8

Natural and Step Responses of *RLC* Circuits

Assessment Problems

AP 8.1 [a] $\frac{1}{(2RC)^2} = \frac{1}{LC}$, therefore $C = 500\,\text{nF}$

[b] $\alpha = 5000 = \frac{1}{2RC}$, therefore $C = 1\,\mu\text{F}$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - \frac{(10^3)(10^6)}{20}} = (-5000 \pm j5000)\ \text{rad/s}$$

[c] $\frac{1}{\sqrt{LC}} = 20{,}000$, therefore $C = 125\,\text{nF}$

$$s_{1,2} = \left[-40 \pm \sqrt{(40)^2 - 20^2}\right] 10^3,$$

$$s_1 = -5.36\,\text{krad/s}, \qquad s_2 = -74.64\,\text{krad/s}$$

AP 8.2
$$\begin{aligned}
i_L &= \frac{1}{50 \times 10^{-3}} \int_0^t [-14e^{-5000x} + 26e^{-20{,}000x}]\,dx + 30 \times 10^{-3} \\
&= 20\left\{ \frac{-14e^{-5000x}}{-5000}\Bigg|_0^t + \frac{26e^{-20{,}000t}}{-20{,}000}\Bigg|_0^t \right\} + 30 \times 10^{-3} \\
&= 56 \times 10^{-3}(e^{-5000t} - 1) - 26 \times 10^{-3}(e^{-20{,}000t} - 1) + 30 \times 10^{-3} \\
&= [56e^{-5000t} - 56 - 26e^{-20{,}000t} + 26 + 30]\,\text{mA} \\
&= 56e^{-5000t} - 26e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0
\end{aligned}$$

AP 8.3 From the given values of R, L, and C, $s_1 = -10\,\text{krad/s}$ and $s_2 = -40\ \text{krad/s}$.

[a] $v(0^-) = v(0^+) = 0$, therefore $i_R(0^+) = 0$

[b] $i_C(0^+) = -(i_L(0^+) + i_R(0^+)) = -(-4+0) = 4\,\text{A}$

[c] $C\dfrac{dv_c(0^+)}{dt} = i_c(0^+) = 4, \qquad \text{therefore} \quad \dfrac{dv_c(0^+)}{dt} = \dfrac{4}{C} = 4\times 10^8\,\text{V/s}$

[d] $v = [A_1e^{-10{,}000t} + A_2e^{-40{,}000t}]\,\text{V}, \qquad t \geq 0^+$

$$v(0^+) = A_1 + A_2, \qquad \frac{dv(0^+)}{dt} = -10{,}000A_1 - 40{,}000A_2$$

$$\text{Therefore} \quad A_1 + A_2 = 0, \qquad -A_1 - 4A_2 = 40{,}000; \qquad A_1 = 40{,}000/3\,\text{V}$$

[e] $A_2 = -40{,}000/3\,\text{V}$

[f] $v = [40{,}000/3][e^{-10{,}000t} - e^{-40{,}000t}]\,\text{V}, \qquad t \geq 0$

AP 8.4 [a] $\dfrac{1}{2RC} = 8000, \qquad \text{therefore} \quad R = 62.5\,\Omega$

[b] $i_R(0^+) = \dfrac{10\,\text{V}}{62.5\,\Omega} = 160\,\text{mA}$

$$i_C(0^+) = -(i_L(0^+) + i_R(0^+)) = -80 - 160 = -240\,\text{mA} = C\frac{dv(0^+)}{dt}$$

$$\text{Therefore} \quad \frac{dv(0^+)}{dt} = \frac{-240\,\text{m}}{C} = -240\,\text{kV/s}$$

[c] $B_1 = v(0^+) = 10\,\text{V}, \qquad \dfrac{dv_c(0^+)}{dt} = \omega_d B_2 - \alpha B_1$

$$\text{Therefore} \quad 6000B_2 - 8000B_1 = -240{,}000, \qquad B_2 = (-80/3)\,\text{V}$$

[d] $i_L = -(i_R + i_C); \qquad i_R = v/R; \qquad i_C = C\dfrac{dv}{dt}$

$$v = e^{-8000t}[10\cos 6000t - \frac{80}{3}\sin 6000t]\,\text{V}$$

$$\text{Therefore} \quad i_R = e^{-8000t}[160\cos 6000t - \frac{1280}{3}\sin 6000t]\,\text{mA}$$

$$i_C = e^{-8000t}[-240\cos 6000t + \frac{460}{3}\sin 6000t]\,\text{mA}$$

$$i_L = 10e^{-8000t}[8\cos 6000t + \frac{82}{3}\sin 6000t]\,\text{mA}, \qquad t \geq 0$$

AP 8.5 [a] $\left(\dfrac{1}{2RC}\right)^2 = \dfrac{1}{LC} = \dfrac{10^6}{4}, \qquad \text{therefore} \quad \dfrac{1}{2RC} = 500, \qquad R = 100\,\Omega$

[b] $0.5CV_0^2 = 12.5\times 10^{-3}, \qquad \text{therefore} \quad V_0 = 50\,\text{V}$

[c] $0.5LI_0^2 = 12.5\times 10^{-3}, \qquad I_0 = 250\,\text{mA}$

[d] $D_2 = v(0^+) = 50, \qquad \frac{dv(0^+)}{dt} = D_1 - \alpha D_2$

$$i_R(0^+) = \frac{50}{100} = 500\,\text{mA}$$

Therefore $\quad i_C(0^+) = -(500 + 250) = -750\,\text{mA}$

Therefore $\quad \frac{dv(0^+)}{dt} = -750 \times \frac{10^{-3}}{C} = -75{,}000\,\text{V/s}$

Therefore $\quad D_1 - \alpha D_2 = -75{,}000; \qquad \alpha = \frac{1}{2RC} = 500, \quad D_1 = -50{,}000\ \text{V/s}$

[e] $v = [50e^{-500t} - 50{,}000te^{-500t}]\,\text{V}$

$$i_R = \frac{v}{R} = [0.5e^{-500t} - 500te^{-500t}]\,\text{A}, \qquad t \geq 0^+$$

AP 8.6 [a] $i_R(0^+) = \frac{V_0}{R} = \frac{40}{500} = 0.08\,\text{A}$

[b] $i_C(0^+) = I - i_R(0^+) - i_L(0^+) = -1 - 0.08 - 0.5 = -1.58\,\text{A}$

[c] $\frac{di_L(0^+)}{dt} = \frac{V_o}{L} = \frac{40}{0.64} = 62.5\,\text{A/s}$

[d] $\alpha = \frac{1}{2RC} = 1000; \qquad \frac{1}{LC} = 1{,}562{,}500; \qquad s_{1,2} = -1000 \pm j750\ \text{rad/s}$

[e] $i_L = i_f + B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t, \qquad i_f = I = -1\,\text{A}$

$i_L(0^+) = 0.5 = i_f + B_1', \qquad$ therefore $\quad B_1' = 1.5\,\text{A}$

$\frac{di_L(0^+)}{dt} = 62.5 = -\alpha B_1' + \omega_d B_2', \qquad$ therefore $\quad B_2' = (25/12)\,\text{A}$

Therefore $\quad i_L(t) = -1 + e^{-1000t}[1.5\cos 750t + (25/12)\sin 750t]\,\text{A}, \qquad t \geq 0$

[f] $v(t) = \frac{L\,di_L}{dt} = 40e^{-1000t}[\cos 750t - (154/3)\sin 750t]V \qquad t \geq 0$

AP 8.7 [a] $i(0^+) = 0$, since there is no source connected to L for $t < 0$.

[b] $v_c(0^+) = v_C(0^-) = \left(\frac{15\,\text{k}}{15\,\text{k} + 9\,\text{k}}\right)(80) = 50\,\text{V}$

[c] $50 + 80i(0^+) + L\frac{di(0^+)}{dt} = 100, \qquad \frac{di(0^+)}{dt} = 10{,}000\,\text{A/s}$

[d] $\alpha = 8000; \qquad \frac{1}{LC} = 100 \times 10^6; \qquad s_{1,2} = -8000 \pm j6000\ \text{rad/s}$

[e] $i = i_f + e^{-\alpha t}[B_1'\cos\omega_d t + B_2'\sin\omega_d t]; \qquad i_f = 0, \quad i(0^+) = 0$

Therefore $\quad B_1' = 0; \qquad \frac{di(0^+)}{dt} = 10{,}000 = -\alpha B_1' + \omega_d B_2'$

Therefore $\quad B_2' = 1.67\,\text{A}; \qquad i = 1.67e^{-8000t}\sin 6000t\,\text{A}, \qquad t \geq 0$

AP 8.8 $v_c(t) = v_f + e^{-\alpha t}[B_1' \cos\omega_d t + B_2' \sin\omega_d t], \qquad v_f = 100\,\text{V}$

$$v_c(0^+) = 50\,\text{V}; \qquad \frac{dv_c(0^+)}{dt} = 0; \qquad \text{therefore} \quad 50 = 100 + B_1'$$

$$B_1' = -50\,\text{V}; \qquad 0 = -\alpha B_1' + \omega_d B_2'$$

$$\text{Therefore} \quad B_2' = \frac{\alpha}{\omega_d}B_1' = \left(\frac{8000}{6000}\right)(-50) = -66.67\,\text{V}$$

$$\text{Therefore} \quad v_c(t) = 100 - e^{-8000t}[50\cos 6000t + 66.67\sin 6000t]\,\text{V}, \qquad t \geq 0$$

Problems

P 8.1 [a] $\alpha = \dfrac{1}{2RC} = \dfrac{10^9}{(10{,}000)(8)} = 12{,}500$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(1.25)(8)} = 10^8$$

$$s_{1,2} = -12{,}500 \pm \sqrt{(1.5625 - 1)10^8} = -12{,}500 \pm 7500$$

$$s_1 = -5000\ \text{rad/s}$$

$$s_2 = -20{,}000\ \text{rad/s}$$

[b] overdamped

[c] $\omega_d = \sqrt{\omega_o^2 - \alpha^2}$

$$\therefore \quad \alpha^2 = \omega_o^2 - \omega_d^2 = 10^8 - 36 \times 10^6 = 0.64 \times 10^8$$

$$\alpha = 0.8 \times 10^4 = 8000$$

$$\frac{1}{2RC} = 8000; \qquad \therefore \quad R = \frac{10^9}{(16{,}000)(8)} = 7812.5\,\Omega$$

[d] $s_1 = -8000 + j6000$ rad/s; $\qquad s_2 = -8000 - j6000$ rad/s

[e] $\alpha = 10^4 = \dfrac{1}{2RC}; \qquad \therefore \quad R = \dfrac{1}{2C(10^4)} = 6250\,\Omega$

P 8.2 [a] $-\alpha + \sqrt{\alpha^2 - \omega_o^2} = -5000$

$$-\alpha - \sqrt{\alpha^2 - \omega_o^2} = -20{,}000$$

$$\therefore \quad -2\alpha = -25{,}000$$

$$\alpha = 12{,}500\,\text{rad/s}$$

$$\frac{1}{2RC} = \frac{10^6}{2R(0.05)} = 12{,}500$$

$$R = 800\,\Omega$$

$$2\sqrt{\alpha^2 - \omega_o^2} = 15{,}000$$

$$4(\alpha^2 - \omega_o^2) = 225 \times 10^6$$

$$\therefore \quad \omega_o = 10{,}000\,\text{rad/s}$$

$$\omega_o^2 = 10^8 = \frac{1}{LC}$$

$$\therefore \quad L = \frac{1}{10^8 C} = 200\,\text{mH}$$

[b] $i_\text{R} = \dfrac{v(t)}{R} = -6.25e^{-5000t} + 25e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0^+$

$$i_\text{C} = C\frac{dv(t)}{dt} = 1.25e^{-5000t} - 20e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_\text{L} = -(i_\text{R} + i_\text{C}) = 5e^{-5000t} - 5e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0^+$$

P 8.3 [a] $\alpha = 4000; \qquad \omega_d = 3000$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$$

$$\therefore \quad \omega_o^2 = \omega_d^2 + \alpha^2 = 9 \times 10^6 + 16 \times 10^6 = 25 \times 10^6$$

$$\frac{1}{LC} = 25 \times 10^6$$

$$L = \frac{1}{(25 \times 10^6)(50 \times 10^{-9})} = 0.8\,\text{H} = 800\,\text{mH}$$

[b] $\alpha = \dfrac{1}{2RC}$

$$\therefore \quad R = \frac{1}{2\alpha C} = \frac{10^9}{(8000)(50)} = 2500\,\Omega$$

[c] $V_o = v(0) = 125\,\text{V}$

[d] $I_o = i_L(0) = -i_R(0) - i_C(0)$

$$i_R(0) = \frac{V_o}{R} = \frac{125}{2.5} \times 10^{-3} = 50\,\text{mA}$$

$$i_C(0) = C\frac{dv}{dt}(0)$$

$$\frac{dv}{dt} = 125\{e^{-4000t}[-3000\sin 3000t - 6000\cos 3000t] -$$

$$4000e^{-4000t}[\cos 3000t - 2\sin 3000t]$$

$$\frac{dv}{dt}(0) = 125\{1(-6000) - 4000\} = -125 \times 10^4$$

$$C\frac{dv}{dt}(0) = -125 \times 10^4(50 \times 10^{-9}) = -6250 \times 10^{-5} = -62.5\,\text{mA}$$

$$\therefore\quad I_o = -50 + 62.5 = 12.5\,\text{mA}$$

[e]
$$\frac{dv}{dt} = 125e^{-4000t}[5000\sin 3000t - 10{,}000\cos 3000t]$$

$$= 625 \times 10^3 e^{-4000t}[\sin 3000t - 2\cos 3000t]$$

$$C\frac{dv}{dt} = 31{,}250 \times 10^{-6} e^{-4000t}(\sin 3000t - 2\cos 3000t)$$

$$i_C(t) = 31.25e^{-4000t}(\sin 3000t - 2\cos 3000t)\,\text{mA}$$

$$i_R(t) = 50e^{-4000t}(\cos 3000t - 2\sin 3000t)\,\text{mA}$$

$$i_L(t) = -i_R(t) - i_C(t)$$

$$= e^{-4000t}(12.5\cos 3000t + 68.75\sin 3000t)\,\text{mA},\qquad t \geq 0$$

CHECK:

$$\frac{di_L}{dt} = \{-4000e^{-4000t}[12.5\cos 3000t + 68.75\sin 3000t]$$

$$+e^{-4000t}[-37.5 \times 10^3 \sin 3000t$$

$$+206.25 \times 10^3 \cos 3000t] \times 10^{-3}$$

$$= e^{-4000t}[156.25\cos 3000t - 312.5\sin 3000t]$$

$$L\frac{di_L}{dt} = e^{-4000t}[125\cos 3000t - 250\sin 3000t]$$

$$= 125e^{-4000t}[\cos 3000t - 2\sin 3000t]\,\text{V}$$

P 8.4 [a] $\left(\frac{1}{2RC}\right)^2 = \frac{1}{LC} = (4000)^2$

$$\therefore \quad C = \frac{1}{(16 \times 10^6)(5)} = 12.5\,\text{nF}$$

$$\frac{1}{2RC} = 4000$$

$$\therefore \quad R = \frac{10^9}{(8000)(12.5)} = 10\,\text{k}\Omega$$

$$v(0) = D_2 = 25\,\text{V}$$

$$i_\text{R}(0) = \frac{25}{10} = 2.5\text{mA}$$

$$i_\text{C}(0) = -2.5 - 5 = -7.5\,\text{mA}$$

$$\frac{dv}{dt}(0) = D_1 - 4000D_2 = \frac{-7.5 \times 10^{-3}}{12.5 \times 10^{-9}} = -6 \times 10^5$$

$$\therefore \quad D_1 = -6 \times 10^5 + 4000(25) = -5 \times 10^5\ \text{V/s}$$

[b] $v = -5 \times 10^5 te^{-4000t} + 25e^{-4000t}$

$$\frac{dv}{dt} = [20 \times 10^8 t - 6 \times 10^5]e^{-4000t}$$

$$i_\text{C} = C\frac{dv}{dt} = 12.5 \times 10^{-9}[20 \times 10^8 t - 6 \times 10^5]e^{-4000t}$$

$$= (25{,}000t - 7.5)e^{-4000t}\,\text{mA}, \qquad t > 0$$

P 8.5 [a] $2\alpha = 200; \qquad \alpha = 100\,\text{rad/s}$

$$2\sqrt{\alpha^2 - \omega_o^2} = 120; \qquad \omega_o = 80\,\text{rad/s}$$

$$C = \frac{1}{2\alpha R} = \frac{1}{200(200)} = 25\,\mu F$$

$$L = \frac{1}{\omega_o^2 C} = \frac{10^6}{(80)^2(25)} = 6.25\,\text{H}$$

$$i_\text{C}(0^+) = A_1 + A_2 = 15\,\text{mA}$$

$$\frac{di_\text{C}}{dt} + \frac{di_\text{L}}{dt} + \frac{di_\text{R}}{dt} = 0$$

$$\frac{di_\text{C}(0)}{dt} = -\frac{di_\text{L}(0)}{dt} - \frac{di_\text{R}(0)}{dt}$$

$$\frac{di_L(0)}{dt} = \frac{0}{6.25} = 0\,\text{A/s}$$

$$\frac{di_R(0)}{dt} = \frac{1}{R}\frac{dv(0)}{dt} = \frac{1}{R}\frac{i_C(0)}{C} = \frac{15 \times 10^{-3}}{(200)(25 \times 10^{-6})} = 3\,\text{A/s}$$

$$\therefore \quad \frac{di_C(0)}{dt} = -3\,\text{A/s}$$

$$\therefore \quad 160A_1 + 40A_2 = 3$$

$$4A_1 + A_2 + = 75 \times 10^{-3}; \qquad \therefore \quad A_1 = 20\,\text{mA}; \qquad A_2 = -5\,\text{mA}$$

$$\therefore \quad i_C = 20e^{-160t} - 5e^{-40t}\,\text{mA}, \qquad t \geq 0$$

[b] By hypothesis

$$v = A_3e^{-160t} + A_4e^{-40t}, \qquad t \geq 0$$

$$v(0) = A_3 + A_4 = 0$$

$$\frac{dv(0)}{dt} = \frac{15 \times 10^{-3}}{25 \times 10^{-6}} = 600\,\text{V/s}$$

$$-160A_3 - 40A_4 = 600; \qquad \therefore \quad A_3 = -5\,\text{V}; \qquad A_4 = 5\,\text{V}$$

$$v = -5e^{-160t} + 5e^{-40t}\,\text{V}, \qquad t \geq 0$$

[c] $i_R(t) = \dfrac{v}{200} = -25e^{-160t} + 25e^{-40t}\,\text{mA}, \qquad t \geq 0^+$

[d] $i_L = -i_R - i_C$

$$i_L = 5e^{-160t} - 20e^{-40t}\,\text{mA}, \qquad t \geq 0$$

P 8.6 [a] $i_R(0) = \dfrac{90}{2000} = 45\text{mA}$

$$i_L(0) = -30\text{mA}$$

$$i_C(0) = -i_L(0) - i_R(0) = 30 - 45 = -15\,\text{mA}$$

[b] $\alpha = \dfrac{1}{2RC} = \dfrac{10^9}{(4000)(10)} = 25{,}000$

$$\omega_o^2 = \frac{1}{LC} = \frac{(10^3)(10^9)}{(250)(10)} = 4 \times 10^8$$

$$s_{1,2} = -25{,}000 \pm \sqrt{6.25 \times 10^8 - 10^8(4)} = -25{,}000 \pm 15{,}000$$

$$s_1 = -10{,}000\ \text{rad/s}; \qquad s_2 = -40{,}000\ \text{rad/s}$$

$$v = A_1e^{-10{,}000t} + A_2e^{-40{,}000t}$$

$$v(0) = A_1 + A_2 = 90$$

$$\frac{dv}{dt}(0) = -10^4 A_1 - 4A_2 \times 10^4 = \frac{-15 \times 10^{-3}}{10 \times 10^{-9}} = -1.5 \times 10^6 \text{V/s}$$

$$-A_1 - 4A_2 = -150$$

$$\therefore \quad -3A_2 = -60; \qquad A_2 = 20; \qquad A_1 = 70$$

$$v = 70e^{-10{,}000t} + 20e^{-40{,}000t}\,\text{V}, \qquad t \geq 0$$

[c] $$i_C = C\frac{dv}{dt}$$

$$= 10 \times 10^{-9}[-70 \times 10^4 e^{-10{,}000t} - 80 \times 10^4 e^{-40{,}000t}]$$

$$= -7e^{-10{,}000t} - 8e^{-40{,}000t}\,\text{mA}$$

$$i_R = 35e^{-10{,}000t} + 10e^{-40{,}000t}\,\text{mA}$$

$$i_L = -i_C - i_R = -28e^{-10{,}000t} - 2e^{-40{,}000t}\,\text{mA}, \quad t \geq 0$$

P 8.7 $$\alpha = \frac{1}{2RC} = \frac{10^9}{(5000)(10)} = 2 \times 10^4$$

$$\alpha^2 = 4 \times 10^8; \qquad \therefore \quad \alpha^2 = \omega_o^2$$

Critical damping:

$$v = D_1 te^{-\alpha t} + D_2 e^{-\alpha t}$$

$$i_R(0^+) = \frac{90}{2500} = 36\,\text{mA}$$

$$i_C(0^+) = -[i_L(0^+) + i_R(0^+)] = -[-30 + 36] = -6\,\text{mA}$$

$$v(0) = D_2 = 90$$

$$\frac{dv}{dt} = D_1[t(-\alpha e^{-\alpha t}) + e^{-\alpha t}] - \alpha D_2 e^{-\alpha t}$$

$$\frac{dv}{dt}(0) = D_1 - \alpha D_2 = \frac{i_C(0)}{C} = \frac{-6 \times 10^{-3}}{10 \times 10^{-9}} = -6 \times 10^5$$

$$D_1 = \alpha D_2 - 6 \times 10^5 = (2 \times 10^4)(90) - 6 \times 10^5 = 120 \times 10^4$$

$$v = (120 \times 10^4 t + 90)e^{-20{,}000t}\,\text{V}, \qquad t \geq 0$$

P 8.8 $$\frac{1}{2RC} = \frac{3 \times 10^9}{(25{,}000)(10)} = 12{,}000$$

$$\frac{1}{LC} = 4 \times 10^8$$

$$s_{1,2} = -12{,}000 \pm j16{,}000\,\text{rad/s}$$

$\therefore$ response is underdamped

$$v(t) = B_1 e^{-12{,}000t} \cos 16{,}000t + B_2 e^{-12{,}000t} \sin 16{,}000t$$

$$v(0^+) = 90\,\text{V} = B_1; \qquad i_\text{R}(0^+) = \frac{90}{(12{,}500/3)} = 21.6\,\text{mA}$$

$$i_\text{C}(0^+) = [-i_\text{L}(0^+) + i_\text{R}(0^+)] = -[-30 + 21.6] = 8.4\,\text{mA}$$

$$\frac{dv(0^+)}{dt} = \frac{8.4 \times 10^{-3}}{10 \times 10^{-9}} = 840{,}000\,\text{V/s}$$

$$\frac{dv(0)}{dt} = -12{,}000B_1 + 16{,}000B_2 = 840{,}000$$

$$\text{or } -3B_1 + 4B_2 = 210; \qquad \therefore \quad B_2 = 120\,\text{V}$$

$$v(t) = 90e^{-12{,}000t} \cos 16{,}000t + 120e^{-12{,}000t} \sin 16{,}000t\,\text{V}, \qquad t \geq 0$$

P 8.9 $$\alpha = 2000/2 = 1000$$

$$R = \frac{1}{2\alpha C} = \frac{10^6}{(2000)(18)} = 27.78\,\Omega$$

$$v(0^+) = -24\,\text{V}$$

$$i_\text{R}(0^+) = \frac{-24}{27.78} = -864\,\text{mA}$$

$$\frac{dv}{dt} = 2400e^{-200t} + 21{,}600e^{-1800t}$$

$$\frac{dv(0^+)}{dt} = 2400 + 21{,}600 = 24{,}000\,\text{V/s}$$

$$i_\text{C}(0^+) = 18 \times 10^{-6}(24{,}000) = 432\,\text{mA}$$

$$i_\text{L}(0^+) = -[i_\text{R}(0^+) + i_\text{C}(0^+)] = -[-864 + 432] = 432\,\text{mA}$$

P 8.10 [a] $\omega_o^2 = \frac{1}{LC} = \frac{10^9}{40} = 25 \times 10^6$

$\omega_o = 5000$ rad/s

$\frac{1}{2RC} = 5000; \qquad R = \frac{1}{10{,}000C}$

$R = \frac{10^9}{8 \times 10^4} = 12.5\,\text{k}\Omega$

[b] $v(t) = D_1 te^{-5000t} + D_2 e^{-5000t}$

$v(0) = -25\,\text{V} = D_2$

$\frac{dv}{dt} = (D_1 t - 25)(-5000e^{-5000t}) + D_1 e^{-5000t}$

$\frac{dv}{dt}(0) = 125 \times 10^3 + D_1 = \frac{i_C(0)}{C}$

$i_C(0) = -i_R(0) - i_L(0)$

$i_R(0) = \frac{-25}{12.5} = -2\,\text{mA}$

$\therefore \quad i_C(0) = 2 - (-1) = 3\,\text{mA}$

$\therefore \quad \frac{dv}{dt}(0) = \frac{3 \times 10^{-3}}{8 \times 10^{-9}} = 0.375 \times 10^6 = 3.75 \times 10^5$

$\therefore \quad 1.25 \times 10^5 + D_1 = 3.75 \times 10^5$

$D_1 = 2.5 \times 10^5 = 25 \times 10^4\,\text{V/s}$

$\therefore \quad v(t) = (25 \times 10^4 t - 25)e^{-5000t}\,\text{V}, \qquad t \geq 0$

[c] $i_C(t) = 0$ when $\frac{dv}{dt}(t) = 0$

$\frac{dv}{dt} = (25 \times 10^4 t - 25)(-5000)e^{-5000t} + e^{-5000t}(25 \times 10^4)$

$= (375{,}000 - 125 \times 10^7 t)e^{-5000t}$

$\frac{dv}{dt} = 0$ when $125 \times 10^7 t_1 = 375{,}000; \qquad \therefore \quad t_1 = 300\,\mu\text{s}$

$v(300\mu\text{s}) = 50e^{-1.5} = 11.16\,\text{V}$

[d] $i_{\rm L}(300\mu{\rm s}) = -i_{\rm R}(300\mu{\rm s}) = \frac{11.16}{12.5} = 0.89\,{\rm mA}$

$$\omega_{\rm C}(300\mu{\rm s}) = 4\times 10^{-9}(11.16)^2 = 497.87\,{\rm nJ}$$

$$\omega_{\rm L}(300\mu{\rm s}) = (2.5)(0.89)^2\times 10^{-6} = 1991.48\,{\rm nJ}$$

$$\omega(300\mu{\rm s}) = \omega_{\rm C} + \omega_{\rm L} = 2489.35\,{\rm nJ}$$

$$\omega(0) = 4\times 10^{-9}(625) + 2.5(10^{-6}) = 5000\,{\rm nJ}$$

$$\%\ \text{remaining} = \frac{2489.35}{5000}(100) = 49.79\%$$

P 8.11 [a] $\alpha = \frac{1}{2RC} = 1\ {\rm rad/s}$

$$\omega_o^2 = \frac{1}{LC} = 10$$

$$\omega_d = \sqrt{10-1} = 3\ {\rm rad/s}$$

$$\therefore\quad v = B_1e^{-t}\cos 3t + B_2e^{-t}\sin 3t$$

$$v(0) = B_1 = 0; \qquad v = B_2e^{-t}\sin 3t$$

$$i_R(0^+) = 0\,{\rm A}; \qquad i_C(0^+) = 3\,{\rm A}; \qquad \frac{dv}{dt}(0^+) = \frac{3}{0.25} = 12\ {\rm V/s}$$

$$12 = -\alpha B_1 + \omega_d B_2 = -1(0) + 3B_2$$

$$\therefore\quad B_2 = 4$$

$$\therefore\quad v = 4e^{-t}\sin 3t\,{\rm V}, \qquad t \geq 0$$

[b] $\frac{dv}{dt} = 4e^{-t}(3\cos 3t - \sin 3t)$

$$\frac{dv}{dt} = 0 \quad \text{when} \quad 3\cos 3t = \sin 3t \quad \text{or} \quad \tan 3t = 3$$

$$\therefore\quad 3t_1 = 1.25, \qquad t_1 = 416.35\,{\rm ms}$$

$$3t_2 = 1.25 + \pi, \qquad t_2 = 1463.55\,{\rm ms}$$

$$3t_3 = 1.25 + 2\pi, \qquad t_3 = 2510.74\,{\rm ms}$$

[c] $t_3 - t_1 = 2094.40\,{\rm ms}; \qquad T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{3} = 2094.40\,{\rm ms}$

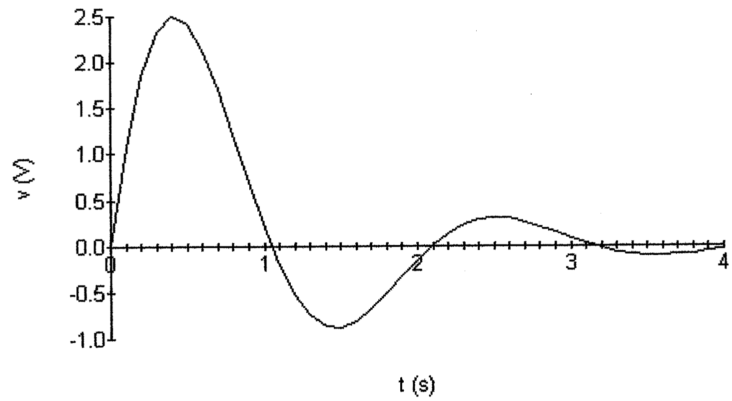
[d] $t_2 - t_1 = 1047.20\,{\rm ms}; \qquad \frac{T_d}{2} = \frac{2094.40}{2} = 1047.20\,{\rm ms}$

[e] $v(t_1) = 4e^{-(0.41635)}\sin 3(0.41635) = 2.50\,\text{V}$

$v(t_2) = 4e^{-(1.46355)}\sin 3(1.46355) = -0.88\,\text{V}$

$v(t_3) = 4e^{-(2.51074)}\sin 3(2.51074) = 0.31\,\text{V}$

[f]



P 8.12 [a] $\alpha = 0; \qquad \omega_d = \omega_o = \sqrt{10} = 3.16\,\text{rad/s}$

$v = B_1\cos\omega_o t + B_2\sin\omega_o t; \qquad v(0) = B_1 = 0; \qquad v = B_2\sin\omega_o t$

$$C\frac{dv}{dt}(0) = -i_L(0) = 3$$

$$12 = -\alpha B_1 + \omega_d B_2 = -0 + \sqrt{10}B_2$$

$$\therefore \quad B_2 = 12/\sqrt{10} = 3.79\,\text{V}$$

$$v = 3.79\sin 3.16t\,\text{V}, \qquad t \geq 0$$

[b] $2\pi f = 3.16; \qquad f = \dfrac{3.16}{2\pi} \cong 0.50\,\text{Hz}$

[c] 3.79 V

P 8.13 From the form of the solution we have

$$v(0) = A_1 + A_2$$

$$\frac{dv(0^+)}{dt} = -\alpha(A_1 + A_2) + j\omega_d(A_1 - A_2)$$

We know both $v(0)$ and $dv(0^+)/dt$ will be real numbers. To facilitate the algebra we let these numbers be K_1 and K_2, respectively. Then our two simultaneous equations are

$$K_1 = A_1 + A_2$$

$$K_2 = (-\alpha + j\omega_d)A_1 + (-\alpha - j\omega_d)A_2$$

The characteristic determinate is

$$\Delta = \begin{vmatrix} 1 & 1 \\ (-\alpha + j\omega_d) & (-\alpha - j\omega_d) \end{vmatrix} = -j2\omega_d$$

The numerator determinates are

$$N_1 = \begin{vmatrix} K_1 & 1 \\ K_2 & (-\alpha - j\omega_d) \end{vmatrix} = -(\alpha + j\omega_d)K_1 - K_2$$

and $N_2 = \begin{vmatrix} 1 & K_1 \\ (-\alpha + j\omega_d) & K_2 \end{vmatrix} = K_2 + (\alpha - j\omega_d)K_1$

It follows that $A_1 = \dfrac{N_1}{\Delta} = \dfrac{\omega_d K_1 - j(\alpha K_1 + K_2)}{2\omega_d}$

and $A_2 = \dfrac{N_2}{\Delta} = \dfrac{\omega_d K_1 + j(\alpha K_1 + K_2)}{2\omega_d}$

We see from these expressions that $A_1 = A_2^*$

P 8.14 By definition, $B_1 = A_1 + A_2$. From the solution to Problem 8.13 we have

$$A_1 + A_2 = \frac{2\omega_d K_1}{2\omega_d} = K_1$$

But K_1 is $v(0)$, therefore, $B_1 = v(0)$, which is identical to Eq. (8.30). By definition, $B_2 = j(A_1 - A_2)$. From Problem 8.13 we have

$$B_2 = j(A_1 - A_2) = \frac{j[-2j(\alpha K_1 + K_2)]}{2\omega_d} = \frac{\alpha K_1 + K_2}{\omega_d}$$

It follows that

$$K_2 = -\alpha K_1 + \omega_d B_2, \quad \text{but} \quad K_2 = \frac{dv(0^+)}{dt} \quad \text{and} \quad K_1 = B_1$$

Thus we have

$$\frac{dv}{dt}(0^+) = -\alpha B_1 + \omega_d B_2,$$

which is identical to Eq. (8.31).

P 8.15 [a] $\alpha = \dfrac{1}{2RC} = 1000\sqrt{2}, \qquad \omega_o = 10^3,$ therefore overdamped

$s_1 = -414.21, \qquad s_2 = -2414.21$

therefore $\quad v = A_1 e^{-414.21t} + A_2 e^{-2414.21t}$

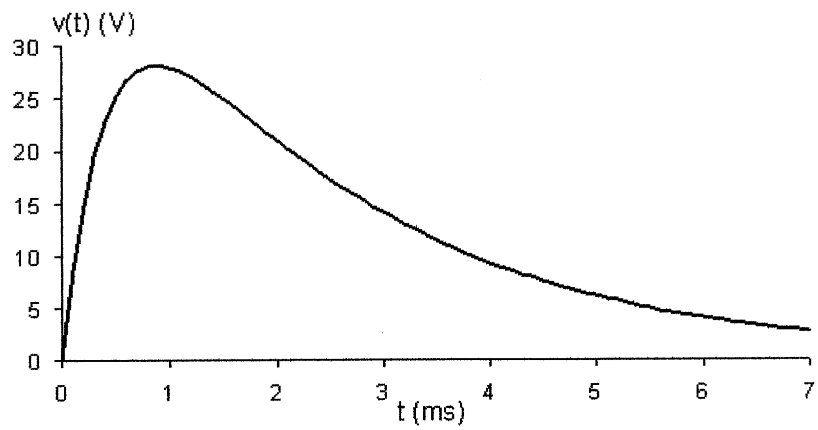
$v(0^+) = 0 = A_1 + A_2; \qquad \left[\dfrac{dv(0^+)}{dt}\right] = \dfrac{i_C(0^+)}{C} = 98{,}000\,\text{V/s}$

Therefore $\quad -414.21A_1 - 2414.21A_2 = 98{,}000$

$A_1 = 49, \quad A_2 = -49$

$v(t) = 49[e^{-414.21t} - e^{-2414.21t}]\,\text{V}, \qquad t \geq 0$

[b]

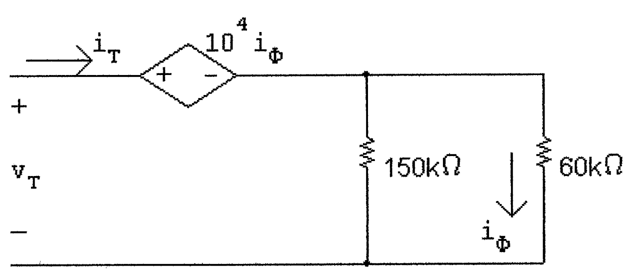


Example 8.4: $v_{\max} \cong 74.1\,\text{V}$ at 1.4 ms

Example 8.5: $v_{\max} \cong 36.1\,\text{V}$ at 1.0 ms

Problem 8.15: $v_{\max} \cong 28.2\,\text{V}$ at 0.9 ms

P 8.16



$$v_T = 10^4 \frac{i_T(150\times10^3)}{210\times10^3} + \frac{(150)(60)10^6}{210\times10^3} i_T$$

$$\frac{v_T}{i_T} = \frac{1500\times10^3}{210} + \frac{9000\times10^3}{210} = \frac{10{,}500}{210}\times10^3 = 50\,\text{k}\Omega$$

$$V_o = \frac{75}{10}(6) = 45\,\text{V}; \qquad I_o = 0$$

$$i_C(0) = -i_R(0) - i_L(0) = -\frac{45}{50{,}000} = -0.9\,\text{mA}$$

$$\frac{i_C(0)}{C} = \frac{-0.9}{1.25} \times 10^6 = -720 \times 10^3$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(8)(1.25)} = 10^8; \qquad \omega_o = 10^4 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^9}{(2)(50)(1.25) \times 10^3} = 8000 \text{ rad/s}$$

$$\omega_d = \sqrt{(100 - 64) \times 10^6} = 6000 \text{ rad/s}$$

$$v_o = B_1 e^{-8000t} \cos 6000t + B_2 e^{-8000t} \sin 6000t$$

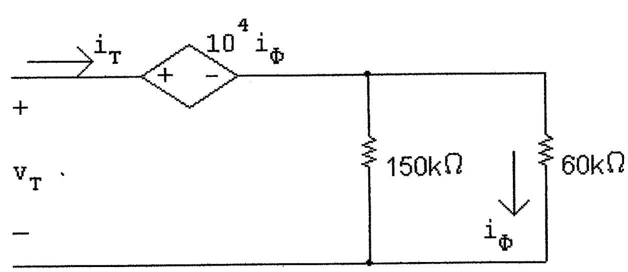
$$v_o(0) = B_1 = 45\,\text{V}$$

$$\frac{dv_o}{dt}(0) = 6000B_2 - 8000B_1 = -720 \times 10^3$$

$$\therefore \;\; 6000B_2 = 8000(45) - 720 \times 10^3; \qquad \therefore \;\; B_2 = -60\,\text{V}$$

$$v_o = 45e^{-8000t} \cos 6000t - 60e^{-8000t} \sin 6000t\,\text{V}, \qquad t \geq 0$$

P 8.17



$$v_T = 10^4 \frac{i_T(150 \times 10^3)}{210 \times 10^3} + \frac{(150)(60)10^6}{210 \times 10^3} i_T$$

$$\frac{v_T}{i_T} = \frac{1500 \times 10^3}{210} + \frac{9000 \times 10^3}{210} = \frac{10{,}500}{210} \times 10^3 = 50\,\text{k}\Omega$$

$$V_o = \frac{75}{10}(6) = 45\,\text{V}; \qquad I_o = 0$$

$$i_C(0) = -i_R(0) - i_L(0) = -\frac{45}{50{,}000} = -0.9\,\text{mA}$$

$$\frac{i_C(0)}{C} = \frac{-0.9\text{m}}{10^{-9}} = -900 \times 10^3$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(10)(10^{-9})} = 10^8; \qquad \omega_o = 10{,}000\text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{(2)(50{,}000)(10^{-9})} = 10{,}000\text{ rad/s}$$

$\alpha^2 = \omega_o^2$ so the response is critically damped

$$v_o = D_1te^{-10{,}000t} + D_2e^{-10{,}000t}$$

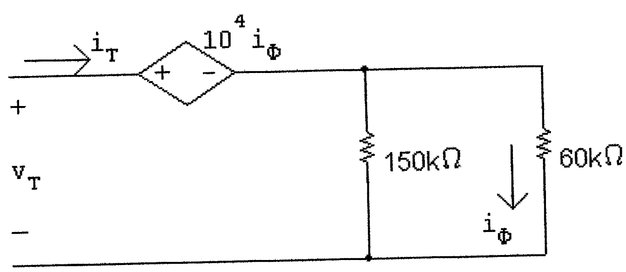
$$v_o(0) = D_2 = 45\,\text{V}$$

$$\frac{dv_o}{dt}(0) = D_1 - \alpha D_2 = -900 \times 10^3$$

$$\therefore\; D_1 = -900 \times 10^3 + (10{,}000)(45); \qquad \therefore\; D_1 = -450{,}000\,\text{V}/s$$

$$v_o = -450{,}000te^{-10{,}000t} + 45e^{-10{,}000t}\,\text{V}, \qquad t \geq 0$$

P 8.18



$$v_T = 10^4\frac{i_T(150 \times 10^3)}{210 \times 10^3} + \frac{(150)(60)10^6}{210 \times 10^3}i_T$$

$$\frac{v_T}{i_T} = \frac{1500 \times 10^3}{210} + \frac{9000 \times 10^3}{210} = \frac{10{,}500}{210} \times 10^3 = 50\,\text{k}\Omega$$

$$V_o = \frac{75}{10}(6) = 45\,\text{V}; \qquad I_o = 0$$

$$i_C(0) = -i_R(0) - i_L(0) = -\frac{45}{50{,}000} = -0.9\,\text{mA}$$

$$\frac{i_C(0)}{C} = \frac{-0.9}{800 \times 10^{-12}} = -1125 \times 10^3$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(12.5)(800 \times 10^{-12})} = 10^8; \qquad \omega_o = 10{,}000 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{(2)(50{,}000)(800 \times 10^{-12})} = 12{,}500 \text{ rad/s}$$

$\alpha^2 > \omega_o^2$ so the response is overdamped

$$v_o = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -12{,}500 \pm \sqrt{(12{,}500)^2 - 10^8} = -12{,}500 \pm 7500$$

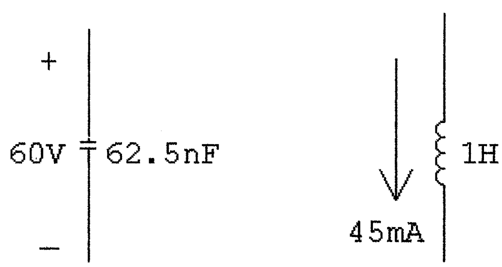
$$\therefore \quad s_{1,2} = -5000 \text{ r/s}, -20{,}000 \text{ r/s}$$

$$A_1 + A_2 = V_o = 45 \quad \text{and} \quad -5000A_1 - 20{,}000A_2 = -1125 \times 10^3$$

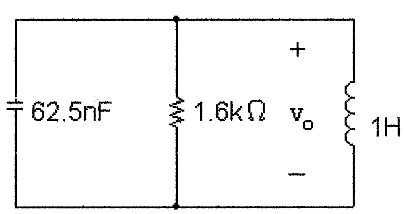
$$\therefore \quad A_1 = -15, \quad A_2 = 60$$

$$v_o = -15e^{-5000t} + 60e^{-20{,}000t} \text{ V}, \qquad t \geq 0$$

P 8.19 $t < 0$: $\quad V_o = 60\,\text{V}, \quad I_o = 45\,\text{mA}$



$t > 0$:



$$i_R(0) = \frac{60}{1600} = 37.5\,\text{mA}; \qquad i_L(0) = 45\,\text{mA}$$

$$i_C(0) = -37.5 - 45 = -82.5\,\text{mA}$$

$$\alpha = \frac{1}{2RC} = \frac{10^9}{3200(62.5)} = 5000 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{62.5} = 16 \times 10^6$$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - 16 \times 10^6} = -5000 \pm 3000$$

$$s_1 = -2000 \text{ rad/s}; \qquad s_2 = -8000 \text{ rad/s}$$

$$\therefore \quad v_o = A_1 e^{-2000t} + A_2 e^{-8000t}$$

$$A_1 + A_2 = v_o(0) = 60$$

$$\frac{dv_o}{dt}(0) = -2000A_1 - 8000A_2 = \frac{-82.5 \times 10^{-3}}{62.5 \times 10^{-9}} = -1320 \times 10^3$$

Solving, $\quad A_1 = -140\,\text{V}, \qquad A_2 = 200\,\text{V}$

$$\therefore \quad v_o = -140e^{-2000t} + 200e^{-8000t}\,\text{V}, \qquad t \geq 0$$

P 8.20 $\quad \omega_o^2 = \dfrac{1}{LC} = \dfrac{16 \times 10^6}{0.64} = 25 \times 10^6$

$$\alpha = \frac{1}{2RC} = \frac{16 \times 10^6}{4000} = 4000 \text{ rad/s}; \qquad \alpha^2 = 16 \times 10^3$$

$$\omega_d = \sqrt{(25 - 16) \times 10^6} = 3000 \text{ rad/s}$$

$$s_{1,2} = -4000 \pm j3000\text{rad/s}$$

$$v_o(t) = B_1 e^{-4000t} \cos 3000t + B_2 e^{-4000t} \sin 3000t$$

$$v_o(0) = B_1 = 60\,\text{V}$$

$$i_R(0) = \frac{60}{2000} = 30\,\text{mA}$$

$$i_L(0) = 45\,\text{mA}$$

$$i_C(0) = -i_R(0) - i_L(0) = -75\,\text{mA}$$

$$\frac{i_C(0)}{C} = (-75 \times 10^{-3})(16 \times 10^6) = -12 \times 10^5$$

$$\frac{dv_o}{dt}(0) = -4000B_1 + 3000B_2 = -12 \times 10^5$$

$$\therefore \quad 3B_2 = 4B_1 - 1200 = 240 - 1200 = -960; \quad \therefore \quad B_2 = -320\,\text{V}$$

$$v_o(t) = 60e^{-4000t}\cos 3000t - 320e^{-4000t}\sin 3000t\,\text{V}, \qquad t \geq 0$$

P 8.21 $\omega_o^2 = \frac{1}{LC} = \frac{16 \times 10^6}{0.16} = 10^8; \qquad \omega_o = 10^4$

$$\alpha = \frac{1}{2RC} = \frac{16 \times 10^6}{1600} = 10^4$$

$$\therefore \quad \alpha^2 = \omega_o^2 \text{ (critical damping)}$$

$$v_o(t) = D_1te^{-10{,}000t} + D_2e^{-10{,}000t}$$

$$v_o(0) = D_2 = 60\,\text{V}$$

$$i_R(0) = \frac{60}{800} = 75\,\text{mA}$$

$$i_L(0) = 45\,\text{mA}$$

$$i_C(0) = -120\,\text{mA}$$

$$\frac{dv_o}{dt}(0) = -10{,}000D_2 + D_1$$

$$\frac{i_C(0)}{C} = (-120 \times 10^{-3})(16 \times 10^6) = -1920 \times 10^3$$

$$D_1 - 10{,}000D_2 = -1920 \times 10^3; \qquad D_1 = -1320 \times 10^3\text{V/s}$$

$$v_o(t) = (60 - 132 \times 10^4t)e^{-10{,}000t}\,\text{V}, \qquad t > 0$$

P 8.22 [a] $v = L\left(\frac{di_L}{dt}\right) = 16[e^{-20{,}000t} - e^{-80{,}000t}]\,\text{V}, \qquad t \geq 0$

[b] $i_R = \frac{v}{R} = 40[e^{-20{,}000t} - e^{-80{,}000t}]\,\text{mA}, \qquad t \geq 0^+$

[c] $i_C = I - i_L - i_R = [-8e^{-20{,}000t} + 32e^{-80{,}000t}]\,\text{mA}, \qquad t \geq 0^+$

P 8.23 [a] $v = L\left(\frac{di_{\rm L}}{dt}\right) = 40e^{-32{,}000t}\sin 24{,}000t\,\text{V}, \qquad t \geq 0$

[b] $i_{\rm C}(t) = I - i_{\rm R} - i_{\rm L} = 24 \times 10^{-3} - \frac{v}{625} - i_{\rm L}$

$$= [24e^{-32{,}000t}\cos 24{,}000t - 32e^{-32{,}000t}\sin 24{,}000t]\,\text{mA}, \qquad t \geq 0^+$$

P 8.24 $v = L\left(\frac{di_{\rm L}}{dt}\right) = 960{,}000te^{-40{,}000t}\,\text{V}, \qquad t \geq 0$

P 8.25 $\omega_o^2 = \frac{1}{LC} = \frac{10^6}{(20)(5)} = 10^4; \qquad \omega_o = 100\text{ rad/s}$

$$\alpha = \frac{1}{2RC} = \frac{10^6}{(1600)(5)} = \frac{10^4}{80} = 125\text{ rad/s}$$

$$s_{1,2} = -125 \pm \sqrt{(125)^2 - 10^4} = -125 \pm 75$$

$$s_1 = -50\text{ rad/s}; \qquad s_2 = -200\text{ rad/s}$$

$$I_f = 15\,\text{mA}$$

$$i_{\rm L} = 15 + A_1'e^{-50t} + A_2'e^{-200t}$$

$$\therefore\ -30 = 15 + A_1' + A_2'; \qquad A_1' + A_2' = -45 \times 10^{-3}$$

$$\frac{di_{\rm L}}{dt} = -50A_1' - 200A_2' = \frac{60}{20} = 3$$

Solving, $A_1' = -40\,\text{mA}; \qquad A_2' = -5\,\text{mA}$

$$i_{\rm L} = 15 - 40e^{-50t} - 5e^{-200t}\,\text{mA}, \quad t \geq 0$$

P 8.26 $\alpha = \frac{1}{2RC} = \frac{10^6}{(2500)(5)} = 80; \qquad \alpha^2 = 6400$

$$\omega_o^2 = 10^4; \qquad \omega_d = \sqrt{10^4 - 6400} = 60\text{ rad/s}$$

$$s_{1,2} = -\alpha \pm j\omega_d = -80 \pm j60\text{ rad/s}$$

$$i_{\rm L} = 15 + B_1'e^{-80t}\cos 60t + B_2'e^{-80t}\sin 60t$$

$$-30 = 15 + B_1' \qquad \therefore\ B_1' = -45\,\text{mA}$$

$$\frac{di_{\rm L}}{dt}(0) = -80B_1' + 60B_2' = 3$$

$$\therefore\ B_2' = -10\,\text{mA}$$

$$i_{\rm L} = 15 - 45e^{-80t}\cos 60t - 10e^{-80t}\sin 60t\,\text{mA}, \quad t \geq 0$$

P 8.27 $\alpha = \frac{1}{2RC} = \frac{10^6}{(2000)(5)} = 100$

$\alpha^2 = 10^4 = \omega_o^2 \qquad$ critical damping

$i_L = I_f + D_1'te^{-100t} + D_2'e^{-100t} = 15 + D_1'te^{-100t} + D_2'e^{-100t}$

$i_L(0) = -30 = 15 + D_2'; \qquad \therefore \; D_2' = -45\,\text{mA}$

$\frac{di_L}{dt}(0) = -100D_2' + D_1' = 3000 \times 10^{-3}$

$\therefore \; D_1' = 3000 \times 10^{-3} + 100(-45 \times 10^{-3}) = -1500 \times 10^{-3}$

$i_L = 15 - 1500te^{-100t} - 45e^{-100t}\,\text{mA}, \quad t \geq 0$

P 8.28 $\alpha = \frac{1}{2RC} = \frac{10^6}{(1600)(6.25)} = 100; \qquad \alpha^2 = 10^4$

$\omega_o^2 = \frac{1}{LC} = \frac{10^6}{(25)(6.25)} = 6400$

$s_{1,2} = -100 \pm \sqrt{10^4 - 6400} = -100 \pm 60$

$s_1 = -40$ rad/s; $\qquad s_2 = -160$ rad/s

$v_o(\infty) = 0 = V_f$

$\therefore \; v_o = A_1'e^{-40t} + A_2'e^{-160t}$

$v_o(0) = 30 = A_1' + A_2'$

Note: $\qquad i_C(0^+) = 0$

$\therefore \; \frac{dv_o}{dt}(0) = 0 = -40A_1' - 160A_2'$

Solving, $\qquad A_1' = 40\,\text{V}, \qquad A_2' = -10\,\text{V}$

$v_o(t) = 40e^{-40t} - 10e^{-160t}\,\text{V}, \qquad t > 0^+$

P 8.29 [a] $i_o = I_f + A_1'e^{-40t} + A_2'e^{-160t}$

$$I_f = \frac{30}{800} = 37.5\text{mA}; \qquad i_o(0) = 0$$

$$0 = 37.5 \times 10^{-3} + A_1' + A_2', \qquad \therefore \quad A_1' + A_2' = -37.5 \times 10^{-3}$$

$$\frac{di_o}{dt}(0) = \frac{30}{25} = -40A_1' - 160A_2'$$

Solving, $\quad A_1' = -40\,\text{mA}; \qquad A_2' = 2.5\,\text{mA}$

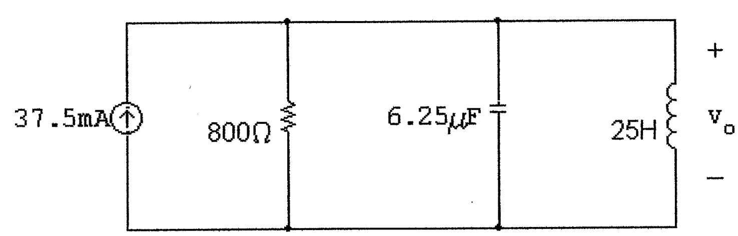
$$i_o = 37.5 - 40e^{-40t} + 2.5e^{-160t}\,\text{mA}, \quad t \geq 0$$

[b] $\dfrac{di_o}{dt} = [1600e^{-40t} - 400e^{-160t}] \times 10^{-3}$

$$L\frac{di_o}{dt} = 25(1.6)e^{-40t} - 25(0.4)e^{-160t}$$

$$\therefore \quad v_o = 40e^{-40t} - 10e^{-160t}\,\text{V}, \quad t \geq 0$$

P 8.30 For $\quad t > 0$



$$\alpha = \frac{1}{2RC} = 100; \qquad \frac{1}{LC} = 6400$$

$$s_{1,2} = -100 \pm 60$$

$$s_1 = -40\,\text{rad/s}; \qquad s_2 = -160\,\text{rad/s}$$

$$v_o = V_f + A_1'e^{-40t} + A_2'e^{-160t}$$

$$V_f = 0; \qquad v_o(0^+) = 0; \qquad i_C(0^+) = 37.5\,\text{mA}$$

$$\therefore \quad A_1' + A_2' = 0$$

$$\frac{dv_o(0^+)}{dt} = \frac{i_C(0^+)}{6.25 \times 10^{-6}} = 6000\,\text{V/s}$$

$$\frac{dv_o(0^+)}{dt} = -40A_1' - 160A_2'$$

$$-40A_1' - 160A_2' = 6000$$

$$A_1' + 4A_2' = -150$$

$$A_1' + A_2' = 0$$

$$\therefore \quad A_1' = 50\,\text{V}; \qquad A_2' = -50\,\text{V}$$

$$v_o = 50e^{-40t} - 50e^{-160t}\,\text{V}, \qquad t \geq 0$$

P 8.31 [a] From the solution to Prob. 8.30 $s_1 = -40\,\text{rad/s}$ and $s_2 = -160\,\text{rad/s}$, therefore

$$i_o = I_f + A_1'e^{-40t} + A_2'e^{-160t}$$

$$I_f = 37.5\,\text{mA}; \qquad i_o(0^+) = 0; \qquad \frac{di_o(0^+)}{dt} = 0$$

$$\therefore \quad 0 = 37.5 + A_1' + A_2'; \qquad -40A_1' - 160A_2' = 0$$

It follows that

$$A_1' = -50\,\text{mA}; \qquad A_2' = 12.5\,\text{mA}$$

$$\therefore \quad i_o = 37.5 - 50e^{-40t} + 12.5e^{-160t}\,\text{mA}, \qquad t \geq 0$$

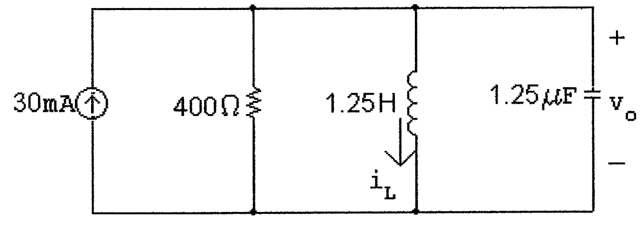
[b] $$\frac{di_o}{dt} = 2e^{-40t} - 2e^{-160t}$$

$$v_o = L\frac{di_o}{dt} = 25[2e^{-40t} - 2e^{-160t}]$$

$$v_o = 50e^{-40t} - 50e^{-160t}\,\text{V}, \qquad t \geq 0$$

P 8.32 $i_L(0^-) = i_L(0^+) = 30\,\text{mA}$

For $t > 0$



$$i_L(0^-) = i_L(0^+) = 30\,\text{mA}$$

$$\alpha = \frac{1}{2RC} = 1000\,\text{rad/s}; \qquad \omega_o^2 = \frac{1}{LC} = 64 \times 10^4$$

$$s_1 = -400\,\text{rad/s} \qquad s_2 = -1600\,\text{rad/s}$$

$$v_o(\infty) = 0 = V_f$$

$$v_o = A_1' e^{-400t} + A_2' e^{-1600t}$$

$$i_C(0^+) = -30 + 30 + 0 = 0$$

$$\therefore \quad \frac{dv_o}{dt} = 0$$

$$\frac{dv_o}{dt}(0) = -400A_1' - 1600A_2'$$

$$\therefore \quad A_1' + 400A_2' = 0; \qquad A_1' + A_2' = 0$$

$$\therefore \quad A_1' = 0; \qquad A_2' = 0$$

$$\therefore \quad v_o = 0 \text{ for } t \geq 0$$

Note: $\quad v_o(0) = 0; \qquad v_o(\infty) = 0; \qquad \dfrac{dv_o(0)}{dt} = 0$

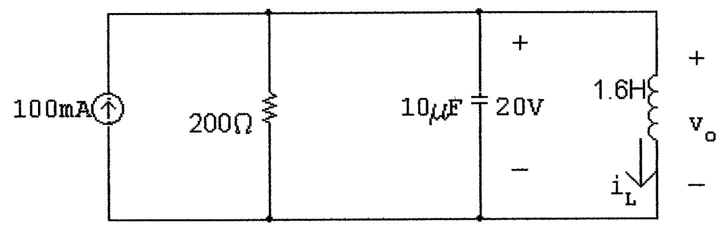
Hence the 30 mA current circulates between the current source and the ideal inductor in the equivalent circuit. In the original circuit the 12 V source sustains a current of 30 mA in the inductor. This is an example of a circuit going directly into steady state when the switch is closed. There is no transient period, or interval.

P 8.33 $\quad t < 0$:

$$v_o(0^-) = v_o(0^+) = \frac{1000}{1250}(25) = 20\,\text{V}$$

$$i_L(0^-) = i_L(0^+) = 0$$

$t > 0$



$$-100 + \frac{20}{0.2} + i_C(0^+) + 0 = 0; \qquad \therefore \; i_C(0^+) = 0$$

$$\frac{1}{2RC} = \frac{10^6}{(400)(10)} = 250\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^6}{10(1.6)} = 62{,}500$$

$\therefore \; \alpha^2 = \omega_o^2$ critically damped

[a] $v_o = V_f + D_1'te^{-250t} + D_2'e^{-250t}$

$V_f = 0$

$$\frac{dv_o(0)}{dt} = -250D_2' + D_1' = 0$$

$v_o(0^+) = 20 = D_2'$

$D_1' = 250D_2' = 5000\,\text{V/s}$

$\therefore \; v_o = 5000te^{-250t} + 20e^{-250t}\,\text{V}, \quad t \geq 0^+$

[b] $i_L = I_f + D_3'te^{-250t} + D_4'e^{-250t}$

$$i_L(0^+) = 0; \qquad I_f = 100\,\text{mA}; \qquad \frac{di_L(0^+)}{dt} = \frac{20}{1.6} = 12.5\ \text{A/s}$$

$\therefore \; 0 = 100 + D_4'; \qquad D_4' = -100\,\text{mA};$

$-250D_4' + D_3' = 12.5; \qquad D_3' = -12.5\,\text{A/s}$

$\therefore \; i_L = 100 - 12{,}500te^{-250t} - 100e^{-250t}\,\text{mA} \qquad t \geq 0$

P 8.34 [a] $w_L = \int_0^\infty p\,dt = \int_0^\infty v_o i_L\,dt$

$v_o = 5000te^{-250t} + 20e^{-250t}\,\text{V}$

$i_L = 0.1 - 12.5te^{-250t} - 0.1e^{-250t}\,\text{A}$

$$p = 2e^{-250t} + 500te^{-250t} - 750te^{-500t} - 62{,}500t^2e^{-500t} - 2e^{-500t}\,\mathrm{W}$$

$$\frac{w_L}{2} = \int_0^\infty e^{-250t}\,dt + 250\int_0^\infty te^{-250t}\,dt - 375\int_0^\infty te^{-500t} -$$

$$31{,}250\int_0^\infty t^2e^{-500t}\,dt - \int_0^\infty e^{-500t}\,dt$$

$$= \frac{e^{-250t}}{-250}\Big|_0^\infty + \frac{250}{(250)^2}e^{-250t}(-250t-1)\Big|_0^\infty -$$

$$\frac{375}{(500)^2}e^{-500t}(-500t-1)\Big|_0^\infty -$$

$$\frac{31{,}250}{(-500)^3}e^{-500t}(500^2t^2 + 1000t + 2)\Big|_0^\infty -$$

$$\frac{e^{-500t}}{(-500)}\Big|_0^\infty$$

All the upper limits evaluate to zero hence

$$\frac{w_L}{2} = \frac{1}{250} + \frac{250}{62{,}500} - \frac{375}{25\times10^4} - \frac{(31{,}250)(2)}{(5)^3 10^6} - \frac{1}{500}$$

$$w_L = 8 + 8 - 3 - 1 - 4 = 8\,\mathrm{mJ}$$

Note this value corresponds to the final energy stored in the inductor, i.e.

$$w_L(\infty) = \frac{1}{2}(1.6)(0.1)^2 = 8\,\mathrm{mJ}.$$

[b] $v = 5000te^{-250t} + 20e^{-250t}\,\mathrm{V}$

$$i_R = \frac{v}{200} = 25te^{-250t} + 0.1e^{-250t}\,\mathrm{A}$$

$$p_R = vi_R = 2e^{-500t}[62{,}500t^2 + 500t + 1]$$

$$w_R = \int_0^\infty p_R\,dt$$

$$\frac{w_R}{2} = 62{,}500\int_0^\infty t^2e^{-500t}\,dt + 500\int_0^\infty te^{-500t}\,dt + \int_0^\infty e^{-500t}\,dt$$

$$= \frac{62{,}500e^{-500t}}{-125\times10^6}[25\times10^4t^2 + 1000t + 2]\Big|_0^\infty +$$

$$\frac{500e^{-500t}}{25\times10^4}(-500t-1)\Big|_0^\infty + \frac{e^{-500t}}{(-500)}\Big|_0^\infty$$

Since all the upper limits evaluate to zero we have

$$\frac{w_R}{2} = \frac{62{,}500(2)}{125\times10^6} + \frac{500}{25\times10^4} + \frac{1}{500}$$

$w_R = 2 + 4 + 4 = 10\,\text{mJ}$

[c] $100 = i_R + i_C + i_L \quad (\text{mA})$

$$i_R + i_L = 25{,}000te^{-250t} + 100e^{-250t} + 100 - 12{,}500te^{-250t} - 100e^{-250t}\,\text{mA}$$
$$= 100 + 12{,}500te^{-250t}\,\text{mA}$$
$$\therefore\quad i_C = 100 - (i_R + i_L) = -12{,}500te^{-250t}\,\text{mA} = -12.5te^{-250t}\,\text{A}$$
$$p_C = vi_C = [5000te^{-250t} + 20e^{-250t}][-12.5te^{-250t}]$$
$$= -250[250t^2e^{-500t} + te^{-500t}]$$

$$\frac{w_C}{-250} = 250\int_0^\infty t^2e^{-500t}\,dt + \int_0^\infty te^{-500t}\,dt$$

$$\frac{w_C}{-250} = \frac{250e^{-500t}}{-125\times 10^6}[25\times 10^4t^2 + 1000t + 2]\Big|_0^\infty + \frac{e^{-500t}}{25\times 10^4}(-500t - 1)\Big|_0^\infty$$

Since all upper limits evaluate to zero we have

$$w_C = \frac{-250(250)(2)}{125\times 10^6} - \frac{250(1)}{25\times 10^4} = -1000\times 10^{-6} - 10\times 10^{-4} = -2\,\text{mJ}$$

Note this 2 mJ corresponds to the initial energy stored in the capacitor, i.e.,

$$w_C(0) = \frac{1}{2}(10\times 10^{-6})(20)^2 = 2\,\text{mJ}.$$

Thus $w_C(\infty) = 0\,\text{mJ}$ which agrees with the final value of $v = 0$.

[d] $i_s = 100\,\text{mA}$

$$p_s(\text{del}) = 100v\,\text{mW}$$
$$= 0.1[5000te^{-250t} + 20e^{-250t}]$$
$$= 2e^{-250t} + 500te^{-250t}\,\text{W}$$
$$\frac{w_s}{2} = \int_0^\infty e^{-250t}\,dt + \int_0^\infty 250te^{-250t}\,dt$$
$$= \frac{e^{-250t}}{-250}\Big|_0^\infty + \frac{250e^{-250t}}{62{,}500}(-250t - 1)\Big|_0^\infty$$
$$= \frac{1}{250} + \frac{1}{250}$$
$$w_s = \frac{2(2)}{250} = \frac{4}{250} = 16\,\text{mJ}$$

[e] $w_L = 8\,\text{mJ} \quad (\text{absorbed})$

$w_R = 10\,\text{mJ} \quad (\text{absorbed})$

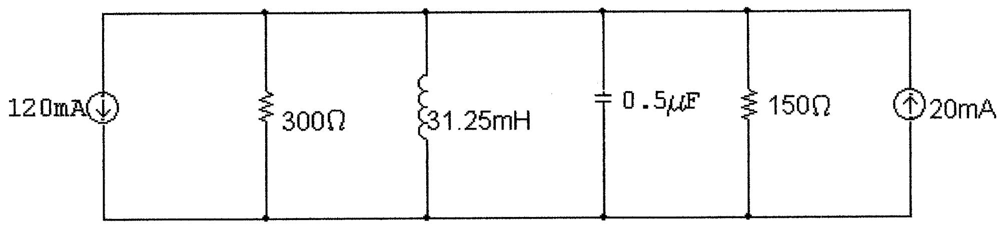
$$w_C = 2\,\text{mJ} \quad \text{(delivered)}$$

$$w_S = 16\,\text{mJ} \quad \text{(delivered)}$$

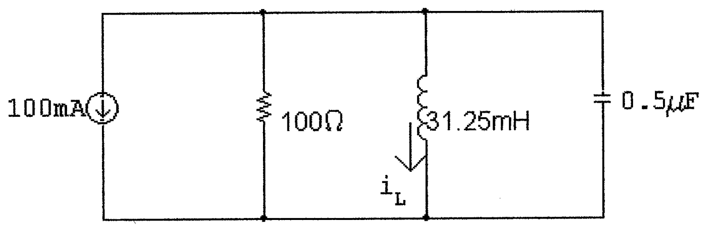
$$\sum w_{\text{del}} = w_{\text{abs}} = 18\,\text{mJ}.$$

P 8.35 $t < 0:$ $\quad i_L = 3/150 = 20\,\text{mA}$

$t > 0:$



$300\|150 = 100\,\Omega$



$$i_L(0) = 20\,\text{mA}, \qquad i_L(\infty) = -100\,\text{mA}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(31.25)(0.5)} = 64 \times 10^6; \qquad \omega_o = 8000\ \text{rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^6}{(200)(0.5)} = 10^4; \qquad \alpha^2 = 100 \times 10^6$$

$$\alpha^2 - \omega_o^2 = (100 - 64)10^6 = 36 \times 10^6$$

$$s_{1,2} = -10{,}000 \pm 6000$$

$$s_1 = -4000\ \text{rad/s}; \qquad s_2 = -16{,}000\ \text{rad/s}$$

$$i_L = I_f + A_1' e^{-4000t} + A_2' e^{-16{,}000t}$$

$$i_L(\infty) = I_f = -100\text{mA}$$

$$i_L(0) = A_1' + A_2' + I_f = 20\,\text{mA}$$

$\therefore \quad A_1' + A_2' - 100 = 20 \quad \text{so} \quad A_1' + A_2' = 120\,\text{mA}$

$\frac{di_L}{dt}(0) = 0 = -4000A_1 - 16{,}000A_2'$

Solving, $\quad A_1' = 160\,\text{mA}, \qquad A_2' = -40\,\text{mA}$

$i_L = -100 + 160e^{-4000t} - 40e^{-16{,}000t}\,\text{mA}, \qquad t \geq 0$

P 8.36 $v_C(0^+) = \frac{1}{2}(240) = 120\,\text{V}$

$i_L(0^+) = 60\,\text{mA}; \qquad i_L(\infty) = \frac{240}{5} \times 10^{-3} = 48\,\text{mA}$

$$\alpha = \frac{1}{2RC} = \frac{10^6}{2(2500)(5)} = 40$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^6}{400} = 2500$$

$\alpha^2 = 1600; \qquad \alpha^2 < \omega_o^2; \qquad \therefore \quad \text{underdamped}$

$s_{1,2} = -40 \pm j\sqrt{2500 - 1600} = -40 \pm j30\ \text{rad/s}$

$$\begin{aligned} i_L &= I_f + B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t \\ &= 48 + B_1'e^{-40t}\cos 30t + B_2'e^{-40t}\sin 30t \end{aligned}$$

$i_L(0) = 48 + B_1'; \qquad B_1' = 60 - 48 = 12\,\text{mA}$

$\frac{di_L}{dt}(0) = 30B_2' - 40B_1' = \frac{120}{80} = 1.5 = 1500 \times 10^{-3}$

$\therefore \quad 30B_2' = 40(12) \times 10^{-3} + 1500 \times 10^{-3}; \qquad B_2' = 66\,\text{mA}$

$\therefore \quad i_L = 48 + 12e^{-40t}\cos 30t + 66e^{-40t}\sin 30t\,\text{mA}, \qquad t \geq 0$

P 8.37 [a] $2\alpha = 5000; \quad \alpha = 2500\,\text{rad/s}$

$\sqrt{\alpha^2 - \omega_o^2} = 1500; \qquad \omega_o^2 = 4 \times 10^6; \qquad \omega_o = 2000\,\text{rad/s}$

$\alpha = \frac{R}{2L} = 2500; \qquad R = 5000L$

$\omega_o^2 = \frac{1}{LC} = 4 \times 10^6; \qquad L = \frac{10^9}{4 \times 10^6(50)} = 5\text{H}$

$R = 25{,}000\,\Omega$

[b] $i(0) = 0$

$$L\frac{di(0)}{dt} = v_c(0); \qquad \frac{1}{2}(50)\times 10^{-9}v_c^2(0) = 90\times 10^{-6}$$

$$\therefore \quad v_c^2(0) = 3600; \qquad v_c(0) = 60\,\text{V}$$

$$\frac{di(0)}{dt} = \frac{60}{5} = 12\,\text{A/s}$$

[c] $i(t) = A_1e^{-1000t} + A_2e^{-4000t}$

$$i(0) = A_1 + A_2 = 0$$

$$\frac{di(0)}{dt} = -1000A_1 - 4000A_2 = 12$$

Solving,

$$\therefore \quad A_1 = 4\,\text{mA}; \qquad A_2 = -4\,\text{mA}$$

$$i(t) = 4e^{-1000t} - 4e^{-4000t}\,\text{mA} \qquad t \ge 0$$

[d] $\dfrac{di(t)}{dt} = -4e^{-1000t} + 16e^{-4000t}$

$$\frac{di}{dt} = 0 \text{ when } 16e^{-4000t} = 4e^{-1000t}$$

$$\text{or } e^{3000t} = 4$$

$$\therefore \quad t = \frac{\ln 4}{3000}\mu\text{s} = 462.10\,\mu\text{s}$$

[e] $i_{\max} = 4e^{-0.4621} - 4e^{-1.8484} = 1.89\,\text{mA}$

[f] $v_L(t) = 5\dfrac{di}{dt} = [-20e^{-1000t} + 80e^{-4000t}]\,\text{V}, \quad t \ge 0^+$

P 8.38 $\alpha = 800\,\text{rad/s}; \qquad \omega_d = 600\,\text{rad/s}$

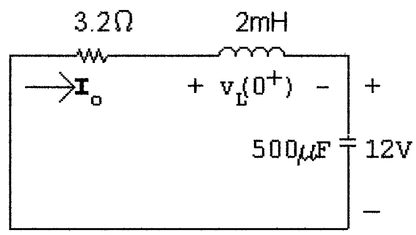
$$\omega_o^2 - \alpha^2 = 36\times 10^4; \qquad \omega_o^2 = 100\times 10^4; \quad w_o = 1000\,\text{rad/s}$$

$$\alpha = \frac{R}{2L} = 800; \qquad R = 1600L$$

$$\frac{1}{LC} = 100\times 10^4; \qquad L = \frac{10^6}{(100\times 10^4)(500)} = 2\,\text{mH}$$

$$\therefore \quad R = 3.2\,\Omega$$

$$i(0^+) = B_1 = 0\,\text{A}; \qquad \text{at } t = 0^+$$



$$12 + 0 + v_L(0^+) = 0; \qquad v_L(0^+) = -12\,\text{V}$$

$$\frac{di(0^+)}{dt} = \frac{-12}{0.002} = -6000\,\text{A/s}$$

$$\therefore \quad \frac{di(0^+)}{dt} = 600B_2 - 800B_1 = -6000$$

$$\therefore \quad 600B_2 = 800B_1 - 6000; \qquad \therefore \quad B_2 = -10\,\text{A}$$

$$\therefore \quad i = -10e^{-800t}\sin 600t\,\text{A}, \quad t \geq 0$$

P 8.39 From Prob. 8.38 we know v_c will be of the form

$$v_c = B_3e^{-800t}\cos 600t + B_4e^{-800t}\sin 600t$$

From Prob. 8.38 we have

$$v_c(0) = 12\,\text{V} = B_3$$

and

$$\frac{dv_c(0)}{dt} = \frac{i_C(0)}{C} = 0$$

$$\frac{dv_c(0)}{dt} = 600B_4 - 800B_3$$

$$\therefore \quad 600B_4 = 800B_3 + 0; \qquad B_4 = 16\,\text{V}$$

$$v_c(t) = 12e^{-800t}\cos 600t + 16e^{-800t}\sin 600t\,\text{V} \qquad t \geq 0$$

P 8.40 [a] $t < 0$:

$$i_o = \frac{120}{8000} = 15\,\text{mA}; \qquad v_o = (5000)(0.015) = 75\,\text{V}$$

$t > 0$:

$$\alpha = \frac{R}{2L} = \frac{5000}{2(1)} = 2500 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(1)(250)} = 4 \times 10^6 = 400 \times 10^4$$

$$\alpha^2 - \omega_o^2 = 625 \times 10^4 - 400 \times 10^4 = 225 \times 10^4$$

$$\therefore \quad s_{1,2} = -2500 \pm 1500$$

$$s_1 = -1000 \text{ rad/s} \qquad s_2 = -4000 \text{ rad/s}$$

$$\therefore \quad i_o(t) = A_1 e^{-1000t} + A_2 e^{-4000t}$$

$$i_o(0) = A_1 + A_2 = 15 \times 10^{-3}$$

$$\frac{di_o}{dt}(0) = -1000A_1 - 4000A_2 = 0$$

Solving, $\quad A_1 = 20\,\text{mA}; \qquad A_2 = -5\,\text{mA}$

$$i_o(t) = 20e^{-1000t} - 5e^{-4000t}\,\text{mA}, \qquad t \geq 0^+$$

[b] $v_o(t) = A_1 e^{-1000t} + A_2 e^{-4000t}$

$$v_o(0) = A_1 + A_2 = 75$$

$$\frac{dv_o}{dt}(0) = -1000A_1 - 4000A_2 = \frac{-15 \times 10^{-3}}{250 \times 10^{-9}}$$

Solving, $\quad A_1 = 80\,\text{V}; \qquad A_2 = -5\,\text{V}$

$$v_o(t) = 80e^{-1000t} - 5e^{-4000t}\,\text{V}, \qquad t \geq 0^+$$

Check:

$$5000i_o + 1\frac{di_o}{dt} = v_o$$

$$5000i_o = 100e^{-1000t} - 25e^{-4000t}$$

$$\frac{di_o}{dt} = -20e^{-1000t} + 20e^{-4000t}$$

$$\therefore \quad 5000i_o + \frac{di_o}{dt} = 80e^{-1000t} - 5e^{-4000t}\,\text{V} \qquad \text{(checks)}$$

P 8.41 [a] $\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(0.25)(160)} = \frac{10^8}{4} = 25 \times 10^6$

$$\alpha = \frac{R}{2L} = \omega_o = 5000\,\text{rad/s}$$

$$\therefore \quad R = (5000)(2)L = 2500\,\Omega$$

[b] $i(0) = i_L(0) = 24\,\text{mA}$

$$v_L(0) = 90 - (0.024)(2500) = 30\,\text{V}$$

$$\frac{di}{dt}(0) = \frac{30}{0.25} = 120\ \text{A/s}$$

[c] $v_C = D_1te^{-5000t} + D_2e^{-5000t}$

$$v_C(0) = D_2 = 90\,\text{V}$$

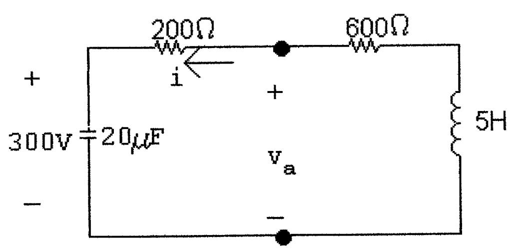
$$\frac{dv_C}{dt}(0) = D_1 - 5000D_2 = \frac{i_C(0)}{C} = \frac{-i_L(0)}{C}$$

$$D_1 - 450{,}000 = -\frac{24 \times 10^{-3}}{160 \times 10^{-9}} = -150{,}000$$

$$\therefore \quad D_1 = 300{,}000\ \text{V/s}$$

$$v_C = 300{,}000te^{-5000t} + 90e^{-5000t}\,\text{V}, \qquad t \geq 0^+$$

P 8.42 [a] For $t > 0$:



Since $i(0^-) = i(0^+) = 0$

$$v_a(0^+) = 300\,\text{V}$$

[b] $v_a = 200i + 5 \times 10^4 \int_0^t i\,dx + 300$

$$\frac{dv_a}{dt} = 200\frac{di}{dt} + 5 \times 10^4 i$$

$$\frac{dv_a(0^+)}{dt} = 200\frac{di(0^+)}{dt} + 5 \times 10^4 i(0^+) = 200\frac{di(0^+)}{dt}$$

$$-L\frac{di(0^+)}{dt} = 300$$

$$\frac{di(0^+)}{dt} = -0.2(300) = -60\,\text{A/s}$$

$$\therefore \quad \frac{dv_a(0^+)}{dt} = -12{,}000\,\text{V/s}$$

[c] $\alpha = \dfrac{R}{2L} = \dfrac{800}{10} = 80\,\text{rad/s}$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^6}{(5)(20)} = 10^4$$

$$s_{1,2} = -80 \pm \sqrt{6400 - 10^4} = -80 \pm j60\,\text{rad/s}$$

Underdamped:

$$v_a = B_1 e^{-80t}\cos 60t + B_2 e^{-80t}\sin 60t$$

$$v_a(0) = B_1 = 300\,\text{V}$$

$$\frac{dv_a(0)}{dt} = -80B_1 + 60B_2 = -12{,}000; \qquad \therefore \quad B_2 = 200\,\text{V}$$

$$v_a = 300e^{-80t}\cos 60t + 200e^{-80t}\sin 60t\,\text{V}, \quad t \geq 0^+$$

P 8.43 $i_L(0^-) = i_L(0^+) = \dfrac{70}{50 + 200} = 280\,\text{mA}$

$$v_c(0^-) = v_c(0^+) = 200(0.280) = 56\,\text{V}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(0.100)(200 \times 10^{-9})} = 50 \times 10^6$$

$$\alpha = \frac{R}{2L} = \frac{200}{2(0.100)} = 1000; \qquad \alpha^2 = 10^6$$

$$\alpha^2 < \omega_o^2 \quad \therefore \quad \text{underdamped}$$

$$s_{1,2} = -1000 \pm j7000\,\text{rad/s}$$

$$i = B_1 e^{-1000t}\cos 7000t + B_2 e^{-1000t}\sin 7000t$$

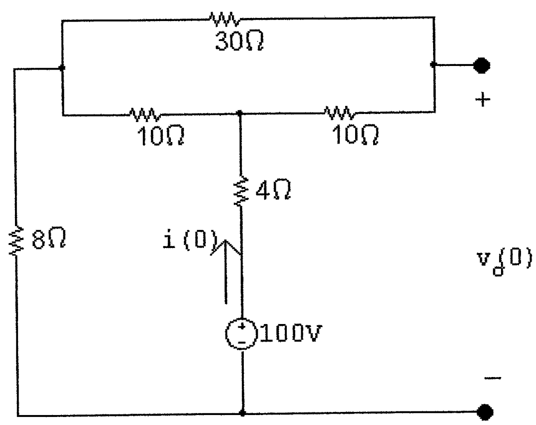
$$i(0) = B_1 = 280\,\text{mA}$$

$$\frac{di}{dt}(0) = 7000B_2 - 1000B_1 = 0$$

$$\therefore \quad B_2 = \frac{1}{7}B_1 = 40\,\text{mA}$$

$$i = 280e^{-1000t}\cos 7000t + 40e^{-1000t}\sin 7000t\,\text{mA}, \qquad t \geq 0^+$$

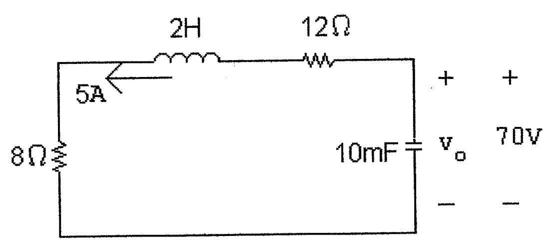
P 8.44 $t < 0$:



$$i(0) = \frac{100}{4+8+8} = \frac{100}{20} = 5\text{ A}$$

$$v_o(0) = 100 - 5(4) - 10(5)\left(\frac{10}{50}\right) = 70\text{ V}$$

$t > 0$:



$$\alpha = \frac{R}{2L} = \frac{20}{4} = 5, \qquad \alpha^2 = 25$$

$$\omega_o^2 = \frac{1}{LC} = \frac{100}{2} = 50$$

$\omega_o^2 > \alpha^2$ underdamped

$$v_o = B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t; \qquad \omega_d = \sqrt{50-25} = 5$$

$$v_o = B_1 e^{-5t}\cos 5t + B_2 e^{-5t}\sin 5t$$

$$v_o(0) = B_1 = 70\text{ V}$$

$$C\frac{dv_o}{dt}(0) = -5, \qquad \frac{dv_o}{dt} = \frac{-5}{10}\times 10^3 = -500\text{ V/s}$$

$$\frac{dv_o}{dt}(0) = -5B_1 + 5B_2 = -500$$

$$5B_2 = -500 + 5B_1 = -500 + 350; \qquad B_2 = -150/5 = -30\text{ V}$$

$$\therefore\ v_o = 70e^{-5t}\cos 5t - 30e^{-5t}\sin 5t\text{ V}, \qquad t \geq 0$$

P 8.45 $\alpha = \dfrac{R}{2L} = 5000\,\text{rad/s}$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{20} = 50 \times 10^6$$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - 50 \times 10^6} = -5000 \pm j5000\,\text{rad/s}$$

$$v_o = V_f + B_1'e^{-5000t}\cos 5000t + B_2'e^{-5000t}\sin 5000t$$

$$v_o(0) = 0 = V_f + B_1'$$

$$v_o(\infty) = 40\,\text{V}; \qquad \therefore \quad B_1' = -40\,\text{V}$$

$$\frac{dv_o(0)}{dt} = 0 = 5000B_2' - 5000B_1'$$

$$\therefore \quad B_2' = B_1' = -40\,\text{V}$$

$$v_o = 40 - 40e^{-5000t}\cos 5000t - 40e^{-5000t}\sin 5000t\,\text{V}, \quad t \geq 0$$

P 8.46 $\alpha = \dfrac{R}{2L} = 5000\,\text{rad/s}$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(0.4)(100 \times 10^{-9})} = 25 \times 10^6 \qquad \therefore \quad \omega_o = 5000\,\text{rad/s}$$

The response is therefore critically damped

$$v_o = V_f + D_1'te^{-5000t} + D_2'e^{-5000t}$$

$$v_o(0) = 0 = V_f + D_2'$$

$$v_o(\infty) = 40\,\text{V}; \qquad \therefore \quad D_2' = -40\,\text{V}$$

$$\frac{dv_o(0)}{dt} = 0 = D_1' - \alpha D_2'$$

$$\therefore \quad D_1' = (5000)(-40) = -200{,}000\ \text{V/s}$$

$$v_o = 40 - 200{,}000te^{-5000t} - 40e^{-5000t}\,\text{V}, \quad t \geq 0$$

P 8.47 $\alpha = \dfrac{R}{2L} = 5000\,\text{rad/s}$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(0.4)(156.25\times 10^{-9})} = 16\times 10^6 \qquad \therefore\ \omega_o = 4000\ \text{rad/s}$$

The response is therefore overdamped

$$s_{1,2} = -5000 \pm \sqrt{5000^2 - 4000^2} = -5000 \pm 3000 = -2000\,\text{rad/s},\ -8000\,\text{rad/s},$$

$$v_o = V_f + A_1'e^{-2000t} + A_2'e^{-8000t}$$

$$v_o(0) = 0 = V_f + A_1' + A_2'$$

$$v_o(\infty) = 40\,\text{V}; \qquad \therefore\ A_1' + A_2' = -40\,\text{V}$$

$$\frac{dv_o(0)}{dt} = 0 = s_1A_1' + S_2A_2' = -2000A_1' - 8000A_2'$$

$$\therefore\ A_1' = -53.33\,\text{V}, \quad A_2' = 13.33\,\text{V}$$

$$v_o = 40 - 53.33e^{-2000t} + 13.33e^{-8000t}\,\text{V}, \quad t \geq 0$$

P 8.48 [a] Let i be the current in the direction of the voltage drop $v_o(t)$. Then by hypothesis

$$i = i_f + B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t$$

$$i_f = i(\infty) = 0, \qquad i(0) = \frac{V_g}{R} = B_1'$$

Therefore $i = B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t$

$$L\frac{di(0)}{dt} = 0, \qquad \text{therefore} \quad \frac{di(0)}{dt} = 0$$

$$\frac{di}{dt} = [(\omega_d B_2' - \alpha B_1')\cos\omega_d t - (\alpha B_2' + \omega_d B_1')\sin\omega_d t]\,e^{-\alpha t}$$

Therefore $\omega_d B_2' - \alpha B_1' = 0; \qquad B_2' = \dfrac{\alpha}{\omega_d}B_1' = \dfrac{\alpha}{\omega_d}\dfrac{V_g}{R}$

Therefore

$$v_o = L\frac{di}{dt} = -\left\{L\left(\frac{\alpha^2 V_g}{\omega_d R} + \frac{\omega_d V_g}{R}\right)\sin\omega_d t\right\}e^{-\alpha t}$$

$$= -\left\{\frac{LV_g}{R}\left(\frac{\alpha^2}{\omega_d} + \omega_d\right)\sin\omega_d t\right\}e^{-\alpha t}$$

$$= -\frac{V_g L}{R}\left(\frac{\alpha^2 + \omega_d^2}{\omega_d}\right)e^{-\alpha t}\sin\omega_d t$$

$$= -\frac{V_g L}{R}\left(\frac{\omega_o^2}{\omega_d}\right)e^{-\alpha t}\sin\omega_d t$$

$$= -\frac{V_g L}{R\omega_d}\left(\frac{1}{LC}\right)e^{-\alpha t}\sin\omega_d t$$

$$v_o = -\frac{V_g}{RC\omega_d}e^{-\alpha t}\sin\omega_d t\,\text{V}, \quad t \geq 0^+$$

[b] $$\frac{dv_o}{dt} = -\frac{V_g}{\omega_d RC}\{\omega_d\cos\omega_d t - \alpha\sin\omega_d t\}e^{-\alpha t}$$

$$\frac{dv_o}{dt} = 0 \quad \text{when} \quad \tan\omega_d t = \frac{\omega_d}{\alpha}$$

Therefore $\omega_d t = \tan^{-1}(\omega_d/\alpha)$ (smallest t)

$$t = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$$

P 8.49 [a] From Problem 8.48 we have

$$v_o = \frac{-V_g}{RC\omega_d}e^{-\alpha t}\sin\omega_d t$$

$$\alpha = \frac{R}{2L} = \frac{120}{0.01} = 12{,}000\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^{12}}{2500} = 400 \times 10^6$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = 16\,\text{krad/s}$$

$$\frac{-V_g}{RC\omega_d} = \frac{-(-600)10^9}{(120)(500)(16) \times 10^3} = 625$$

$$\therefore \quad v_o = 625e^{-12{,}000t}\sin 16{,}000t\,\text{V}$$

[b] From Problem 8.48

$$t_d = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right) = \frac{1}{16{,}000}\tan^{-1}\left(\frac{16{,}000}{12{,}000}\right)$$

$$t_d = 57.96\,\mu\text{s}$$

[c] $v_{\text{max}} = 625e^{-0.012(57.96)}\sin[(0.016)(57.96)] = 249.42\,\text{V}$

[d] $R = 12\,\Omega; \qquad \alpha = 1200\,\text{rad/s}$

$$\omega_d = 19{,}963.97\,\text{rad/s}$$

$$v_o = 5009.02e^{-1200t}\sin 19{,}963.97t\,\text{V}, \quad t \geq 0$$

$$t_d = 75.67\,\mu\text{s}$$

$$v_{\text{max}} = 4565.96\,\text{V}$$

P 8.50 $i_C(0) = 0; \qquad v_o(0) = 200\,\text{V}$

$$\alpha = \frac{R}{2L} = \frac{4}{2(0.04)} = 50\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^3}{0.4} = 2500$$

$\therefore \quad \alpha^2 = \omega_o^2; \qquad$ critical damping

$$v_o(t) = V_f + D_1' te^{-50t} + D_2' e^{-50t}$$

$$V_f = 100\,\text{V}$$

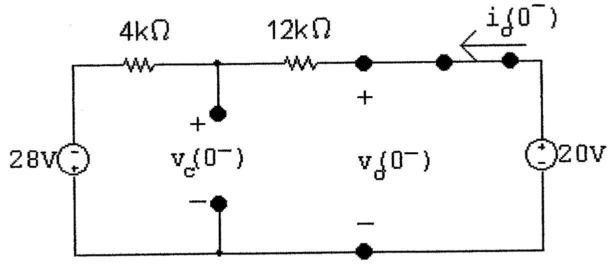
$$v_o(0) = 100 + D_2' = 200; \qquad D_2' = 100\,\text{V}$$

$$\frac{dv_o}{dt}(0) = -50D_2' + D_1' = 0$$

$$D_1' = 50D_2' = 5000\ \text{V/s}$$

$$v_o = 100 + 5000te^{-50t} + 100e^{-50t}\,\text{V}, \quad t \geq 0$$

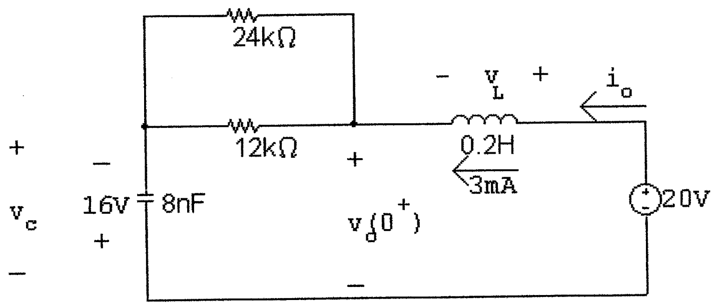
P 8.51 [a] $t < 0$:



$$i_o(0^-) = \frac{48}{16{,}000} = 3\,\text{mA}$$

$$v_C(0^-) = 20 - (12{,}000)(0.003) = -16\,\text{V}$$

$t = 0^+$:



$$12\,\text{k}\Omega \| 24\,\text{k}\Omega = 8\,\text{k}\Omega$$

$$\therefore \quad v_o(0^+) = (0.003)(8000) - 16 = 24 - 16 = 8\,\text{V}$$

and $v_L(0^+) = 20 - 8 = 12\,\text{V}$

[b] $v_o(t) = 8000 i_o + v_C$

$$\frac{dv_o}{dt}(t) = 8000\frac{di_o}{dt} + \frac{dv_C}{dt}$$

$$\frac{dv_o}{dt}(0^+) = 8000\frac{di_o}{dt}(0^+) + \frac{dv_C}{dt}(0^+)$$

$$v_L(0^+) = L\frac{di_o}{dt}(0^+)$$

$$\frac{di_o}{dt}(0^+) = \frac{v_L(0^+)}{L} = \frac{12}{0.2} = 60\ \text{A/s}$$

$$C\frac{dv_c}{dt}(0^+) = i_o(0^+)$$

$$\therefore \quad \frac{dv_c}{dt}(0^+) = \frac{3 \times 10^{-3}}{8 \times 10^{-9}} = 375{,}000$$

$$\therefore \quad \frac{dv_o}{dt}(0^+) = 8000(60) + 375{,}000 = 855{,}000\ \text{V/s}$$

[c] $\omega_o^2 = \frac{1}{LC} = \frac{10^9}{1.6} = 625 \times 10^6; \qquad \omega_o = 25{,}000$ rad/s

$$\alpha = \frac{R}{2L} = \frac{8000}{0.4} = 20{,}000 \text{ rad/s}; \qquad \alpha^2 = 400 \times 10^6$$

$$\alpha^2 < \omega_o^2 \qquad \text{underdamped}$$

$$s_{1,2} = -20{,}000 \pm j15{,}000 \text{ rad/s}$$

$$v_o(t) = V_f + B_1'e^{-20{,}000t}\cos 15{,}000t + B_2'e^{-20{,}000t}\sin 15{,}000t$$

$$V_f = v_o(\infty) = 20\,\text{V}$$

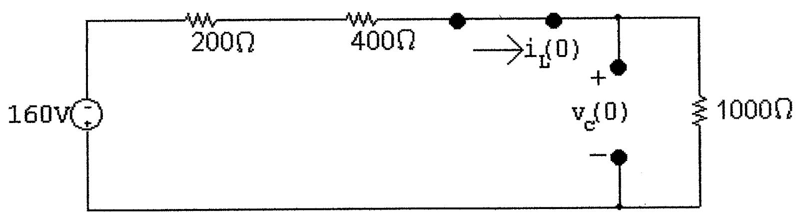
$$8 = 20 + B_1'; \qquad B_1' = -12\,\text{V}$$

$$-20{,}000B_1' + 15{,}000B_2' = 855{,}000$$

Solving, $\qquad B_2' = 41$ V

$$\therefore\quad v_o(t) = 20 - 12e^{-20{,}000t}\cos 15{,}000t + 41e^{-20{,}000t}\sin 15{,}000t\,\text{V}, \qquad t \geq 0^+$$

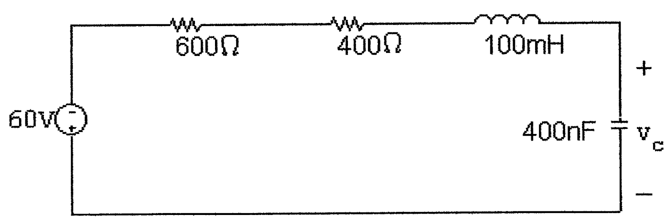
P 8.52 $t < 0$:



$$i_L(0) = \frac{-160}{1600} = -100\,\text{mA}$$

$$v_C(0) = 1000 i_L(0) = -100\,\text{V}$$

$t > 0$:



$$\alpha = \frac{R}{2L} = \frac{1000}{200} \times 10^3 = 5000 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{(10^9)(10^3)}{(100)(400)} = \frac{10^8}{4} = 25 \times 10^6$$

$\omega_o = 5000$ rad/s $\quad \therefore \quad$ critical damping

$$v_C(t) = V_f + D_1'te^{-5000t} + D_2'e^{-5000t}$$

$$v_C(0) = -100\,\text{V}; \qquad V_f = -60\,\text{V}$$

$$\therefore \quad -100 = -60 + D_2'; \qquad D_2' = -40\,\text{V}$$

$$C\frac{dv_C}{dt}(0) = i_L(0) = -100 \times 10^{-3}$$

$$\frac{dv_C}{dt}(0) = \frac{-100 \times 10^{-3}}{400 \times 10^{-9}} = -250{,}000 \text{ V/s}$$

$$\therefore \quad D_1' = 5000(-40) - 250{,}000 = -450{,}000$$

$$v_C(t) = -60 - 450{,}000te^{-5000t} - 40e^{-5000t}\,\text{V}, \qquad t \geq 0$$

P 8.53 [a] $v_c = V_f + [B_1' \cos\omega_d t + B_2' \sin\omega_d t]\, e^{-\alpha t}$

$$\frac{dv_c}{dt} = [(\omega_d B_2' - \alpha B_1')\cos\omega_d t - (\alpha B_2' + \omega_d B_1')\sin\omega_d t]e^{-\alpha t}$$

Since the initial stored energy is zero,

$$v_c(0^+) = 0 \quad \text{and} \quad \frac{dv_c(0^+)}{dt} = 0$$

It follows that $B_1' = -V_f$ and $B_2' = \dfrac{\alpha B_1'}{\omega_d}$

When these values are substituted into the expression for $[dv_c/dt]$, we get

$$\frac{dv_c}{dt} = \left(\frac{\alpha^2}{\omega_d} + \omega_d\right) V_f e^{-\alpha t} \sin\omega_d t$$

But $V_f = V$ and $\dfrac{\alpha^2}{\omega_d} + \omega_d = \dfrac{\alpha^2 + \omega_d^2}{\omega_d} = \dfrac{\omega_o^2}{\omega_d}$

Therefore $\dfrac{dv_c}{dt} = \left(\dfrac{\omega_o^2}{\omega_d}\right) Ve^{-\alpha t}\sin\omega_d t$

[b] $\dfrac{dv_c}{dt} = 0$ when $\sin\omega_d t = 0$, or $\omega_d t = n\pi$

where $n = 1, 2, 3, \ldots$

Therefore $t = \dfrac{n\pi}{\omega_d}$

[c] When $t_n = \dfrac{n\pi}{\omega_d}$, $\cos\omega_d t_n = \cos n\pi = (-1)^n$

and $\sin\omega_d t = \sin n\pi = 0$

Therefore $v_c(t_n) = V[1 - (-1)^n e^{-\alpha n\pi/\omega_d}]$

[d] It follows from [c] that

$$v(t_1) = V + Ve^{-(\alpha\pi/\omega_d)} \quad \text{and} \quad v_c(t_3) = V + Ve^{-(3\alpha\pi/\omega_d)}$$

$$\text{Therefore} \quad \frac{v_c(t_1) - V}{v_c(t_3) - V} = \frac{e^{-(\alpha\pi/\omega_d)}}{e^{-(3\alpha\pi/\omega_d)}} = e^{(2\alpha\pi/\omega_d)}$$

$$\text{But} \quad \frac{2\pi}{\omega_d} = t_3 - t_1 = T_d, \quad \text{thus} \quad \alpha = \frac{1}{T_d}\ln\frac{[v_c(t_1) - V]}{[v_c(t_3) - V]}$$

P 8.54 $\alpha = \dfrac{1}{T_d}\ln\left\{\dfrac{v_c(t_1) - V}{v_c(t_3) - V}\right\}$; $\quad T_d = t_3 - t_1 = \dfrac{3\pi}{12} - \dfrac{\pi}{12} = \dfrac{2\pi}{12}$ ms

$$\alpha = \frac{12{,}000}{2\pi}\ln\left[\frac{13.505}{0.985}\right] = 5000; \qquad \omega_d = \frac{2\pi}{T_d} = 12{,}000\,\text{rad/s}$$

$$\omega_o^2 = \omega_d^2 + \alpha^2 = 144\times 10^6 + 25\times 10^6 = 169\times 10^6$$

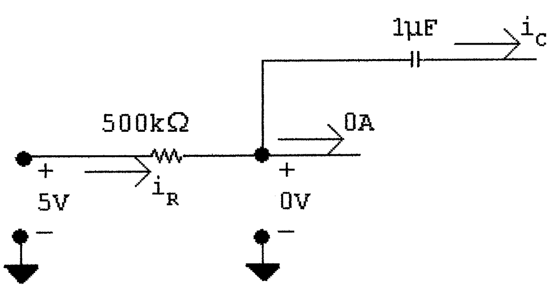
$$L = \frac{1}{(169)(0.2)} = 29.6\,\text{mH}; \qquad R = 2\alpha L = 295.86\,\Omega$$

P 8.55 At $t = 0$ the voltage across each capacitor is zero. It follows that since the operational amplifiers are ideal, the current in the 500 kΩ is zero. Therefore there cannot be an instantaneous change in the current in the 1 μF capacitor. Since the capacitor current equals $C(dv_o/dt)$, the derivative must be zero.

P 8.56 [a] From Example 8.13 $\dfrac{d^2v_o}{dt^2} = 2$

$$\text{therefore} \quad \frac{dg(t)}{dt} = 2, \quad g(t) = \frac{dv_o}{dt}$$

$$g(t) - g(0) = 2t; \qquad g(t) = 2t + g(0); \qquad g(0) = \frac{dv_o(0)}{dt}$$



$$i_R = \frac{5}{500}\times 10^{-3} = 10\,\mu\text{A} = i_C = -C\frac{dv_o(0)}{dt}$$

$$\frac{dv_o(0)}{dt} = \frac{-10 \times 10^{-6}}{1 \times 10^{-6}} = -10 = g(0)$$

$$\frac{dv_o}{dt} = 2t - 10$$

$$dv_o = 2t\,dt - 10\,dt$$

$$v_o - v_o(0) = t^2 - 10t; \qquad v_o(0) = 8\,\text{V}$$

$$v_o = t^2 - 10t + 8, \qquad 0 \le t \le t_{\text{sat}}$$

[b] $t^2 - 10t + 8 = -9$

$$t^2 - 10t + 17 = 0$$

$$t \cong 2.17\,\text{s}$$

P 8.57 Part (1) — Example 8.14, with R_1 and R_2 removed:

[a] $R_a = 100\,\text{k}\Omega$; $\quad C_1 = 0.1\,\mu\text{F}$; $\quad R_b = 25\,\text{k}\Omega$; $\quad C_2 = 1\,\mu\text{F}$

$$\frac{d^2v_o}{dt^2} = \left(\frac{1}{R_aC_1}\right)\left(\frac{1}{R_bC_2}\right)v_g; \qquad \frac{1}{R_aC_1} = 100 \quad \frac{1}{R_bC_2} = 40$$

$$v_g = 250 \times 10^{-3}; \qquad \text{therefore} \quad \frac{d^2v_o}{dt^2} = 1000$$

[b] Since $v_o(0) = 0 = \dfrac{dv_o(0)}{dt}$, our solution is $v_o = 500t^2$

The second op-amp will saturate when

$$v_o = 6\,\text{V}, \quad \text{or} \quad t_{\text{sat}} = \sqrt{6/500} \cong 0.1095\,\text{s}$$

[c] $\dfrac{dv_{o1}}{dt} = -\dfrac{1}{R_aC_1}v_g = -25$

[d] Since $v_{o1}(0) = 0$, $\quad v_{o1} = -25t\,\text{V}$

At $t = 0.1095\,\text{s}$, $\quad v_{o1} \cong -2.74\,\text{V}$

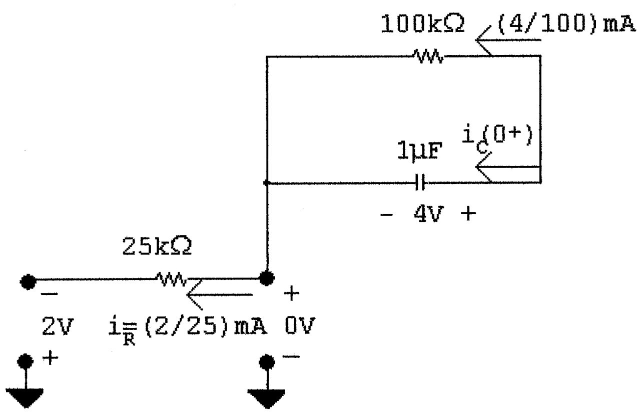
Therefore the second amplifier saturates before the first amplifier saturates. Our expressions are valid for $0 \le t \le 0.1095\,\text{s}$. Once the second op-amp saturates, our linear model is no longer valid.

Part (2) — Example 8.14 with $v_{o1}(0) = -2\,\text{V}$ and $v_o(0) = 4\,\text{V}$:

[a] Initial conditions will not change the differential equation; hence the equation is the same as Example 8.14.

[b] $v_o = 5 + A_1'e^{-10t} + A_2'e^{-20t}$ (from Example 8.14)

$v_o(0) = 4 = 5 + A_1' + A_2'$



$$\frac{4}{100} + i_C(0^+) - \frac{2}{25} = 0$$

$$i_C(0^+) = \frac{4}{100}\,\text{mA} = C\frac{dv_o(0^+)}{dt}$$

$$\frac{dv_o(0^+)}{dt} = \frac{0.04 \times 10^{-3}}{10^{-6}} = 40\,\text{V/s}$$

$$\frac{dv_o}{dt} = -10A_1'e^{-10t} - 20A_2'e^{-20t}$$

$$\frac{dv_o}{dt}(0^+) = -10A_1' - 20A_2' = 40$$

Therefore $-A_1' - 2A_2' = 4$ and $A_1' + A_2' = -1$
Thus, $A_1' = 2$ and $A_2' = -3$

$v_o = 5 + 2e^{-10t} - 3e^{-20t}$ V

[c] Same as Example 8.14:

$$\frac{dv_{o1}}{dt} + 20v_{o1} = -25$$

[d] From Example 8.14:

$v_{o1}(\infty) = -1.25$ V; $v_1(0) = -2$ V (given)

Therefore

$v_{o1} = -1.25 + (-2 + 1.25)e^{-20t} = -1.25 - 0.75e^{-20t}$ V

P 8.58 [a] $$\frac{d^2v_o}{dt^2} = \frac{1}{R_1C_1R_2C_2}v_g$$

$$\frac{1}{R_1C_1R_2C_2} = \frac{10^{-6}}{(50)(20)(2)(4) \times 10^{-6} \times 10^{-6}} = 125$$

$$\therefore \quad \frac{d^2v_o}{dt^2} = 125v_g$$

$0 \le t \le 0.2^-$:

$v_g = 400\,\text{mV}$

$$\frac{d^2v_o}{dt^2} = 50$$

Let $g(t) = \dfrac{dv_o}{dt}$, then $\dfrac{dg}{dt} = 50$ or $dg = 50\,dt$

$$\int_{g(0)}^{g(t)} dx = 50\int_0^t dy$$

$$g(t) - g(0) = 50t, \quad g(0) = \frac{dv_o}{dt}(0) = 0$$

$$g(t) = \frac{dv_o}{dt} = 50t$$

$dv_o = 50t\,dt$

$$\int_{v_o(0)}^{v_o(t)} dx = 50\int_0^t x\,dx; \qquad v_o(t) - v_o(0) = 25t^2, \quad v_o(0) = 0$$

$v_o(t) = 25t^2\,\text{V}, \quad 0 \le t \le 0.2^-$

$$\frac{dv_{o1}}{dt} = -\frac{1}{R_1C_1}v_g = -10v_g = -4$$

$dv_{o1} = -4\,dt$

$$\int_{v_{o1}(0)}^{v_{o1}(t)} dx = -4\int_0^t dy$$

$v_{o1}(t) - v_{o1}(0) = -4t, \qquad v_{o1}(0) = 0$

$v_{o1}(t) = -4t\,\text{V}, \qquad 0 \le t \le 0.2^-$

$0.2^+ \le t \le t_{\text{sat}}$:

$$\frac{d^2v_o}{dt^2} = -12.5, \qquad \text{let} \quad g(t) = \frac{dv_o}{dt}$$

$$\frac{dg(t)}{dt} = -12.5; \qquad dg(t) = -12.5\,dt$$

$$\int_{g(0.2^+)}^{g(t)} dx = -12.5\int_{0.2}^t dy$$

$$g(t) - g(0.2^+) = -12.5(t - 0.2) = -12.5t + 2.5$$

$$g(0.2^+) = \frac{dv_o(0.2^+)}{dt}$$

$$C\frac{dv_o}{dt}(0.2^+) = \frac{0 - v_{o1}(0.2^+)}{20 \times 10^3}$$

$$v_{o1}(0.2^+) = v_o(0.2^-) = -4(0.2) = -0.80\,\text{V}$$

$$\therefore \quad C\frac{dv_{o1}(0.2^+)}{dt} = \frac{0.80}{20 \times 10^3} = 40\,\mu\text{A}$$

$$\frac{dv_{o1}}{dt}(0.2^+) = \frac{40 \times 10^{-6}}{4 \times 10^{-6}} = 10\,\text{V/s}$$

$$\therefore \quad g(t) = -12.5t + 2.5 + 10 = -12.5t + 12.5 = \frac{dv_o}{dt}$$

$$\therefore \quad dv_o = -12.5t\,dt + 12.5\,dt$$

$$\int_{v_o(0.2^+)}^{v_o(t)} dx = \int_{0.2^+}^{t} -12.5y\,dy + \int_{0.2^+}^{t} 12.5\,dy$$

$$v_o(t) - v_o(0.2^+) = -6.25y^2 \Big|_{0.2}^{t} + 12.5y \Big|_{0.2}^{t}$$

$$v_o(t) = v_o(0.2^+) - 6.25t^2 + 0.25 + 12.5t - 2.5$$

$$v_o(0.2^+) = v_o(0.2^-) = 1\,\text{V}$$

$$\therefore \quad v_o(t) = -6.25t^2 + 12.5t - 1.25\,\text{V}, \qquad 0.2^+ \le t \le t_{\text{sat}}$$

$$\frac{dv_{o1}}{dt} = -10(-0.1) = 1, \qquad 0.2^+ \le t \le t_{\text{sat}}$$

$$dv_{o1} = dt; \qquad \int_{v_{o1}(0.2^+)}^{v_{o1}(t)} dx = \int_{0.2^+}^{t} dy$$

$$v_{o1}(t) - v_{o1}(0.2^+) = t - 0.2; \qquad v_{o1}(0.2^+) = v_{o1}(0.2^-) = -0.8\,\text{V}$$

$$\therefore \quad v_{o1}(t) = t - 1\,\text{V}, \qquad 0.2^+ \le t \le t_{\text{sat}}$$

Summary:

$0 \le t \le 0.2^-\text{s}: \qquad v_{o1} = -4t\,\text{V}, \quad v_o = 25t^2\,\text{V}$

$0.2^+\text{s} \le t \le t_{\text{sat}}: \qquad v_{o1} = t - 1\,\text{V}, \quad v_o = -6.25t^2 + 12.5t - 1.25\,\text{V}$

[b] $-10 = -6.25t_{\text{sat}}^2 + 12.5t_{\text{sat}} - 1.25$

$$\therefore \quad 6.25t_{\text{sat}}^2 - 12.5t_{\text{sat}} - 8.75 = 0$$

$$t_{\text{sat}}^2 - 2t_{\text{sat}} - 1.4 = 0$$

$$t_{\text{sat}} = 1 \pm \sqrt{2 + 1.4} = 1 \pm 1.844$$

$$\therefore \quad t_{\text{sat}} = 2.844\,\text{sec}$$

$$v_{o1}(t_{\text{sat}}) = 1.844 - 1 = 0.844\,\text{V}$$

P 8.59 $\tau_1 = (0.25 \times 10^6)(2 \times 10^{-6}) = 0.50\,\text{s}$

$$\frac{1}{\tau_1} = 2; \qquad \tau_2 = (0.25 \times 10^6)(4 \times 10^{-6}) = 1\,\text{s}; \qquad \therefore \quad \frac{1}{\tau_2} = 1$$

$$\therefore \quad \frac{d^2v_o}{dt^2} + 3\frac{dv_o}{dt} + 2v_o = 50$$

$$s^2 + 3s + 2 = 0$$

$$(s+1)(s+2) = 0; \qquad s_1 = -1, \quad s_2 = -2$$

$$v_o = V_f + A_1'e^{-t} + A_2'e^{-2t}; \qquad V_f = \frac{50}{2} = 25\,\text{V}$$

$$v_o = 25 + A_1'e^{-t} + A_2'e^{-2t}$$

$$v_o(0) = 0 = 25 + A_1' + A_2'; \qquad \frac{dv_o}{dt}(0) = 0 = -A_1' - 2A_2'$$

$$\therefore \quad A_1' = -50, \qquad A_2' = 25\,\text{V}$$

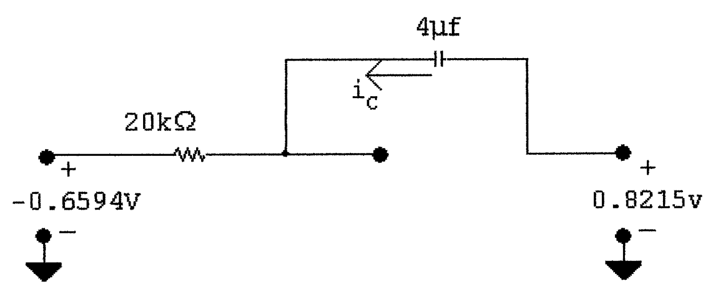
$$v_o(t) = 25 - 50e^{-t} + 25e^{-2t}\,\text{V}, \qquad 0 \le t \le 0.2\,\text{s}$$

$$\frac{dv_{o1}}{dt} + 2v_{o1} = -4; \qquad \therefore \quad v_{o1} = -2 + 2e^{-2t}\,\text{V}, \quad 0 \le t \le 0.2\,\text{s}$$

$$v_o(0.2) = 25 - 50e^{-0.2} + 25e^{-0.4} = 0.8215\,\text{V}$$

$$v_{o1}(0.2) = -2 + 2e^{-0.4} = -0.6594\,\text{V}$$

At $t = 0.2\,\text{s}$



$$i_C = \frac{0 + 0.6594}{20 \times 10^3} = 32.97\,\mu\text{A}$$

$$C\frac{dv_o}{dt} = 32.97\,\mu\text{A}; \qquad \frac{dv_o}{dt} = \frac{32.97}{4} = 8.24\,\text{V/s}$$

$0.2\,\text{s} \leq t < \infty$:

$$\frac{d^2v_o}{dt^2} + 3\frac{dv_o}{dt} + 2 = -12.5$$

$$v_o(\infty) = -6.25$$

$$\therefore \quad v_o = -6.25 + A_1'e^{-(t-0.2)} + A_2'e^{-2(t-0.2)}$$

$$0.8215 = -6.25 + A_1' + A_2'$$

$$\frac{dv_o}{dt}(0.2) = 8.24 = -A_1' - 2A_2'$$

$$\therefore \quad A_1' + A_2' = 7.07; \qquad -A_1' - 2A_2' = 8.24$$

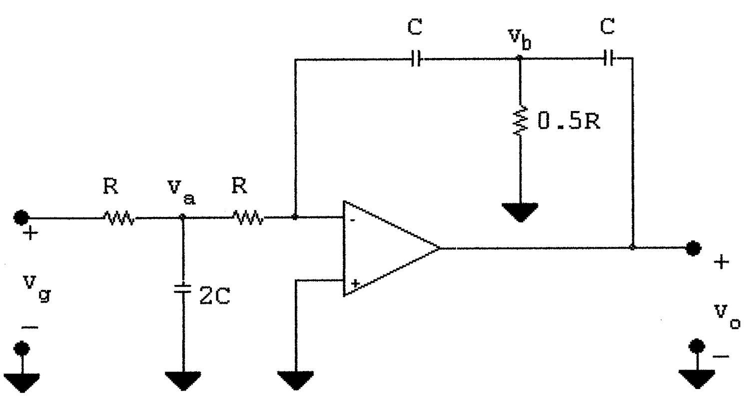
$$A_1' = 22.38; \qquad A_2' = -15.31$$

$$\therefore \quad v_o = -6.25 + 22.38e^{-(t-0.2)} - 15.31e^{-2(t-0.2)}\,\text{V}, \qquad 0.2 \leq t < \infty$$

$$\frac{dv_{o1}}{dt} + 2v_{o1} = 1$$

$$\therefore \quad v_{o1} = 0.5 + (-0.6594 - 1)e^{-2(t-0.2)} = 0.5 - 1.66e^{-2(t-0.2)}\,\text{V}, \qquad 0.2 \leq t < \infty$$

P 8.60 [a]



$$2C\frac{dv_a}{dt} + \frac{v_a - v_g}{R} + \frac{v_a}{R} = 0$$

(1) Therefore $\dfrac{dv_a}{dt} + \dfrac{v_a}{RC} = \dfrac{v_g}{2RC}$; $\qquad \dfrac{0 - v_a}{R} + C\dfrac{d(0 - v_b)}{dt} = 0$

(2) Therefore $\frac{dv_b}{dt} + \frac{v_a}{RC} = 0, \quad v_a = -RC\frac{dv_b}{dt}$

$$\frac{2v_b}{R} + C\frac{dv_b}{dt} + C\frac{d(v_b - v_o)}{dt} = 0$$

(3) Therefore $\frac{dv_b}{dt} + \frac{v_b}{RC} = \frac{1}{2}\frac{dv_o}{dt}$

From (2) we have $\frac{dv_a}{dt} = -RC\frac{d^2v_b}{dt^2}$ and $v_a = -RC\frac{dv_b}{dt}$

When these are substituted into (1) we get

(4) $-RC\frac{d^2v_b}{dt^2} - \frac{dv_b}{dt} = \frac{v_g}{2RC}$

Now differentiate (3) to get

(5) $\frac{d^2v_b}{dt^2} + \frac{1}{RC}\frac{dv_b}{dt} = \frac{1}{2}\frac{d^2v_o}{dt^2}$

But from (4) we have

(6) $\frac{d^2v_b}{dt^2} + \frac{1}{RC}\frac{dv_b}{dt} = -\frac{v_g}{2R^2C^2}$

Now substitute (6) into (5)

$$\frac{d^2v_o}{dt^2} = -\frac{v_g}{R^2C^2}$$

[b] When $R_1C_1 = R_2C_2 = RC$: $\frac{d^2v_o}{dt^2} = \frac{v_g}{R^2C^2}$

The two equations are the same except for a reversal in algebraic sign.

[c] Two integrations of the input signal with one operational amplifier.

P 8.61 [a] $f(t) = \text{inertial force} + \text{frictional force} + \text{spring force}$

$$= M[d^2x/dt^2] + D[dx/dt] + Kx$$

[b] $\frac{d^2x}{dt^2} = \frac{f}{M} - \left(\frac{D}{M}\right)\left(\frac{dx}{dt}\right) - \left(\frac{K}{M}\right)x$

Given $v_A = \frac{d^2x}{dt^2}$, then

$$v_B = -\frac{1}{R_1C_1}\int_0^t \left(\frac{d^2x}{dy^2}\right) dy = -\frac{1}{R_1C_1}\frac{dx}{dt}$$

$$v_C = -\frac{1}{R_2C_2}\int_0^t v_B\, dy = \frac{1}{R_1R_2C_1C_2}x$$

$$v_D = -\frac{R_3}{R_4}\cdot v_B = \frac{R_3}{R_4R_1C_1}\frac{dx}{dt}$$

$$v_E = \left[\frac{R_5+R_6}{R_6}\right]v_C = \left[\frac{R_5+R_6}{R_6}\right]\cdot\frac{1}{R_1R_2C_1C_2}\cdot x$$

$$v_F = \left[\frac{-R_8}{R_7}\right]f(t), \qquad v_A = -(v_D+v_E+v_F)$$

Therefore $\dfrac{d^2x}{dt^2} = \left[\dfrac{R_8}{R_7}\right]f(t) - \left[\dfrac{R_3}{R_4R_1C_1}\right]\dfrac{dx}{dt} - \left[\dfrac{R_5+R_6}{R_6R_1R_2C_1C_2}\right]x$

Therefore $M = \dfrac{R_7}{R_8}$, $\quad D = \dfrac{R_3R_7}{R_8R_4R_1C_1}$ and $K = \dfrac{R_7(R_5+R_6)}{R_8R_6R_1R_2C_1C_2}$

Box Number	Function
1	inverting and scaling
2	inverting and scaling
3	integrating and scaling
4	integrating and scaling
5	inverting and scaling
6	noninverting and scaling

P 8.62 [a] Given that the current response is underdamped we know i will be of the form

$$i = I_f + [B_1' \cos\omega_d t + B_2' \sin\omega_d t]e^{-\alpha t}$$

where $$\alpha = \frac{R}{2L}$$

and $$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{\frac{1}{LC} - \alpha^2}$$

The capacitor will force the final value of i to be zero, therefore $I_f = 0$.
By hypothesis $i(0^+) = V_{dc}/R$ therefore $B_1' = V_{dc}/R$.
At $t = 0^+$ the voltage across the primary winding is zero hence $di(0^+)/dt = 0$.
From our equation for i we have

$$\frac{di}{dt} = [(\omega_d B_2' - \alpha B_1')\cos\omega_d t - (\omega_d B_1' + \alpha B_2')\sin\omega_d t]e^{-\alpha t}$$

Hence

$$\frac{di(0^+)}{dt} = \omega_d B_2' - \alpha B_1' = 0$$

Thus

$$B_2' = \frac{\alpha}{\omega_d} B_1' = \frac{\alpha V_{dc}}{\omega_d R}$$

It follows directly that

$$i = \frac{V_{dc}}{R}\left[\cos\omega_d t + \frac{\alpha}{\omega_d}\sin\omega_d t\right] e^{-\alpha t}$$

[b] Since $\omega_d B_1' - \alpha B_1' = 0$ it follows that

$$\frac{di}{dt} = -(\omega_d B_1' + \alpha B_2')e^{-\alpha t}\sin\omega_d t$$

But $\alpha B_2' = \dfrac{\alpha^2 V_{dc}}{\omega_d R}$ and $\omega_d B_1' = \dfrac{\omega_d V_{dc}}{R}$

Therefore

$$\omega_d B_1' + \alpha B_2' = \frac{\omega_d V_{dc}}{R} + \frac{\alpha^2 V_{dc}}{\omega_d R} = \frac{V_{dc}}{R}\left[\frac{\omega_d^2 + \alpha^2}{\omega_d}\right]$$

But $\omega_d^2 + \alpha^2 = \omega_o^2 = \dfrac{1}{LC}$

Hence

$$\omega_d B_1' + \alpha B_2' = \frac{V_{dc}}{\omega_d RLC}$$

Now since $v_1 = L\dfrac{di}{dt}$ we get

$$v_1 = -L\frac{V_{dc}}{\omega_d RLC}e^{-\alpha t}\sin\omega_d t = -\frac{V_{dc}}{\omega_d RC}e^{-\alpha t}\sin\omega_d t$$

[c] $v_c = V_{dc} - iR - L\dfrac{di}{dt}$

$$iR = V_{dc}\left(\cos\omega_d t + \frac{\alpha}{\omega_d}\sin\omega_d t\right)e^{-\alpha t}$$

$$v_c = V_{dc} - V_{dc}\left(\cos\omega_d t + \frac{\alpha}{\omega_d}\sin\omega_d t\right)e^{-\alpha t} + \frac{V_{dc}}{\omega_d RC}e^{-\alpha t}\sin\omega_d t$$

$$= V_{dc} - V_{dc}e^{-\alpha t}\cos\omega_d t + \left(\frac{V_{dc}}{\omega_d RC} - \frac{\alpha V_{dc}}{\omega_d}\right)e^{-\alpha t}\sin\omega_d t$$

$$= V_{dc}\left[1 - e^{-\alpha t}\cos\omega_d t + \frac{1}{\omega_d}\left(\frac{1}{RC} - \alpha\right)e^{-\alpha t}\sin\omega_d t\right]$$

$$= V_{dc}\left[1 - e^{-\alpha t}\cos\omega_d t + Ke^{-\alpha t}\sin\omega_d t\right]$$

P 8.63 $v_{sp} = V_{dc}\left[1 - \frac{a}{\omega_d RC}e^{-\alpha t}\sin\omega_d t\right]$

$$\frac{dv_{sp}}{dt} = \frac{-aV_{dc}}{\omega_d RC}\frac{d}{dt}[e^{-\alpha t}\sin\omega_d t]$$

$$= \frac{-aV_{dc}}{\omega_d RC}[-\alpha e^{-\alpha t}\sin\omega_d t + \omega_d\cos\omega_d t e^{-\alpha t}]$$

$$= \frac{aV_{dc}e^{-\alpha t}}{\omega_d RC}[\alpha\sin\omega_d t - \omega_d\cos\omega_d t]$$

$$\frac{dv_{sp}}{dt} = 0 \quad \text{when} \quad \alpha\sin\omega_d t = \omega_d\cos\omega_d t$$

$$\text{or} \quad \tan\omega_d t = \frac{\omega_d}{\alpha}; \qquad \omega_d t = \tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$$

$$\therefore\ t_{\max} = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$$

Note that because $\tan\theta$ is periodic, i.e., $\tan\theta = \tan(\theta \pm n\pi)$, where n is an integer, there are an infinite number of solutions for t where $dv_{sp}/dt = 0$, that is

$$t = \frac{\tan^{-1}(\omega_d/\alpha) \pm n\pi}{\omega_d}$$

Because of $e^{-\alpha t}$ in the expression for v_{sp} and knowing $t \geq 0$ we know v_{sp} will be maximum when t has its smallest positive value. Hence

$$t_{\max} = \frac{\tan^{-1}(\omega_d/\alpha)}{\omega_d}.$$

P 8.64 [a] $v_c = V_{dc}[1 - e^{-\alpha t}\cos\omega_d t + Ke^{-\alpha t}\sin\omega_d t]$

$$\frac{dv_c}{dt} = V_{dc}\frac{d}{dt}[1 + e^{-\alpha t}(K\sin\omega_d t - \cos\omega_d t)]$$

$$= V_{dc}\{(-\alpha e^{-\alpha t})(K\sin\omega_d t - \cos\omega_d t) + e^{-\alpha t}[\omega_d K\cos\omega_d t + \omega_d\sin\omega_d t]\}$$

$$= V_{dc}e^{-\alpha t}[(\omega_d - \alpha K)\sin\omega_d t + (\alpha + \omega_d K)\cos\omega_d t]$$

$$\frac{dv_c}{dt} = 0 \quad \text{when} \quad (\omega_d - \alpha K)\sin\omega_d t = -(\alpha + \omega_d K)\cos\omega_d t$$

or $\tan\omega_d t = \left[\dfrac{\alpha+\omega_d K}{\alpha K-\omega_d}\right]$

$$\therefore \quad \omega_d t \pm n\pi = \tan^{-1}\left[\frac{\alpha+\omega_d K}{\alpha K-\omega_d}\right]$$

$$t_c = \frac{1}{\omega_d}\left\{tan^{-1}\left(\frac{\alpha+\omega_d K}{\alpha K-\omega_d}\right) \pm n\pi\right\}$$

$$\alpha = \frac{R}{2L} = \frac{4\times 10^3}{6} = 666.67\,\text{rad/s}$$

$$\omega_d = \sqrt{\frac{10^9}{1.2} - (666.67)^2} = 28{,}859.81\,\text{rad/s}$$

$$K = \frac{1}{\omega_d}\left(\frac{1}{RC} - \alpha\right) = 21.63$$

$$t_c = \frac{1}{\omega_d}\left\{\tan^{-1}(-43.29) + n\pi\right\} = \frac{1}{\omega_d}\{-1.55+n\pi\}$$

The smallest positive value of t occurs when $n = 1$, therefore

$t_{c\,\text{max}} = 55.23\,\mu\text{s}$

[b] $v_c(t_{c\,\text{max}}) = 12[1 - e^{-\alpha t_{c\,\text{max}}}\cos\omega_d t_{c\,\text{max}} + Ke^{-\alpha t_{c\,\text{max}}}\sin\omega_d t_{c\,\text{max}}]$

$= 262.42\,\text{V}$

[c] From the text example the voltage across the spark plug reaches its maximum value in 53.63 μs. If the spark plug does not fire the capacitor voltage peaks in 55.23 μs. When v_{sp} is maximum the voltage across the capacitor is 262.15 V. If the spark plug does not fire the capacitor voltage reaches 262.42 V.

P 8.65 [a] $w = \frac{1}{2}L[i(0^+)]^2 = \frac{1}{2}(5)(16)\times 10^{-3} = 40\,\text{mJ}$

[b] $\alpha = \dfrac{R}{2L} = \dfrac{3\times 10^3}{10} = 300\,\text{rad/s}$

$$\omega_d = \sqrt{\frac{10^9}{1.25} - (300)^2} = 28{,}282.68\,\text{rad/s}$$

$$\frac{1}{Rc} = \frac{10^6}{0.75} = \frac{4\times 10^6}{3}$$

$$t_{\text{max}} = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right) = 55.16\,\mu\text{s}$$

$$v_{sp}\,(t_{\text{max}}) = 12 - \frac{12(50)(4\times 10^6)}{3(28{,}282.68)}e^{-\alpha t_{\text{max}}}\sin\omega_d t_{\text{max}} = -27{,}808.04\,\text{V}$$

[c] $v_c\,(t_{\max}) = 12[1 - e^{-\alpha t_{\max}} \cos \omega_d t_{\max} + K e^{-\alpha t_{\max}} \sin \omega_d t_{\max}]$

$$K = \frac{1}{\omega_d}\left[\frac{1}{RC} - \alpha\right] = 47.13$$

$$v_c\,(t_{\max}) = 568.15\,\text{V}$$