9

Sinusoidal Steady State Analysis

Assessment Problems

AP 9.1 **[a]** $\mathbf{V} = 170\underline{/-40^\circ}$ V

[b] $10\sin(1000t + 20^\circ) = 10\cos(1000t - 70^\circ)$

$$\therefore \qquad \mathbf{I} = 10\underline{/-70^\circ}\text{ A}$$

[c] $\mathbf{I} = 5\underline{/36.87^\circ} + 10\underline{/-53.13^\circ}$

$$= 4 + j3 + 6 - j8 = 10 - j5 = 11.18\underline{/-26.57^\circ}\text{ A}$$

[d] $\sin(20{,}000\pi t + 30^\circ) = \cos(20{,}000\pi t - 60^\circ)$

Thus,

$$\mathbf{V} = 300\underline{/45^\circ} - 100\underline{/-60^\circ} = 212.13 + j212.13 - (50 - j86.60)$$

$$= 162.13 + j298.73 = 339.90\underline{/61.51^\circ}\text{ mV}$$

AP 9.2 **[a]** $v = 18.6\cos(\omega t - 54^\circ)$ V

[b] $\mathbf{I} = 20\underline{/45^\circ} - 50\underline{/-30^\circ} = 14.14 + j14.14 - 43.3 + j25$

$$= -29.16 + j39.14 = 48.81\underline{/126.68^\circ}$$

Therefore $\quad i = 48.81\cos(\omega t + 126.68^\circ)$ mA

[c] $\mathbf{V} = 20 + j80 - 30\underline{/15^\circ} = 20 + j80 - 28.98 - j7.76$

$$= -8.98 + j72.24 = 72.79\underline{/97.08^\circ}$$

$$v = 72.79\cos(\omega t + 97.08^\circ)\text{ V}$$

AP 9.3 **[a]** $\omega L = (10^4)(20 \times 10^{-3}) = 200\,\Omega$

[b] $Z_L = j\omega L = j200\,\Omega$

[c] $\mathbf{V}_L = \mathbf{I}Z_L = (10\underline{/30^\circ})(200\underline{/90^\circ}) \times 10^{-3} = 2\underline{/120^\circ}\,\text{V}$

[d] $v_L = 2\cos(10{,}000t + 120^\circ)\,\text{V}$

AP 9.4 **[a]** $X_C = \dfrac{-1}{\omega C} = \dfrac{-1}{4000(5 \times 10^{-6})} = -50\,\Omega$

[b] $Z_C = jX_C = -j50\,\Omega$

[c] $\mathbf{I} = \dfrac{\mathbf{V}}{Z_C} = \dfrac{30\underline{/25^\circ}}{50\underline{/-90^\circ}} = 0.6\underline{/115^\circ}\,\text{A}$

[d] $i = 0.6\cos(4000t + 115^\circ)\,\text{A}$

AP 9.5 $\mathbf{I}_1 = 100\underline{/25^\circ} = 90.63 + j42.26$

$\mathbf{I}_2 = 100\underline{/145^\circ} = -81.92 + j57.36$

$\mathbf{I}_3 = 100\underline{/-95^\circ} = -8.72 - j99.62$

$\mathbf{I}_4 = -(\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3) = (0 + j0)\,\text{A},$ therefore $i_4 = 0\,\text{A}$

AP 9.6 **[a]** $\mathbf{I} = \dfrac{125\underline{/-60^\circ}}{|Z|\underline{/\theta_z}} = \dfrac{125}{|Z|}\underline{/(-60 - \theta_Z)^\circ}$

But $-60 - \theta_Z = -105^\circ \quad \therefore \quad \theta_Z = 45^\circ$

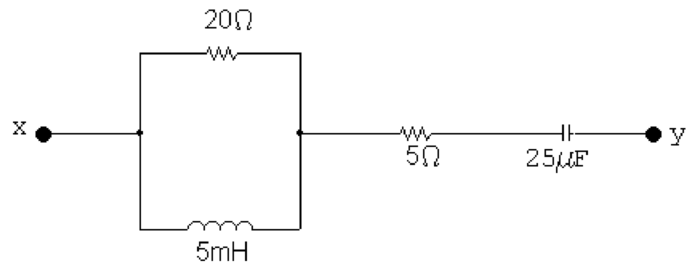
$Z = 90 + j160 + jX_C$

$\therefore \quad X_C = -70\,\Omega; \qquad X_C = -\dfrac{1}{\omega C} = -70$

$\therefore \quad C = \dfrac{1}{(70)(5000)} = 2.86\,\mu\text{F}$

[b] $\mathbf{I} = \dfrac{\mathbf{V}_s}{Z} = \dfrac{125\underline{/-60^\circ}}{(90 + j90)} = 0.982\underline{/-105^\circ}A; \qquad \therefore \quad |\mathbf{I}| = 0.982\,\text{A}$

AP 9.7 **[a]**



$$\omega = 2000\,\text{rad/s}$$

$$\omega L = 10\,\Omega, \qquad \frac{-1}{\omega C} = -20\,\Omega$$

$$Z_{xy} = 20\|j10 + 5 + j20 = \frac{20(j10)}{(20 + j10)} + 5 - j20$$

$$= 4 + j8 + 5 - j20 = (9 - j12)\,\Omega$$

[b] $\omega L = 40\,\Omega, \qquad \dfrac{-1}{\omega C} = -5\,\Omega$

$$Z_{xy} = 5 - j5 + 20\|j40 = 5 - j5 + \left[\frac{(20)(j40)}{20 + j40}\right]$$

$$= 5 - j5 + 16 + j8 = (21 + j3)\,\Omega$$

[c] $Z_{xy} = \left[\dfrac{20(j\omega L)}{20 + j\omega L}\right] + \left(5 - \dfrac{j10^6}{25\omega}\right)$

$$= \frac{20\omega^2 L^2}{400 + \omega^2 L^2} + \frac{j400\omega L}{400 + \omega^2 L^2} + 5 - \frac{j10^6}{25\omega}$$

The impedance will be purely resistive when the j terms cancel, i.e.,

$$\frac{400\omega L}{400 + \omega^2 L^2} = \frac{10^6}{25\omega}$$

Solving for ω yields $\omega = 4000\,\text{rad/s}$.

[d] $Z_{xy} = \dfrac{20\omega^2 L^2}{400 + \omega^2 L^2} + 5 = 10 + 5 = 15\,\Omega$

AP 9.8 The frequency 4000 rad/s was found to give $Z_{\text{xy}} = 15\,\Omega$ in Assessment Problem 9.7. Thus,

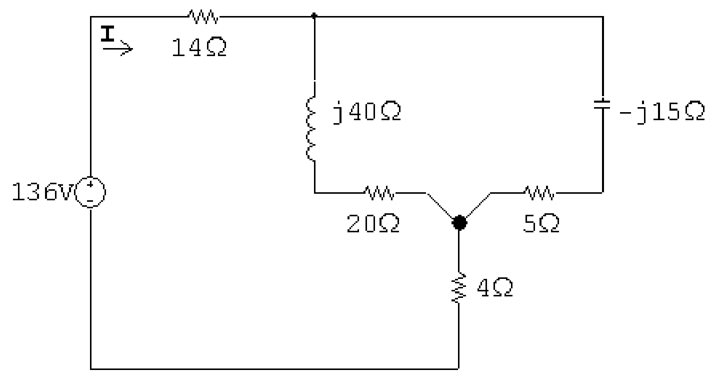
$$\mathbf{V} = 150\underline{/0^\circ}, \qquad \mathbf{I}_s = \frac{\mathbf{V}}{Z_{\text{xy}}} = \frac{150\underline{/0^\circ}}{15} = 10\underline{/0^\circ}\,\text{A}$$

Using current division,

$$\mathbf{I}_L = \frac{20}{20 + j20}(10) = 5 - j5 = 7.07\underline{/-45^\circ}\,\text{A}$$

$$i_L = 7.07\cos(4000t - 45^\circ)\,\text{A}, \qquad I_m = 7.07\,\text{A}$$

AP 9.9 After replacing the delta made up of the $50\,\Omega$, $40\,\Omega$, and $10\,\Omega$ resistors with its equivalent wye, the circuit becomes



The circuit is further simplified by combining the parallel branches,

$$(20 + j40)\|(5 - j15) = (12 - j16)\,\Omega$$

Therefore $\quad \mathbf{I} = \dfrac{136\angle 0^\circ}{14 + 12 - j16 + 4} = 4\angle 28.07^\circ\,\text{A}$

AP 9.10 $\mathbf{V}_1 = 240\angle 53.13^\circ = 144 + j192\,\text{V}$

$$\mathbf{V}_2 = 96\angle -90^\circ = -j96\,\text{V}$$

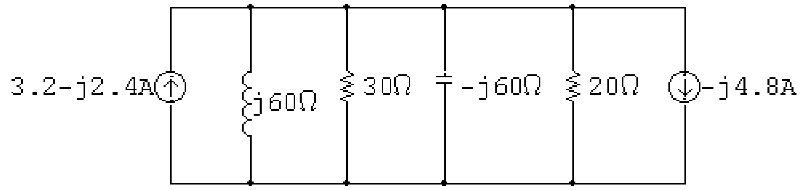
$$j\omega L = j(4000)(15 \times 10^{-3}) = j60\,\Omega$$

$$\frac{1}{j\omega C} = -j\frac{6 \times 10^6}{(4000)(25)} = -j60\,\Omega$$

Perform source transformations:

$$\frac{\mathbf{V}_1}{j60} = \frac{144 + j192}{j60} = 3.2 - j2.4\,\text{A}$$

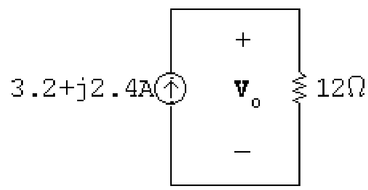
$$\frac{\mathbf{V}_2}{20} = -j\frac{96}{20} = -j4.8\,\text{A}$$



Combine the parallel impedances:

$$Y = \frac{1}{j60} + \frac{1}{30} + \frac{1}{-j60} + \frac{1}{20} = \frac{j5}{j60} = \frac{1}{12}$$

$$Z = \frac{1}{Y} = 12\,\Omega$$



$$\mathbf{V}_o = 12(3.2 + j2.4) = 38.4 + j28.8\,\text{V} = 48\angle 36.87^\circ\,\text{V}$$

$$v_o = 48\cos(4000t + 36.87^\circ)\,\text{V}$$

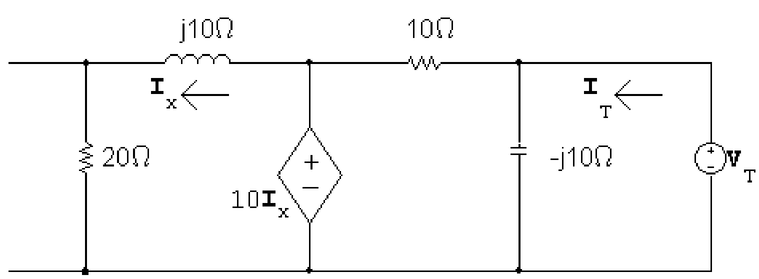
AP 9.11 Use the lower node as the reference node. Let $\mathbf{V}_1$ = node voltage across the $20\,\Omega$ resistor and $\mathbf{V}_{\text{Th}}$ = node voltage across the capacitor. Writing the node voltage equations gives us

$$\frac{\mathbf{V}_1}{20} - 2\angle 45^\circ + \frac{\mathbf{V}_1 - 10\mathbf{I}_x}{j10} = 0 \quad \text{and} \quad \mathbf{V}_{\text{Th}} = \frac{-j10}{10 - j10}(10\mathbf{I}_x)$$

We also have

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{20}$$

Solving these equations for $\mathbf{V}_{\text{Th}}$ gives $\mathbf{V}_{\text{Th}} = 10\angle 45^\circ V$. To find the Thévenin impedance, we remove the independent current source and apply a test voltage source at the terminals a, b. Thus



It follows from the circuit that

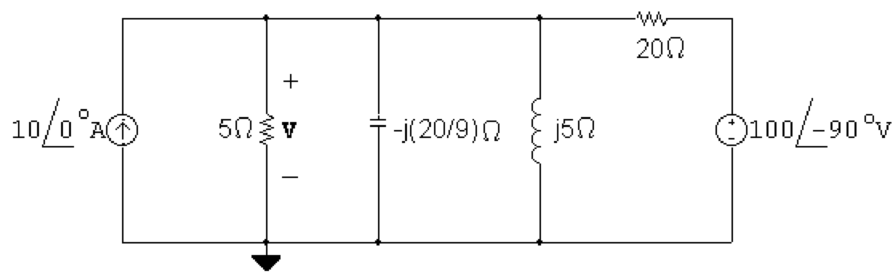
$$10\mathbf{I}_x = (20 + j10)\mathbf{I}_x$$

Therefore

$$\mathbf{I}_x = 0 \quad \text{and} \quad \mathbf{I}_T = \frac{\mathbf{V}_T}{-j10} + \frac{\mathbf{V}_T}{10}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T}, \quad \text{therefore} \quad Z_{\text{Th}} = (5 - j5)\,\Omega$$

AP 9.12 The phasor domain circuit is as shown in the following diagram:



The node voltage equation is

$$-10 + \frac{\mathbf{V}}{5} + \frac{\mathbf{V}}{-j(20/9)} + \frac{\mathbf{V}}{j5} + \frac{\mathbf{V} - 100\underline{/-90^\circ}}{20} = 0$$

Therefore $\mathbf{V} = 10 - j30 = 31.62\underline{/-71.57^\circ}$

Therefore $v = 31.62\cos(50{,}000t - 71.57^\circ)$ V

AP 9.13 Let $\mathbf{I}_{\text{a}}$, $\mathbf{I}_{\text{b}}$, and $\mathbf{I}_{\text{c}}$ be the three clockwise mesh currents going from left to right. Summing the voltages around meshes a and b gives

$$33.8 = (1 + j2)\mathbf{I}_{\text{a}} + (3 - j5)(\mathbf{I}_{\text{a}} - \mathbf{I}_{\text{b}})$$

and

$$0 = (3 - j5)(\mathbf{I}_{\text{b}} - \mathbf{I}_{\text{a}}) + 2(\mathbf{I}_{\text{b}} - \mathbf{I}_{\text{c}}).$$

But

$$\mathbf{V}_x = -j5(\mathbf{I}_{\text{a}} - \mathbf{I}_{\text{b}}),$$

therefore

$$\mathbf{I}_{\text{c}} = -0.75[-j5(\mathbf{I}_{\text{a}} - \mathbf{I}_{\text{b}})].$$

Solving for $\mathbf{I} = \mathbf{I}_{\text{a}} = 29 + j2 = 29.07\underline{/3.95^\circ}$ A.

AP 9.14 **[a]** $M = 0.4\sqrt{0.0625} = 0.1\,\text{H}, \qquad \omega M = 80\,\Omega$

$$Z_{22} = 40 + j800(0.125) + 360 + j800(0.25) = (400 + j300)\,\Omega$$

Therefore $|Z_{22}| = 500\,\Omega, \qquad Z_{22}^* = (400 - j300)\,\Omega$

$$Z_r = \left(\frac{80}{500}\right)^2 (400 - j300) = (10.24 - j7.68)\,\Omega$$

[b] $\mathbf{I}_1 = \dfrac{245.20}{184 + 100 + j400 + Z_r} = 0.50\underline{/-53.13^\circ}\,\text{A}$

$$i_1 = 0.5\cos(800t - 53.13^\circ)\,\text{A}$$

[c] $\mathbf{I}_2 = \left(\dfrac{j\omega M}{Z_{22}}\right)\mathbf{I}_1 = \dfrac{j80}{500\underline{/36.87^\circ}}(0.5\underline{/-53.13^\circ}) = 0.08\underline{/0^\circ}\,\text{A}$

$$i_2 = 80\cos 800t\,\text{mA}$$

AP 9.15 $\mathbf{I}_1 = \dfrac{\mathbf{V}_s}{Z_1 + Z_2/a^2} = \dfrac{25 \times 10^3\underline{/0^\circ}}{1500 + j6000 + (25)^2(4 - j14.4)}$

$$= 4 + j3 = 5\underline{/36.87^\circ}\,\text{A}$$

$$\mathbf{V}_1 = \mathbf{V}_s - Z_1\mathbf{I}_1 = 25{,}000\underline{/0^\circ} - (4 + j3)(1500 + j6000)$$

$$= 37{,}000 - j28{,}500$$

$$\mathbf{V}_2 = -\frac{1}{25}\mathbf{V}_1 = -1480 + j1140 = 1868.15\underline{/142.39^\circ}\,\text{V}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{Z_2} = \frac{1868.15\underline{/142.39^\circ}}{4 - j14.4} = 125\underline{/-143.13^\circ}\,\text{A}$$

Also, $I_2 = -25I_1$

Problems

P 9.12 **[a]** 1000Hz

[b] $\theta_v = 0^\circ$

[c] $\mathbf{I} = \dfrac{200\underline{/0^\circ}}{j\omega L} = \dfrac{200}{\omega L}\underline{/-90^\circ} = 25\underline{/-90^\circ}; \qquad \theta_i = -90^\circ$

[d] $\dfrac{200}{\omega L} = 25; \qquad \omega L = \dfrac{200}{25} = 8\,\Omega$

[e] $L = \dfrac{8}{2\pi(1000)} = 1.27\,\text{mH}$

[f] $Z_L = j\omega L = j8\,\Omega$

P 9.13 **[a]** $\omega = 2\pi f = 314{,}159.27\,\text{rad/s}$

[b] $\mathbf{I} = \dfrac{\mathbf{V}}{Z_C} = \dfrac{10\times 10^{-3}\underline{/0^\circ}}{1/j\omega C} = j\omega C(10\times 10^{-3})\underline{/0^\circ} = 10\times 10^{-3}\omega C\underline{/90^\circ}$

$\therefore \quad \theta_i = 90^\circ$

[c] $628.32\times 10^{-6} = 10\times 10^{-3}\,\omega C$

$$\frac{1}{\omega C} = \frac{10\times 10^{-3}}{628.32\times 10^{-6}} = 15.92\,\Omega, \quad \therefore \quad X_{\text{C}} = -15.92\,\Omega$$

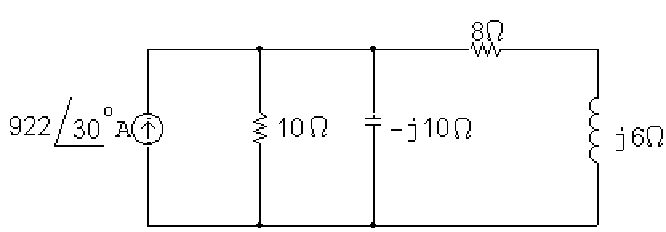
[d] $C = \dfrac{1}{15.92(\omega)} = \dfrac{1}{(15.92)(100\pi\times 10^3)}$

$C = 0.2\,\mu\text{F}$

[e] $Z_c = j\left(\dfrac{-1}{\omega C}\right) = -j15.92\,\Omega$

P 9.14 **[a]** $j\omega L = j(2\times 10^4)(300\times 10^{-6}) = j6\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{1}{(2\times 10^4)(5\times 10^{-6})} = -j10\,\Omega; \qquad \mathbf{I}_g = 922\underline{/30^\circ}\,\text{A}$$



[b] $\mathbf{V}_o = 922\underline{/30^\circ} Z_e$

$$Z_e = \frac{1}{Y_e}; \qquad Y_e = \frac{1}{10} + j\frac{1}{10} + \frac{1}{8 + j6}$$

$$Y_e = 0.18 + j0.04\,\mathrm{S}$$

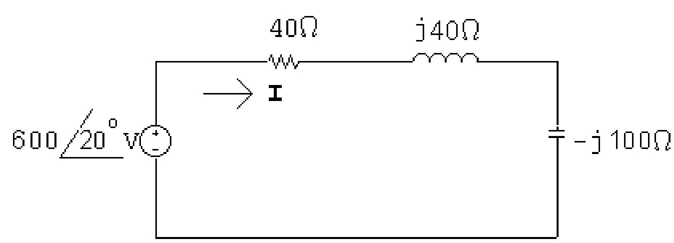
$$Z_e = \frac{1}{0.18 + j0.04} = 5.42\underline{/-12.53^\circ}\,\Omega$$

$$\mathbf{V}_o = (922\underline{/30^\circ})(5.42\underline{/-12.53^\circ}) = 5000.25\underline{/17.47^\circ}\,\mathrm{V}$$

[c] $v_o = 5000.25\cos(2 \times 10^4 t + 17.47^\circ)\,\mathrm{V}$

P 9.15 **[a]** $Z_L = j(8000)(5 \times 10^{-3}) = j40\,\Omega$

$$Z_C = \frac{-j}{(8000)(1.25 \times 10^{-6})} = -j100\,\Omega$$



[b] $\mathbf{I} = \dfrac{600\underline{/20^\circ}}{40 + j40 - j100} = 8.32\underline{/76.31^\circ}\,\mathrm{A}$

[c] $i = 8.32\cos(8000t + 76.31^\circ)\,\mathrm{A}$

P 9.16 $Z = 4 + j(50)(0.24) - j\dfrac{1}{(50)(0.0025)} = 4 + j4 = 5.66\underline{/45^\circ}\,\Omega$

$$\mathbf{I}_o = \frac{\mathbf{V}}{Z} = \frac{0.1\underline{/-90^\circ}}{5.66\underline{/45^\circ}} = 17.68\underline{/-135^\circ}\,\mathrm{mA}$$

$$i_o(t) = 17.68\cos(50t - 135^\circ)\,\mathrm{mA}$$

P 9.17 **[a]** $Y = \frac{1}{3+j4} + \frac{1}{16-j12} + \frac{1}{-j4}$

$= 0.12 - j0.16 + 0.04 + j0.03 + j0.25$

$= 0.16 + j0.12 = 200\underline{/36.87^\circ}$ mS

[b] $G = 160$ mS

[c] $B = 120$ mS

[d] $\mathbf{I} = 8\underline{/0^\circ}$ A, $\quad \mathbf{V} = \frac{\mathbf{I}}{Y} = \frac{8}{0.2\underline{/36.87^\circ}} = 40\underline{/-36.87^\circ}$ V

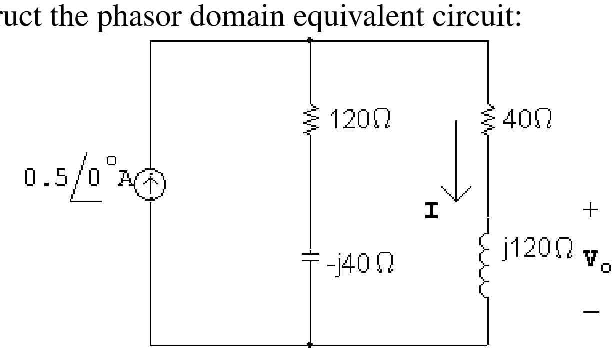
$$\mathbf{I}_C = \frac{\mathbf{V}}{Z_C} = \frac{40\underline{/-36.87^\circ}}{4\underline{/-90^\circ}} = 10\underline{/53.13^\circ}\text{ A}$$

$i_C = 10\cos(\omega t + 53.13^\circ)$ A, $\quad I_m = 10$ A

P 9.18 $Z_L = j(2000)(60 \times 10^{-3}) = j120\,\Omega$

$$Z_C = \frac{-j}{(2000)(12.5 \times 10^{-6})} = -j40\,\Omega$$

Construct the phasor domain equivalent circuit:



Using current division:

$$\mathbf{I} = \frac{(120 - j40)}{120 - j40 + 40 + j120}(0.5) = 0.25 - j0.25\text{ A}$$

$\mathbf{V}_o = j120\mathbf{I} = 30 + j30 = 42.43\underline{/45^\circ}$ V

$v_o = 42.43\cos(2000t + 45^\circ)$ V

P 9.19 **[a]** $\mathbf{V}_g = 300\underline{/78^\circ}$; $\mathbf{I}_g = 6\underline{/33^\circ}$

$$\therefore \quad Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = \frac{300\underline{/78^\circ}}{6\underline{/33^\circ}} = 50\underline{/45^\circ}\,\Omega$$

[b] i_g lags v_g by 45°:

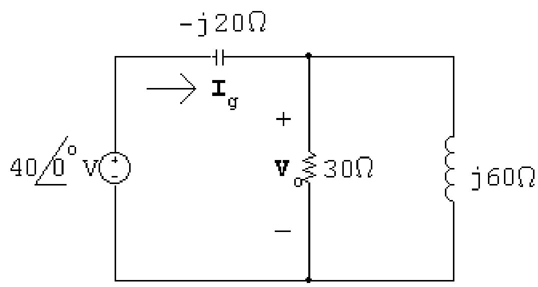
$$2\pi f = 5000\pi; \qquad f = 2500\,\text{Hz}; \qquad T = 1/f = 400\,\mu\text{s}$$

$$\therefore \quad i_g \text{ lags } v_g \text{ by } \frac{45^\circ}{360^\circ}(400\,\mu\text{s}) = 50\,\mu\text{s}$$

P 9.20 $$\frac{1}{j\omega C} = \frac{1}{(1\times 10^{-6})(50\times 10^{3})} = -j20\,\Omega$$

$$j\omega L = j50\times 10^{3}(1.2\times 10^{-3}) = j60\,\Omega$$

$$\mathbf{V}_g = 40\underline{/0^\circ}\,\text{V}$$



$$Z_e = -j20 + 30\|j60 = 24 - j8\,\Omega$$

$$\mathbf{I}_g = \frac{40\underline{/0^\circ}}{24 - j8} = 1.5 + j0.5\,\text{mA}$$

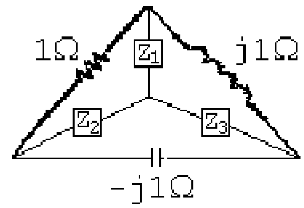
$$\mathbf{V}_o = (30\|j60)\mathbf{I}_g = \frac{30(j60)}{30 + j60}(1.5 + j0.5) = 30 + j30 = 42.43\underline{/45^\circ}\,\text{V}$$

$$v_o = 42.43\cos(50{,}000t + 45^\circ)\,\text{V}$$

P 9.35 Simplify the top triangle using series and parallel combinations:

$$(1 + j1)\|(1 - j1) = 1\,\Omega$$

Convert the lower left delta to a wye:

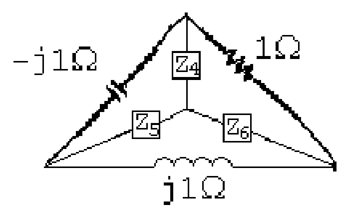


$$Z_1 = \frac{(j1)(1)}{1 + j1 - j1} = j1\,\Omega$$

$$Z_2 = \frac{(-j1)(1)}{1 + j1 - j1} = -j1\,\Omega$$

$$Z_3 = \frac{(j1)(-j1)}{1 + j1 - j1} = 1\,\Omega$$

Convert the lower right delta to a wye:

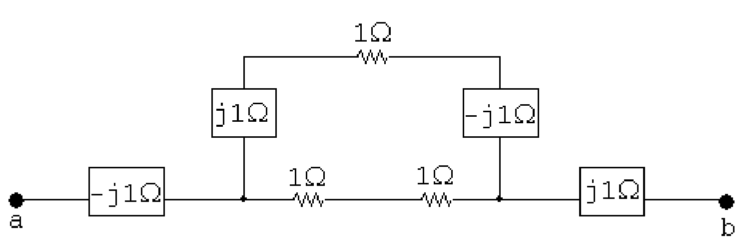


$$Z_4 = \frac{(-j1)(1)}{1 + j1 - j1} = -j1\,\Omega$$

$$Z_5 = \frac{(-j1)(j1)}{1 + j1 - j1} = 1\,\Omega$$

$$Z_6 = \frac{(j1)(1)}{1 + j1 - j1} = j1\,\Omega$$

The resulting circuit is shown below:



Simplify the middle portion of the circuit by making series and parallel combinations:

$$(1 + j1 - j1)\|(1 + 1) = 1\|2 = 2/3\,\Omega$$

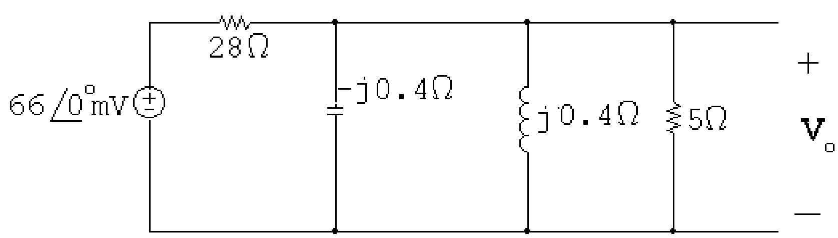
$$Z_{\rm ab} = -j1 + 2/3 + j1 = 2/3\,\Omega$$

P 9.39 $\mathbf{I}_s = 3\underline{/0^\circ}\,\text{mA}$

$$\frac{1}{j\omega C} = -j0.4\,\Omega$$

$$j\omega L = j0.4\,\Omega$$

After source transformation we have



$$\mathbf{V}_o = \frac{-j0.4\|j0.4\|5}{28 + -j0.4\|j0.4\|5}(66 \times 10^{-3}) = 10\,\text{mV}$$

$$v_o = 10\cos 200t\,\text{mV}$$

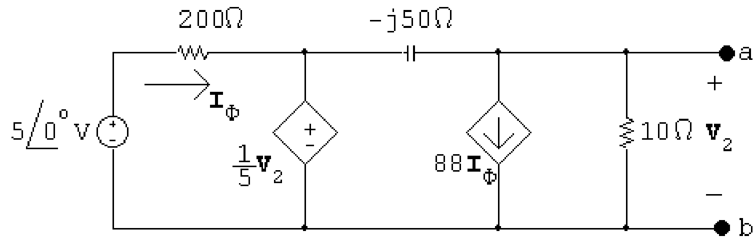
P 9.42 Using voltage division:

$$\mathbf{V}_{\rm Th} = \frac{36}{36 + j60 - j48}(240) = 216 - j72 = 227.68\underline{/-18.43^\circ}\,\text{V}$$

Remove the source and combine impedances in series and in parallel:

$$Z_{\rm Th} = 36\|(j60 - j48) = 3.6 + j10.8\,\Omega$$

P 9.43 Open circuit voltage:



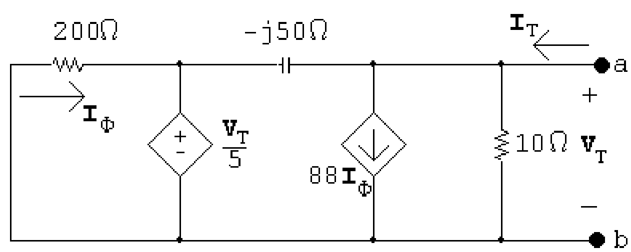
$$\frac{\mathbf{V}_2}{10} + 88\mathbf{I}_\phi + \frac{\mathbf{V}_2 - \frac{1}{5}\mathbf{V}_2}{-j50} = 0$$

$$\mathbf{I}_\phi = \frac{5 - (\mathbf{V}_2/5)}{200}$$

Solving,

$$\mathbf{V}_2 = -66 + j88 = 110\underline{/126.87^\circ}\,\text{V} = \mathbf{V}_{\text{Th}}$$

Find the Thévenin equivalent impedance using a test source:



$$\mathbf{I}_T = \frac{\mathbf{V}_T}{10} + 88\mathbf{I}_\phi + \frac{0.8\mathbf{V}_t}{-j50}$$

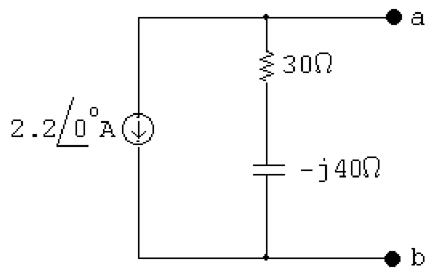
$$\mathbf{I}_\phi = \frac{-\mathbf{V}_T/5}{200}$$

$$\mathbf{I}_T = \mathbf{V}_T\left(\frac{1}{10} - 88\frac{1/5}{200} + \frac{0.8}{-j50}\right)$$

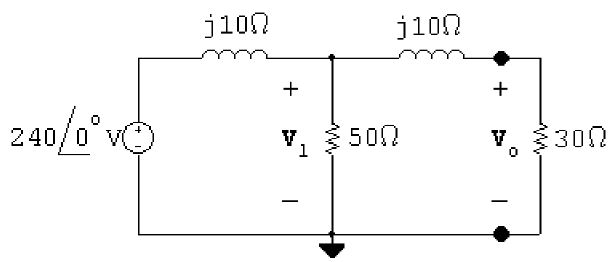
$$\therefore \qquad \frac{\mathbf{V}_T}{\mathbf{I}_T} = 30 - j40 = Z_{\text{Th}}$$

$$\mathbf{I}_{\text{N}} = \frac{\mathbf{V}_{\text{Th}}}{Z_{\text{Th}}} = \frac{-66 + j88}{30 - j40} = -2.2 + j0\,\text{A} = 2.2\underline{/180^\circ}\,\text{A}$$

The Norton equivalent circuit:



P 9.51



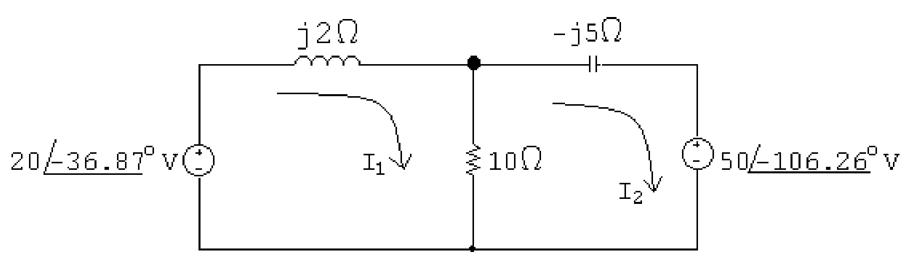
$$\frac{\mathbf{V}_1 - 240}{j10} + \frac{\mathbf{V}_1}{50} + \frac{\mathbf{V}_1}{30 + j10} = 0$$

Solving for $\mathbf{V}_1$ yields

$$\mathbf{V}_1 = 198.63\underline{/-24.44^\circ}\ \mathrm{V}$$

$$\mathbf{V}_o = \frac{30}{30 + j10}(\mathbf{V}_1) = 188.43\underline{/-42.88^\circ}\ \mathrm{V}$$

P 9.54 The circuit with the mesh currents identified is shown below:



The mesh current equations are:

$$-20\underline{/-36.87^\circ} + j2\mathbf{I}_1 + 10(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$50\underline{/-106.26^\circ} + 10(\mathbf{I}_2 - \mathbf{I}_1) - j5\mathbf{I}_2 = 0$$

In standard form:

$$\mathbf{I}_1(10 + j2) + \mathbf{I}_2(-10) = 20\underline{/-36.87^\circ}$$

$$\mathbf{I}_1(-10) + \mathbf{I}_2(10 - j5) = -50\underline{/-106.26^\circ} = 50\underline{/73.74^\circ}$$

Solving on a calculator yields:

$\mathbf{I}_1 = -6 + j10\text{A};$ $\quad\quad$ $\mathbf{I}_2 = -9.6 + j10\text{A}$

Thus,

$$\mathbf{V}_o = 10(\mathbf{I}_1 - \mathbf{I}_2) = 36\text{V}$$

and

$$v_o(t) = 36\cos 2000t\text{V}$$

P 9.56 Write a KCL equation at the top node:

$$\frac{\mathbf{V}_o}{-j8} + \frac{\mathbf{V}_o - 2.4\mathbf{I}_\Delta}{j4} + \frac{\mathbf{V}_o}{5} - (10 + j10) = 0$$

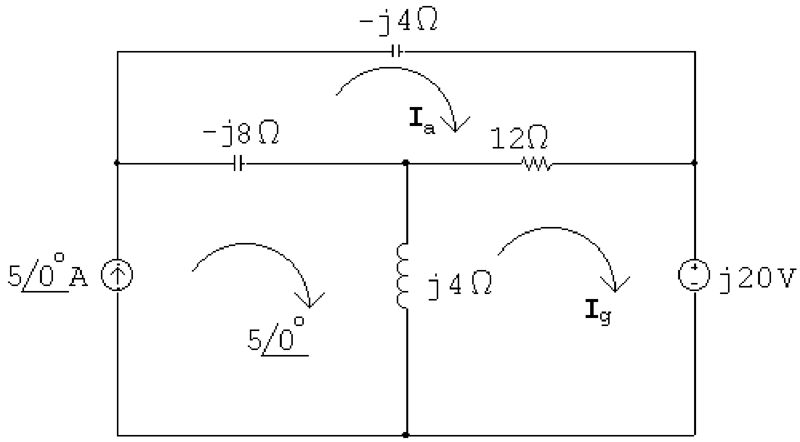
The constraint equation is:

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j8}$$

Solving,

$$\mathbf{V}_o = j80 = 80\underline{/90^\circ}\text{ V}$$

P 9.59



$$(12 - j12)\mathbf{I}_\mathrm{a} - 12\mathbf{I}_g - 5(-j8) = 0$$

$$-12\mathbf{I}_\mathrm{a} + (12 + j4)\mathbf{I}_g + j20 - 5(j4) = 0$$

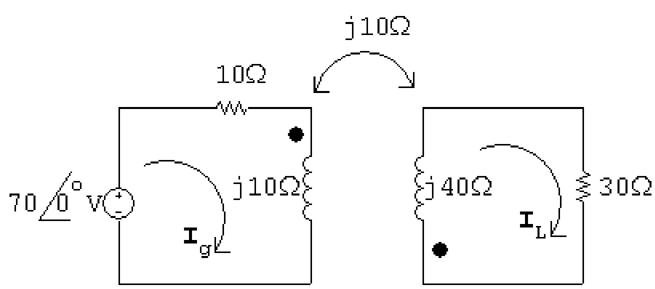
Solving,

$$\mathbf{I}_g = 4 - j2 = 4.47\underline{/-26.57^\circ}\,\mathrm{A}$$

P 9.66 **[a]** $j\omega L_1 = j(5000)(2 \times 10^{-3}) = j10\,\Omega$

$$j\omega L_2 = j(5000)(8 \times 10^{-3}) = j40\,\Omega$$

$$j\omega M = j10\,\Omega$$



$$70 = (10 + j10)\mathbf{I}_g + j10\mathbf{I}_L$$

$$0 = j10\mathbf{I}_g + (30 + j40)\mathbf{I}_L$$

Solving,

$\mathbf{I}_g = 4 - j3\,\mathrm{A}$; $\mathbf{I}_L = -1\,\mathrm{A}$

$$i_g = 5\cos(5000t - 36.87^\circ)\,\mathrm{A}$$

$$i_\mathrm{L} = 1\cos(5000t - 180^\circ)\,\mathrm{A}$$

[b] $k = \dfrac{M}{\sqrt{L_1 L_2}} = \dfrac{2}{\sqrt{16}} = 0.5$

[c] When $t = 100\pi\ \mu$s,

$$5000t = (5000)(100\pi) \times 10^{-6} = 0.5\pi = \pi/2\,\text{rad} = 90^\circ$$

$$i_g(100\pi\mu s) = 5\cos(53.13^\circ) = 3\,\text{A}$$

$$i_L(100\pi\mu s) = 1\cos(-90^\circ) = 0\,\text{A}$$

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2 = \frac{1}{2}(2 \times 10^{-3})(9) + 0 + 0 = 9\,\text{mJ}$$

When $t = 200\pi\ \mu$s,

$$5000t = \pi\,\text{rad} = 180^\circ$$

$$i_g(200\pi\mu s) = 5\cos(180^\circ - 36.87^\circ) = -4\,\text{A}$$

$$i_L(200\pi\mu s) = 1\cos(180^\circ - 180^\circ) = 1\,\text{A}$$

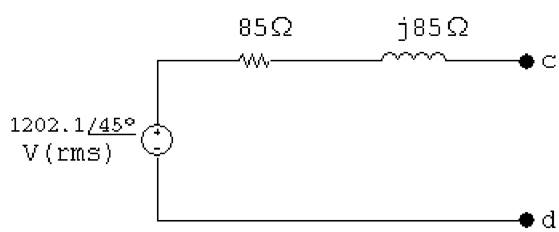
$$w = \frac{1}{2}(2 \times 10^{-3})(16) + \frac{1}{2}(8 \times 10^{-3})(1) + 2 \times 10^{-3}(-4)(1) = 12\,\text{mJ}$$

P 9.67 Remove the voltage source to find the equivalent impedance:

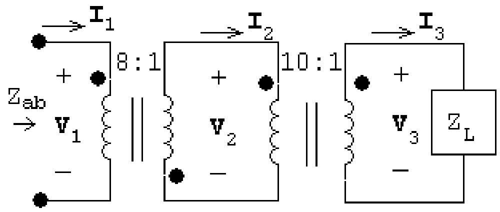
$$Z_{\text{Th}} = 45 + j125 + \left(\frac{20}{|5 + j5|}\right)^2 (5 - j5) = 85 + j85\,\Omega$$

Using voltage division:

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_{\text{cd}} = j20\mathbf{I}_1 = j20\left(\frac{425}{5 + j5}\right) = 850 + j850\,\text{V} = 1202.1\underline{/45^\circ}\ \text{V}$$



P 9.71



$$Z_L = \frac{\mathbf{V}_3}{\mathbf{I}_3} = 80\underline{/60^\circ}\ \Omega$$

$$\frac{\mathbf{V}_2}{10} = \frac{\mathbf{V}_3}{1}; \qquad 10\mathbf{I}_2 = 1\mathbf{I}_3$$

$$\frac{\mathbf{V}_1}{8} = -\frac{\mathbf{V}_2}{1}; \qquad 8\mathbf{I}_1 = -1\mathbf{I}_2$$

$$Z_{\rm ab} = \frac{\mathbf{V}_1}{\mathbf{I}_1}$$

Substituting,

$$Z_{\rm ab} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{-8\mathbf{V}_2}{-\mathbf{I}_2/8} = \frac{8^2\mathbf{V}_2}{\mathbf{I}_2}$$

$$= \frac{8^2(10\mathbf{V}_3)}{\mathbf{I}_3/10} = \frac{(8)^2(10)^2\mathbf{V}_3}{\mathbf{I}_3} = (8)^2(10)^2 Z_L = 512,000\underline{/60^\circ}\ \Omega$$