA}, \qquad v_5(0^-) = 4\,\text{V}, \qquad v_1(0^-) = 8\,\text{V}$$

There are two time constants in the circuit, one for each RC subcircuit. τ_5 is the time constant for the $5\,\mu$F – $20\,\text{k}\Omega$ subcircuit, and τ_1 is the time constant for the $1\,\mu$F – $40\,\text{k}\Omega$ subcircuit:

$\tau_5 = (20 \times 10^3)(5 \times 10^{-6}) = 100\,\text{ms}; \qquad \tau_1 = (40 \times 10^3)(1 \times 10^{-6}) = 40\,\text{ms}$

Therefore,

$v_5(t) = v_5(0^-)e^{-t/\tau_5} = 4e^{-t/0.1} = 4e^{-10t}\,\text{V}, \quad t \geq 0$

$v_1(t) = v_1(0^-)e^{-t/\tau_1} = 8e^{-t/0.04} = 8e^{-25t}\,\text{V}, \quad t \geq 0$

Finally,

$v_o(t) = v_1(t) + v_5(t) = [8e^{-25t} + 4e^{-10t}]\,\text{V}, \qquad t \geq 0$

[b] Find the value of the voltage at 60 ms for each subcircuit and use the voltage to find the energy at 60 ms:

$v_1(60\,\text{ms}) = 8e^{-25(0.06)} \cong 1.79\,\text{V}, \qquad v_5(60\,\text{ms}) = 4e^{-10(0.06)} \cong 2.20\,\text{V}$

$w_1(60\,\text{ms}) = \frac{1}{2}Cv_1^2(60\,\text{ms}) = \frac{1}{2}(1 \times 10^{-6})(1.79)^2 \cong 1.59\,\mu\text{J}$

$w_5(60\,\text{ms}) = \frac{1}{2}Cv_5^2(60\,\text{ms}) = \frac{1}{2}(5 \times 10^{-6})(2.20)^2 \cong 12.05\,\mu\text{J}$

$w(60\,\text{ms}) = 1.59 + 12.05 = 13.64\,\mu\text{J}$

Find the initial energy from the initial voltage:

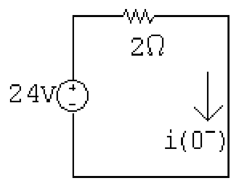
$w(0) = w_1(0) + w_2(0) = \frac{1}{2}(1 \times 10^{-6})(8)^2 + \frac{1}{2}(5 \times 10^{-6})(4)^2 = 72\,\mu\text{J}$

Now calculate the energy dissipated at 60 ms and compare it to the initial energy:

$w_{\text{diss}} = w(0) - w(60\,\text{ms}) = 72 - 13.64 = 58.36\,\mu\text{J}$

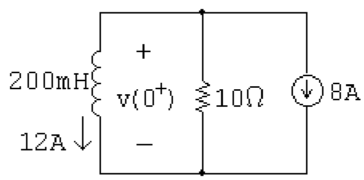
% dissipated $= (58.36 \times 10^{-6}/72 \times 10^{-6})(100) = 81.05\,\%$

AP 7.5 **[a]** Use the circuit at $t < 0$, shown below, to calculate the initial current in the inductor:



$i(0^-) = 24/2 = 12\,\text{A} = i(0^+)$
Note that $i(0^-) = i(0^+)$ because the current in an inductor is continuous.

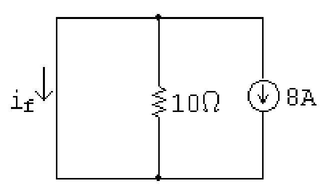
[b] Use the circuit at $t = 0^+$, shown below, to calculate the voltage drop across the inductor at 0^+. Note that this is the same as the voltage drop across the $10\,\Omega$ resistor, which has current from two sources — 8 A from the current source and 12 A from the initial current through the inductor.



$v(0^+) = -10(8 + 12) = -200\,\text{V}$

[c] To calculate the time constant we need the equivalent resistance seen by the inductor for $t > 0$. Only the $10\,\Omega$ resistor is connected to the inductor for $t > 0$. Thus,
$\tau = L/R = (200 \times 10^{-3}/10) = 20\,\text{ms}$

[d] To find $i(t)$, we need to find the final value of the current in the inductor. When the switch has been in position a for a long time, the circuit reduces to the one below:

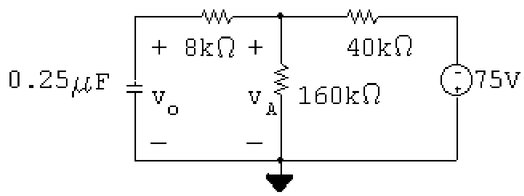


Note that the inductor behaves as a short circuit and all of the current from the 8 A source flows through the short circuit. Thus,
$i_f = -8\,\text{A}$
Now,
$$i(t) = i_f + [i(0^+) - i_f]e^{-t/\tau} = -8 + [12 - (-8)]e^{-t/0.02}$$
$$= -8 + 20e^{-50t}\,\text{A}, \quad t \geq 0$$

[e] To find $v(t)$, use the relationship between voltage and current for an inductor:

$$v(t) = L\frac{di(t)}{dt} = (200 \times 10^{-3})(-50)(20e^{-50t}) = -200e^{-50t}\,\text{V}, \qquad t \geq 0^+$$

AP 7.6 **[a]**



From Example 7.6,

$v_o(t) = -60 + 90e^{-100t}$ V

Write a KVL equation at the top node and use it to find the relationship between v_o and v_A:

$$\frac{v_A - v_o}{8000} + \frac{v_A}{160{,}000} + \frac{v_A + 75}{40{,}000} = 0$$

$$20v_A - 20v_o + v_A + 4v_A + 300 = 0$$

$$25v_A = 20v_o - 300$$

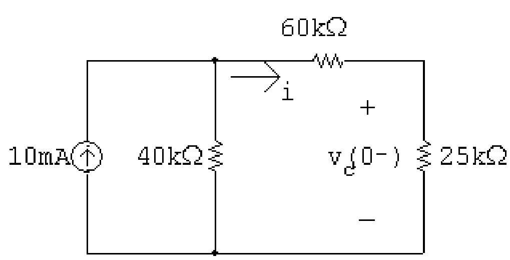
$$v_A = 0.8v_o - 12$$

Use the above equation for v_A in terms of v_o to find the expression for v_A:

$$v_A(t) = 0.8(-60 + 90e^{-100t}) - 12 = -60 + 72e^{-100t}\ \text{V}, \qquad t \geq 0^+$$

[b] $t \geq 0^+$, since there is no requirement that the voltage be continuous in a resistor.

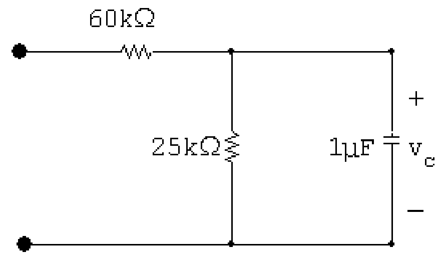
AP 7.7 **[a]** Use the circuit shown below, for $t < 0$, to calculate the initial voltage drop across the capacitor:



$$i = \left(\frac{40 \times 10^3}{125 \times 10^3}\right)(10 \times 10^{-3}) = 3.2\,\text{mA}$$

$$v_c(0^-) = (3.2 \times 10^{-3})(25 \times 10^3) = 80\,\text{V} \quad \text{so} \quad v_c(0^+) = 80\,\text{V}$$

Now use the next circuit, valid for $0 \leq t \leq 10\,\text{ms}$, to calculate $v_c(t)$ for that interval:

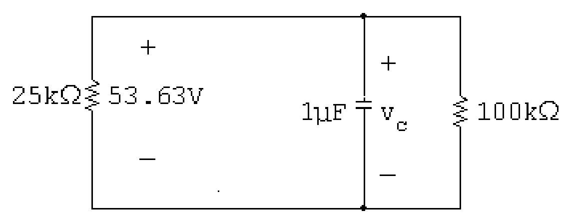


For $\quad 0 \leq t \leq 100\,\text{ms}$:

$\tau = RC = (25 \times 10^3)(1 \times 10^{-6}) = 25\,\text{ms}$

$v_c(t) = v_c(0^-)e^{t/\tau} = 80e^{-40t}\,\text{V}, \qquad 0 \leq t \leq 10\,\text{ms}$

[b] Calculate the starting capacitor voltage in the interval $t \geq 10\,\text{ms}$, using the capacitor voltage from the previous interval:
$v_c(0.01) = 80e^{-40(0.01)} = 53.63\,\text{V}$
Now use the next circuit, valid for $t \geq 10\,\text{ms}$, to calculate $v_c(t)$ for that interval:



For $\quad t \geq 10\,\text{ms}$:

$R_{\text{eq}} = 25\,\text{k}\Omega \| 100\,\text{k}\Omega = 20\,\text{k}\Omega$

$\tau = R_{\text{eq}}C = (20 \times 10^3)(1 \times 10^{-6}) = 0.02\,\text{s}$

Therefore $\quad v_c(t) = v_c(0.01^+)e^{-(t-0.01)/\tau} = 53.63e^{-50(t-0.01)}\,\text{V}, \qquad t \geq 0.01\,\text{s}$

[c] To calculate the energy dissipated in the 25 kΩ resistor, integrate the power absorbed by the resistor over all time. Use the expression $p = v^2/R$ to calculate the power absorbed by the resistor.

$$w_{25\,\text{k}} = \int_0^{0.01} \frac{[80e^{-40t}]^2}{25{,}000}dt + \int_{0.01}^{\infty} \frac{[53.63e^{-50(t-0.01)}]^2}{25{,}000}dt = 2.91\,\text{mJ}$$

[d] Repeat the process in part (c), but recognize that the voltage across this resistor is non-zero only for the second interval:

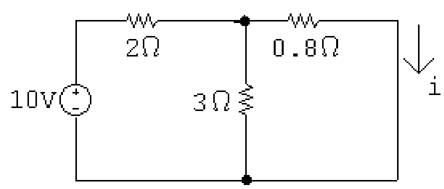
$$w_{100\,\text{k}\Omega} = \int_{0.01}^{\infty} \frac{[53.63e^{-50(t-0.01)}]^2}{100{,}000}dt = 0.29\,\text{mJ}$$

We can check our answers by calculating the initial energy stored in the capacitor. All of this energy must eventually be dissipated by the $25\,\text{k}\Omega$ resistor and the $100\,\text{k}\Omega$ resistor.

Check: $\quad w_{\text{stored}} = (1/2)(1 \times 10^{-6})(80)^2 = 3.2\,\text{mJ}$

$w_{\text{diss}} = 2.91 + 0.29 = 3.2\,\text{mJ}$

AP 7.8 **[a]** Note – the $30\,\Omega$ resistor should be a $3\,\Omega$ resistor; the resistor in parallel with the 8 A current source should be $9\,\Omega$.
Prior to switch a closing at $t = 0$, there are no sources connected to the inductor; thus, $i(0^-) = 0$.
At the instant A is closed, $i(0^+) = 0$.
For $0 \leq t \leq 1$ s,



The equivalent resistance seen by the 10 V source is $2 + (3\|0.8)$. The current leaving the 10 V source is

$$\frac{10}{2 + (3\|0.8)} = 3.8\,\text{A}$$

The final current in the inductor, which is equal to the current in the $0.8\,\Omega$ resistor is

$$i(\infty) = \frac{3}{3 + 0.8}(3.8) = 3\,\text{A}$$

The resistance seen by the inductor is calculated to find the time constant:

$$0.8 + (2\|3) = 2\,\Omega \qquad \tau = \frac{L}{R} = \frac{2}{2} = 1\,\text{s}$$

Therefore,

$$i = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau} = 3 - 3e^{-t}\,\text{A}, \quad 0 \leq t \leq 1\,\text{s}$$

For part (b) we need the value of $i(t)$ at $t = 1$ s:

$$i(1) = 3 - 3e^{-1} = 1.896\,\text{A}$$

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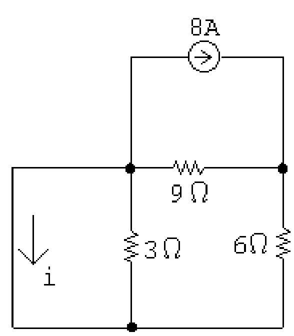
[b] For $t > 1$ s



Use current division to find the final value of the current:

$$i = \frac{9}{9 + 6}(-8) = -4.8\,\text{A}$$

The equivalent resistance seen by the inductor is used to calculate the time constant:

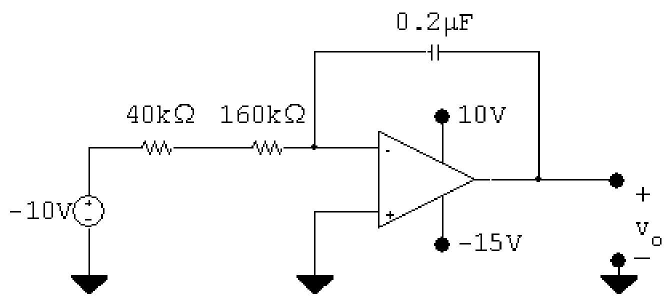
$$3\|(9+6) = 2.5\,\Omega \qquad \tau = \frac{L}{R} = \frac{2}{2.5} = 0.8\,\text{s}$$

Therefore,

$$i = i(\infty) + [i(1^+) - i(\infty)]e^{-(t-1)/\tau}$$

$$= -4.8 + 6.696e^{-1.25(t-1)}\,\text{A}, \quad t \geq 1\,\text{s}$$

AP 7.9 $0 \leq t \leq 32\,\text{ms}$:

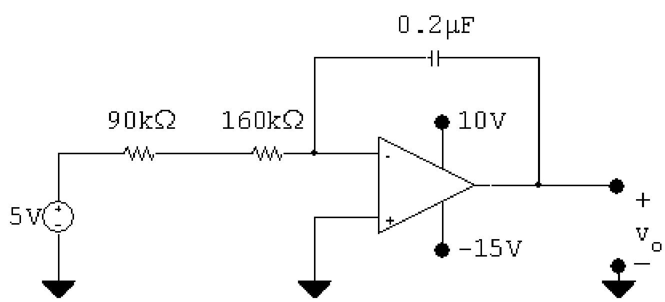


$$v_o = -\frac{1}{RC_f}\int_0^{32\times10^{-3}} -10\,dt + 0 = -\frac{1}{RC_f}(-10t)\Big|_0^{32\times10^{-3}} = -\frac{1}{RC_f}(-320\times10^{-3})$$

$$RC_f = (200\times10^3)(0.2\times10^{-6}) = 40\times10^{-3} \quad \text{so} \quad \frac{1}{RC_f} = 25$$

$$v_o = -25(-320\times10^{-3}) = 8\,\text{V}$$

$t \geq 32$ ms:



$$v_o = -\frac{1}{RC_f}\int_{32\times10^{-3}}^{t} 5\,dy + 8 = -\frac{1}{RC_f}(5y)\Big|_{32\times10^{-3}}^{t} + 8 = -\frac{1}{RC_f}5(t-32\times10^{-3}) + 8$$

$$RC_f = (250\times10^3)(0.2\times10^{-6}) = 50\times10^{-3} \quad \text{so} \quad \frac{1}{RC_f} = 20$$

$v_o = -20(5)(t - 32 \times 10^{-3}) + 8 = -100t + 11.2$

The output will saturate at the negative power supply value:

$-15 = -100t + 11.2 \quad \therefore \quad t = 262\,\text{ms}$

AP 7.10 **[a]** Use RC circuit analysis to determine the expression for the voltage at the non-inverting input:

$v_p = V_f + [V_o - V_f]e^{-t/\tau} = -2 + (0 + 2)e^{-t/\tau}$

$\tau = (160 \times 10^3)(10 \times 10^{-9}) = 10^{-3}; \qquad 1/\tau = 625$

$v_p = -2 + 2e^{-625t}\,\text{V}; \qquad v_n = v_p$

Write a KVL equation at the inverting input, and use it to determine v_o:

$$\frac{v_n}{10{,}000} + \frac{v_n - v_o}{40{,}000} = 0$$

$\therefore \quad v_o = 5v_n = 5v_p = -10 + 10e^{-625t}\,\text{V}$

The output will saturate at the negative power supply value:

$-10 + 10e^{-625t} = -5; \quad e^{-625t} = 1/2; \quad t = \ln 2/625 = 1.11\,\text{ms}$

[b] Use RC circuit analysis to determine the expression for the voltage at the non-inverting input:

$v_p = V_f + [V_o - V_f]e^{-t/\tau} = -2 + (1 + 2)e^{-625t} = -2 + 3e^{-625t}\,\text{V}$

The analysis for v_o is the same as in part (a):

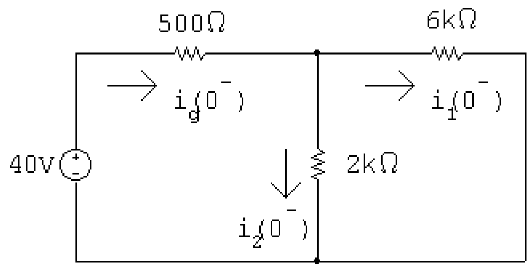
$v_o = 5v_p = -10 + 15e^{-625t}\,\text{V}$

The output will saturate at the negative power supply value:

$-10 + 15e^{-625t} = -5; \qquad e^{-625t} = 1/3; \quad t = \ln 3/625 = 1.76\,\text{ms}$

Problems

P 7.1 **[a]** $t < 0$



$$2\,\text{k}\Omega \| 6\,\text{k}\Omega = 1.5\text{k}\Omega$$

Find the current from the voltage source by combining the resistors in series and parallel and using Ohm's law:

$$i_g(0^-) = \frac{40}{(1500 + 500)} = 20\,\text{mA}$$

Find the branch currents using current division:

$$i_1(0^-) = \frac{2000}{8000}(0.02) = 5\,\text{mA}$$

$$i_2(0^-) = \frac{6000}{8000}(0.02) = 15\,\text{mA}$$

[b] The current in an inductor is continuous. Therefore,

$$i_1(0^+) = i_1(0^-) = 5\,\text{mA}$$

$$i_2(0^+) = -i_1(0^+) = -5\,\text{mA} \qquad \text{(when switch is open)}$$

[c] $\tau = \dfrac{L}{R} = \dfrac{0.4 \times 10^{-3}}{8 \times 10^3} = 5 \times 10^{-5}\,\text{s}; \qquad \dfrac{1}{\tau} = 20{,}000$

$$i_1(t) = i_1(0^+)e^{-t/\tau} = 5e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0$$

[d] $i_2(t) = -i_1(t) \qquad \text{when} \quad t \geq 0^+$

$$\therefore \quad i_2(t) = -5e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0^+$$

[e] The current in a resistor can change instantaneously. The switching operation forces $i_2(0^-)$ to equal $15\,\text{mA}$ and $i_2(0^+) = -5\,\text{mA}$.

P 7.2 **[a]** $i(0) = 60\,\text{V}/(10\,\Omega + 5\,\Omega) = 4\,\text{A}$

[b] $\tau = \dfrac{L}{R} = \dfrac{4}{45+5} = 80\,\text{ms}$

[c] $i = 4e^{-t/0.08} = 4e^{-12.5t}\,\text{A}, \qquad t \geq 0$

$$v_1 = -45i = -180e^{-12.5t}\,\text{V} \qquad t \geq 0^+$$

$$v_2 = L\frac{di}{dt} = (4)(-12.5)(4e^{-12.5t}) = -200e^{-12.5t}\,\text{V} \qquad t \geq 0^+$$

[d] $p_{\text{diss}} = i^2(45) = 720e^{-25t}\,\text{W}$

$$w_{\text{diss}} = \int_0^t 720e^{-25x}\,dx = 720\frac{e^{-25x}}{-25}\Big|_0^t = 28.8 - 28.8e^{-25t}\,\text{J}$$

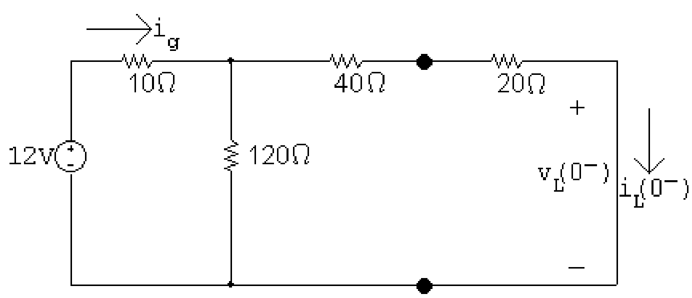
$$w_{\text{diss}}(40\,\text{ms}) = 28.8 - 28.8e^{-1} = 18.205\,\text{J}$$

$$w(0) = \frac{1}{2}(4)(4)^2 = 32\,\text{J}$$

$$\%\ \text{dissipated} = \frac{18.205}{32}(100) = 56.89\%$$

P 7.3 **[a]** $i_o(0^-) = 0$ since the switch is open for $t < 0$.

[b] For $t = 0^-$ the circuit is:

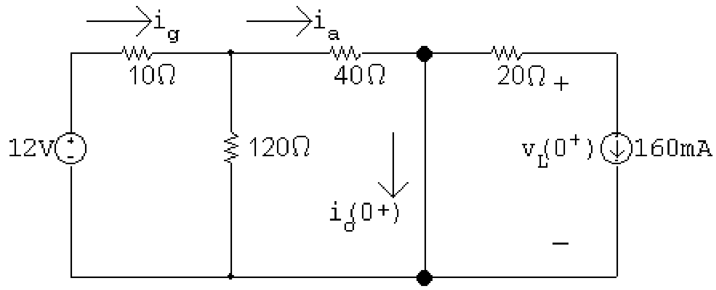


$$120\,\Omega \| 60\,\Omega = 40\,\Omega$$

$$\therefore \quad i_g = \frac{12}{10+40} = 0.24\,\text{A} = 240\,\text{mA}$$

$$i_L(0^-) = \left(\frac{120}{180}\right) i_g = 160\,\text{mA}$$

[c] For $t = 0^+$ the circuit is:



$$120\,\Omega \| 40\,\Omega = 30\,\Omega$$

$$\therefore \quad i_g = \frac{12}{10 + 30} = 0.30\,\text{A} = 300\,\text{mA}$$

$$i_{\text{a}} = \left(\frac{120}{160}\right) 300 = 225\,\text{mA}$$

$$\therefore \quad i_o(0^+) = 225 - 160 = 65\,\text{mA}$$

[d] $i_L(0^+) = i_L(0^-) = 160\,\text{mA}$

[e] $i_o(\infty) = i_{\text{a}} = 225\,\text{mA}$

[f] $i_L(\infty) = 0,$ since the switch short circuits the branch containing the 20 Ω resistor and the 100 mH inductor.

[g] $\tau = \dfrac{L}{R} = \dfrac{100 \times 10^{-3}}{20} = 5\,\text{ms}; \qquad \dfrac{1}{\tau} = 200$

$$\therefore \quad i_L = 0 + (160 - 0)e^{-200t} = 160e^{-200t}\,\text{mA}, \qquad t \geq 0$$

[h] $v_L(0^-) = 0$ since for $t < 0$ the current in the inductor is constant

[i] Refer to the circuit at $t = 0^+$ and note:

$$20(0.16) + v_L(0^+) = 0; \qquad \therefore \quad v_L(0^+) = -3.2\,\text{V}$$

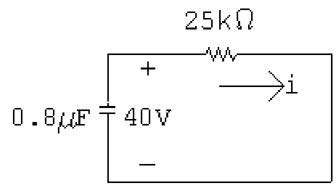
[j] $v_L(\infty) = 0,$ since the current in the inductor is a constant at $t = \infty$.

[k] $v_L(t) = 0 + (-3.2 - 0)e^{-200t} = -3.2e^{-200t}\,\text{V}, \qquad t \geq 0^+$

[l] $i_o = i_{\text{a}} - i_L = 225 - 160e^{-200t}\,\text{mA}, \qquad t \geq 0^+$

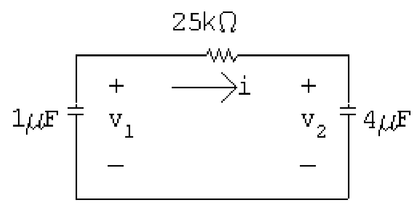
P 7.21 **[a]** $v_1(0^-) = v_1(0^+) = 40\,\text{V}$ $\qquad v_2(0^+) = 0$

$C_{\text{eq}} = (1)(4)/5 = 0.8\,\mu\text{F}$



$\tau = (25 \times 10^3)(0.8 \times 10^{-6}) = 20\text{ms}; \qquad \frac{1}{\tau} = 50$

$i = \frac{40}{25{,}000}e^{-50t} = 1.6e^{-50t}\,\text{mA}, \qquad t \geq 0^+$



$v_1 = \frac{-1}{10^{-6}}\int_0^t 1.6 \times 10^{-3}e^{-50x}\,dx + 40 = 32e^{-50t} + 8\,\text{V}, \qquad t \geq 0$

$v_2 = \frac{1}{4 \times 10^{-6}}\int_0^t 1.6 \times 10^{-3}e^{-50x}\,dx + 0 = -8e^{-50t} + 8\,\text{V}, \qquad t \geq 0$

[b] $w(0) = \frac{1}{2}(10^{-6})(40)^2 = 800\,\mu\text{J}$

[c] $w_{\text{trapped}} = \frac{1}{2}(10^{-6})(8)^2 + \frac{1}{2}(4 \times 10^{-6})(8)^2 = 160\,\mu\text{J}.$

The energy dissipated by the 25 kΩ resistor is equal to the energy dissipated by the two capacitors; it is easier to calculate the energy dissipated by the capacitors (final voltage on the equivalent capacitor is zero):

$w_{\text{diss}} = \frac{1}{2}(0.8 \times 10^{-6})(40)^2 = 640\,\mu\text{J}.$

Check: $\quad w_{\text{trapped}} + w_{\text{diss}} = 160 + 640 = 800\,\mu\text{J}; \qquad w(0) = 800\,\mu\text{J}.$

P 7.22 **[a]** Calculate the initial voltage drop across the capacitor:

$v(0) = (2.7\,\text{k}\|3.3\,\text{k})(40\,\text{mA}) = (1485)(40 \times 10^{-3}) = 59.4\,\text{V}$

The equivalent resistance seen by the capacitor is

$R_e = 3\,\text{k}\|(2.4\,\text{k} + 3.6\,\text{k}) = 3\,\text{k}\|6\,\text{k} = 2\,\text{k}\Omega$

$\tau = R_eC = (2000)(0.5) \times 10^{-6} = 1000\,\mu\text{s}; \qquad \frac{1}{\tau} = 1000$

$v = v(0)e^{-t/\tau} = 59.4e^{-1000t}\,\text{V} \qquad t \geq 0$

$i_o = \frac{v}{2.4\,\text{k} + 3.6\,\text{k}} = 9.9e^{-1000t}\,\text{mA}, \quad t \geq 0^+$

[b] $w(0) = \frac{1}{2}(0.5\times10^{-6})(59.4)^2 = 882.09\,\mu\text{J}$

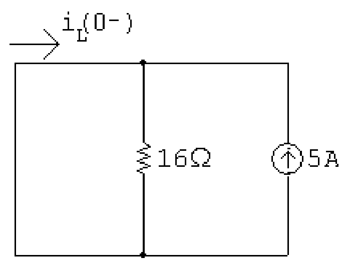
$$i_{3k} = \frac{59.4e^{-1000t}}{3000} = 19.8e^{-1000t}\text{ mA}$$

$$p_{3k} = [(19.8\times10^{-3})e^{-1000t}]^2(3000) = 1.176e^{-2000t}$$

$$w_{3k}(500\,\mu\text{s}) = 1.176\frac{e^{-2000x}}{-2000}\Big|_0^{500\times10^{-6}} = \frac{1.176}{-2000}(e^{-1}-1) = 371.72\,\mu\text{J}$$

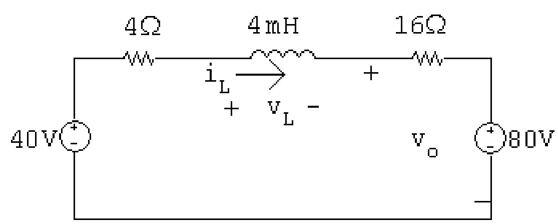
$$\% = \frac{371.72}{882.09}\times100 = 42.14\%$$

P 7.33 **[a]** $t < 0$



$$i_L(0^-) = -5\text{ A}$$

$t > 0$



$$i_L(\infty) = \frac{40-80}{4+16} = -2\text{ A}$$

$$\tau = \frac{L}{R} = \frac{4\times10^{-3}}{4+16} = 200\,\mu\text{s}; \qquad \frac{1}{\tau} = 5000$$

$$i_L = i_L(\infty) + [i_L(0^+) - i_L(\infty)]e^{-t/\tau}$$

$$= -2 + (-5+2)e^{-5000t} = -2 - 3e^{-5000t}\text{ A}, \qquad t \ge 0$$

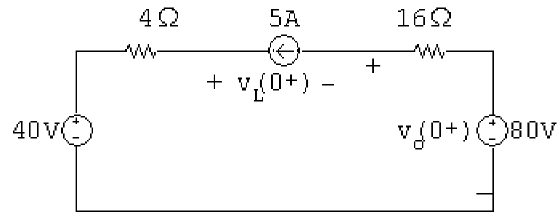
$$v_o = 16i_L + 80 = 16(-2-3e^{-5000t}) + 80 = 48 - 48e^{-5000t}\text{ V}, \qquad t \ge 0^+$$

[b] $v_L = L\dfrac{di_L}{dt} = 4 \times 10^{-3}(-5000)[-3e^{-5000t}] = 60e^{-5000t}\,\text{V}, \qquad t \geq 0^+$

$v_L(0^+) = 60\,\text{V}$

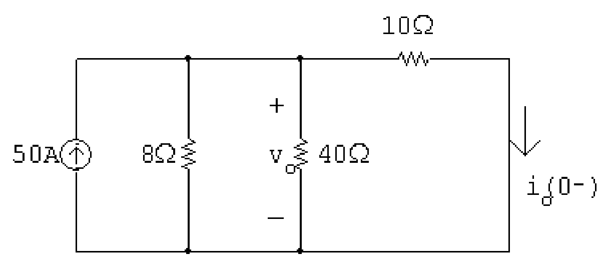
From part (a) $\quad v_o(0^+) = 0\,\text{V}$

Check: at $t = 0^+$ the circuit is:



$v_L(0^+) = 40 + (5\,\text{A})(4\,\Omega) = 60\,\text{V}, \qquad v_o(0^+) = 80 - (16\,\Omega)(5\,\text{A}) = 0\,\text{V}$

P 7.34 **[a]** $t < 0$



KVL equation at the top node:

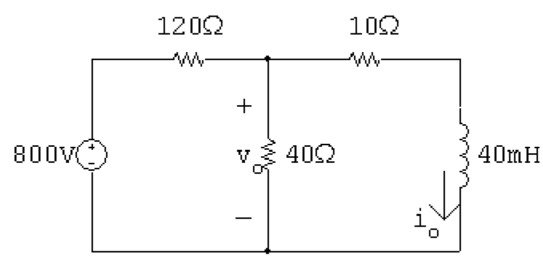
$$50 = \frac{v_o}{8} + \frac{v_o}{40} + \frac{v_o}{10}$$

Multiply by 40 and solve:

$2000 = (5 + 1 + 4)v_o; \qquad v_o = 200\,\text{V}$

$\therefore \quad i_o(0^-) = \dfrac{v_o}{10} = 200/10 = 20\,\text{A}$

$t > 0$



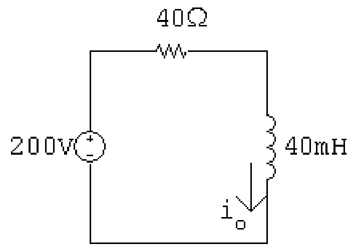
Use voltage division to find the Thévenin voltage:

$$V_{\text{Th}} = v_o = \frac{40}{40 + 120}(800) = 200\,\text{V}$$

Remove the voltage source and make series and parallel combinations of resistors to find the equivalent resistance:

$R_{\text{Th}} = 10 + 120\|40 = 10 + 30 = 40\,\Omega$

The simplified circuit is:



$$\tau = \frac{L}{R} = \frac{40 \times 10^{-3}}{40} = 1\,\text{ms}; \qquad \frac{1}{\tau} = 1000$$

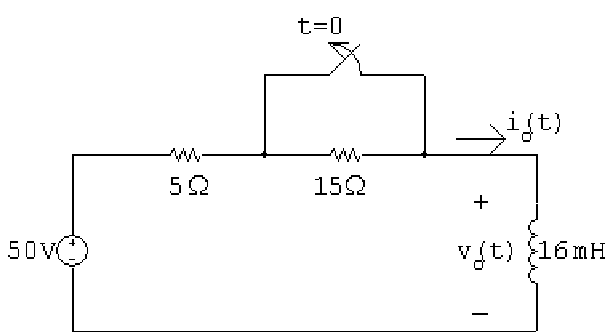
$$i_o(\infty) = \frac{200}{40} = 5\,\text{A}$$

$$\therefore \quad i_o = i_o(\infty) + [i_o(0^+) - i_o(\infty)]e^{-t/\tau}$$

$$= 5 + (20 - 5)e^{-1000t} = 5 + 15e^{-1000t}\,\text{A}, \qquad t \geq 0$$

[b]
$$v_o = 10i_o + L\frac{di_o}{dt}$$
$$= 10(5 + 15e^{-1000t}) + 0.04(-1000)(15e^{-1000t})$$
$$= 50 + 150e^{-1000t} - 600e^{-1000t}$$
$$v_o = 50 - 450e^{-1000t}\,\text{V}, \qquad t \geq 0^+$$

P 7.35 After making a Thévenin equivalent we have



For $t < 0$, the $15\,\Omega$ resistor is bypassed:

$$i_o(0^-) = i_o(0^+) = 50/5 = 10\,\text{A}$$

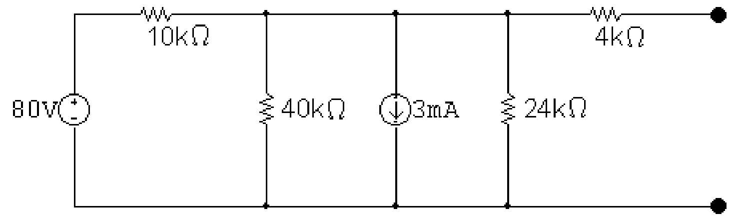
$$\tau = \frac{L}{R} = \frac{16 \times 10^{-3}}{5 + 15} = 8 \times 10^{-4}; \qquad \frac{1}{\tau} = 1250$$

$$i(\infty) = \frac{V}{R_{\text{eq}}} = \frac{50}{5 + 15} = 2.5\,\text{A}$$

$$i_o = i_o(\infty) + [i_o(0^+) - i_o(\infty)]e^{-t/\tau} = 2.5 + (10 - 2.5)e^{-1250t} = 2.5 + 7.5e^{-1250t}\,\text{A}, t \geq 0$$

$$v_o = L\frac{di_o}{dt} = 16 \times 10^{-3}(-1250)(7.5e^{-1250t}) = -150e^{-1250t}\,\text{V}, \quad t \geq 0^+$$

P 7.47 For $t < 0$



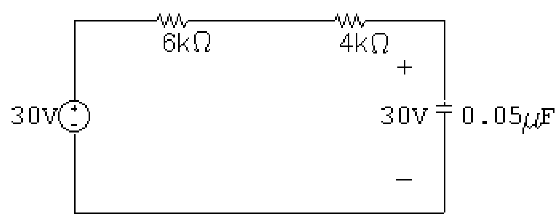
Simplify the circuit:

$80/10{,}000 = 8\,\text{mA}, \qquad 10\,\text{k}\Omega \| 40\,\text{k}\Omega \| 24\,\text{k}\Omega = 6\,\text{k}\Omega$

$8\,\text{mA} - 3\,\text{mA} = 5\,\text{mA}$

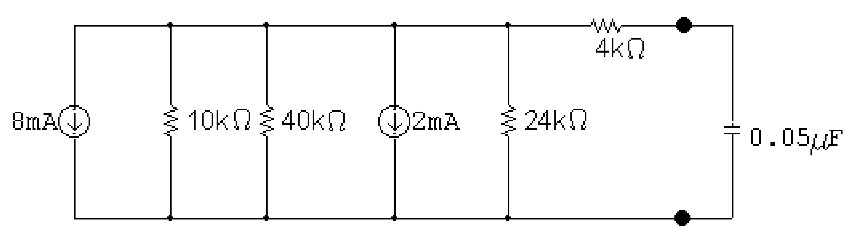
$5\,\text{mA} \times 6\,\text{k}\Omega = 30\,\text{V}$

Thus, for $t < 0$



$\therefore \quad v_o(0^-) = v_o(0^+) = 30\,\text{V}$

$t > 0$



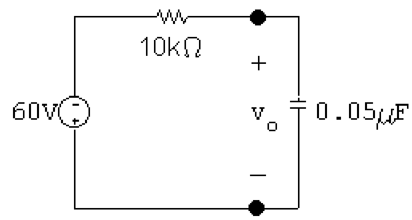
Simplify the circuit:

$8\,\text{mA} + 2\,\text{mA} = 10\,\text{mA}$

$10\,\text{k} \| 40\,\text{k} \| 24\,\text{k} = 6\,\text{k}\Omega$

$(10\,\text{mA})(6\,\text{k}\Omega) = 60\,\text{V}$

Thus, for $t > 0$



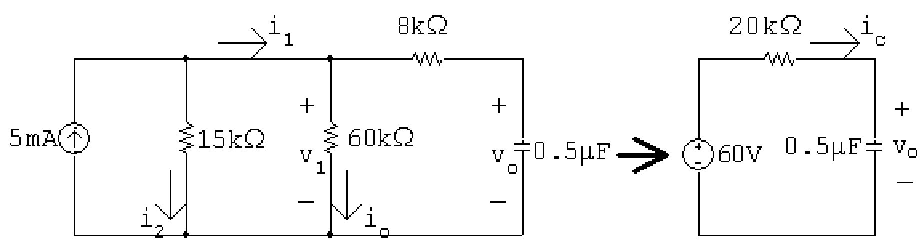
$v_o(\infty) = -60\,\text{V}$

$\tau = RC = (10\,\text{k})(0.05\,\mu) = 0.5\,\text{ms}; \qquad \frac{1}{\tau} = 2000$

$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau} = -60 + [30 - (-60)]e^{-2000t}$

$\quad = -60 + 90e^{-2000t}\,\text{V} \qquad t \geq 0$

P 7.48 **[a]** Simplify the circuit for $t > 0$ using source transformation:



Since there is no source connected to the capacitor for $t < 0$

$v_o(0^-) = v_o(0^+) = 0\,\text{V}$

From the simplified circuit,

$v_o(\infty) = 60\,\text{V}$

$\tau = RC = (20 \times 10^3)(0.5 \times 10^{-6}) = 10\,\text{ms} \qquad 1/\tau = 100$

$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau} = (60 - 60e^{-100t})\,\text{V}, \qquad t \geq 0$

[b] $i_c = C\frac{dv_o}{dt}$

$i_c = 0.5 \times 10^{-6}(-100)(-60e^{-100t}) = 3e^{-100t}\,\text{mA}$

$v_1 = 8000i_c + v_o = (8000)(3 \times 10^{-3})e^{-100t} + (60 - 60e^{-100t}) = 60 - 36e^{-100t}\,\text{V}$

$i_o = \frac{v_1}{60 \times 10^3} = 1 - 0.6e^{-100t}\,\text{mA}, \qquad t \geq 0^+$

[c] $i_1(t) = i_o + i_c = 1 + 2.4e^{-100t}\,\text{mA} \qquad t \geq 0^+$

[d] $i_2(t) = \dfrac{v_1}{15 \times 10^3} = 4 - 2.4e^{-100t}\,\text{mA} \qquad t \geq 0^+$

[e] $i_1(0^+) = 1 + 2.4 = 3.4\,\text{mA}$

At $t = 0^+$:

$$R_e = 15\,\text{k}\|60\,\text{k}\|8\,\text{k} = 4800\,\Omega$$

$$v_1(0^+) = (5 \times 10^{-3})(4800) = 24\,\text{V}$$

$$i_1(0^+) = \frac{v_1(0^+)}{60{,}000} + \frac{v_1(0^+)}{8000} = 0.4\,\text{m} + 3\,\text{m} = 3.4\text{mA} \quad \text{(checks)}$$

P 7.69 Use voltage division to find the initial voltage:

$$v_o(0) = \frac{60}{40 + 60}(50) = 30\,\text{V}$$

Use Ohm's law to find the final value of voltage:

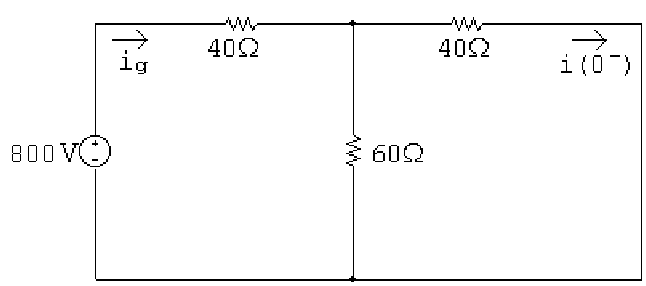
$$v_o(\infty) = (-5\,\text{mA})(20\,\text{k}\Omega) = -100\,\text{V}$$

$$\tau = RC = (20 \times 10^3)(250 \times 10^{-9}) = 5\,\text{ms}; \qquad \frac{1}{\tau} = 200$$

$$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau}$$

$$= -100 + (30 + 100)e^{-200t} = -100 + 130e^{-200t}\,\text{V}, \qquad t \geq 0$$

P 7.70 **[a]** $t < 0$:



Using Ohm's law,

$$i_g = \frac{800}{40 + 60\|40} = 12.5\,\text{A}$$

Using current division,

$$i(0^-) = \frac{60}{60 + 40}(12.5) = 7.5\,\text{A} = i(0^+)$$

[b] $0 \le t \le 1\,$ms:

$$i = i(0^+)e^{-t/\tau} = 7.5e^{-t/\tau}$$

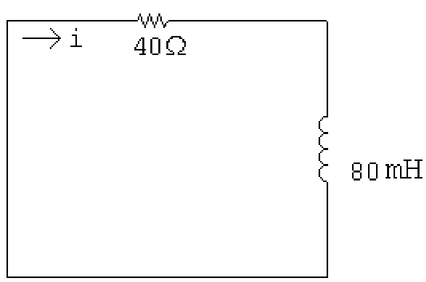
$$\frac{1}{\tau} = \frac{R}{L} = \frac{40 + 120\|60}{80 \times 10^{-3}} = 1000$$

$$i = 7.5e^{-1000t}$$

$$i(200\mu\text{s}) = 7.5e^{-10^3(200\times10^{-6})} = 7.5e^{-0.2} = 6.14\,\text{A}$$

[c] $i(1\text{ms}) = 7.5e^{-1} = 2.7591\,\text{A}$

$1\,\text{ms} \le t < \infty$



$$\frac{1}{\tau} = \frac{R}{L} = \frac{40}{80 \times 10^{-3}} = 500$$

$$i = i(1\,\text{ms})e^{-(t-1\,ms)/\tau} = 2.7591e^{-500(t-0.001)}\,\text{A}$$

$$i(6\text{ms}) = 2.7591e^{-500(0.005)} = 2.7591e^{-2.5} = 226.48\,\text{mA}$$

[d] $0 \le t \le 1\,$ms:

$$i = 7.5e^{-1000t}$$

$$v = L\frac{di}{dt} = (80 \times 10^{-3})(-1000)(7.5e^{-1000t}) = -600e^{-1000t}\,\text{V}$$

$$v(1^-\text{ms}) = -600e^{-1} = -220.73\,\text{V}$$

[e] $1\,\text{ms} \le t \le \infty$:

$$i = 2.7591e^{-500(t-0.001)}$$

$$v = L\frac{di}{dt} = (80 \times 10^{-3})(-500)(2.591e^{-500(t-0.001)})$$

$$= -110.4e^{-500(t-0.001)}\,\text{V}$$

$$v(1^+\text{ms}) = -110.4\,\text{V}$$

P 7.89 Use voltage division to find the voltage at the non-inverting terminal:

$$v_p = \frac{80}{100}(-45) = -36\,\text{V} = v_n$$

Write a KCL equation at the inverting terminal:

$$\frac{-36-14}{80{,}000} + 2.5\times10^{-6}\frac{d}{dt}(-36-v_o) = 0$$

$$\therefore \qquad 2.5\times10^{-6}\frac{dv_o}{dt} = \frac{-50}{80{,}000}$$

Separate the variables and integrate:

$$\frac{dv_o}{dt} = -250 \qquad \therefore \qquad dv_o = -250dt$$

$$\int_{v_o(0)}^{v_o(t)} dx = -250\int_0^t dy \qquad \therefore \qquad v_o(t) - v_o(0) = -250t$$

$$v_o(0) = -36+56 = 20\,\text{V}$$

$$v_o(t) = -250t + 20$$

Find the time when the voltage reaches 0:

$$0 = -250t + 20 \qquad \therefore \qquad t = \frac{20}{250} = 80\,\text{ms}$$

P 7.90 The equation for an integrating amplifier:

$$v_o = \frac{1}{RC}\int_0^t (v_\text{b} - v_\text{a})\,dy + v_o(0)$$

Find the values and substitute them into the equation:

$$RC = (100\times10^3)(0.05\times10^{-6}) = 5\,\text{ms}$$

$$\frac{1}{RC} = 200; \qquad v_\text{b} - v_\text{a} = -15-(-7) = -8\,\text{V}$$

$$v_o(0) = -4+12 = 8\,\text{V}$$

$$v_o = 200\int_0^t -8\,dx + 8 = (-1600t+8)\,\text{V}, \quad 0\le t\le t_\text{sat}$$

RC circuit analysis for v_2:

$$v_2(0^+) = -4\,\text{V}; \qquad v_2(\infty) = -15\,\text{V}; \qquad \tau = RC = (100\,\text{k})(0.05\,\mu) = 5\,\text{ms}$$

$$v_2 = v_2(\infty) + [v_2(0^+) - v_2(\infty)]e^{-t/\tau}$$

$$= -15 + (-4 + 15)e^{-200t} = -15 + 11e^{-200t}\,\text{V}, \quad 0 \le t \le t_{\text{sat}}$$

$$v_f + v_2 = v_o \quad \therefore \quad v_f = v_o - v_2 = 23 - 1600t - 11e^{-200t}\,\text{V}, \quad 0 \le t \le t_{\text{sat}}$$

Note that

$$-1600t_{\text{sat}} + 8 = -20 \quad \therefore \quad t_{\text{sat}} = \frac{-28}{-1600} = 17.5\,\text{ms}$$

so the op amp operates in its linear region until it saturates at 17.5 ms.