9

Sinusoidal Steady State Analysis

Assessment Problems

AP 9.1 [a] $\mathbf{V} = 170\underline{/-40^\circ}\,\text{V}$

[b] $10\sin(1000t + 20^\circ) = 10\cos(1000t - 70^\circ)$

$$\therefore \qquad \mathbf{I} = 10\underline{/-70^\circ}\,\text{A}$$

[c] $\mathbf{I} = 5\underline{/36.87^\circ} + 10\underline{/-53.13^\circ}$

$$= 4 + j3 + 6 - j8 = 10 - j5 = 11.18\underline{/-26.57^\circ}\,\text{A}$$

[d] $\sin(20{,}000\pi t + 30^\circ) = \cos(20{,}000\pi t - 60^\circ)$

Thus,

$$\mathbf{V} = 300\underline{/45^\circ} - 100\underline{/-60^\circ} = 212.13 + j212.13 - (50 - j86.60)$$

$$= 162.13 + j298.73 = 339.90\underline{/61.51^\circ}\,\text{mV}$$

AP 9.2 [a] $v = 18.6\cos(\omega t - 54^\circ)\,\text{V}$

[b] $\mathbf{I} = 20\underline{/45^\circ} - 50\underline{/-30^\circ} = 14.14 + j14.14 - 43.3 + j25$

$$= -29.16 + j39.14 = 48.81\underline{/126.68^\circ}$$

Therefore $\quad i = 48.81\cos(\omega t + 126.68^\circ)\,\text{mA}$

[c] $\mathbf{V} = 20 + j80 - 30\underline{/15^\circ} = 20 + j80 - 28.98 - j7.76$

$$= -8.98 + j72.24 = 72.79\underline{/97.08^\circ}$$

$$v = 72.79\cos(\omega t + 97.08^\circ)\,\text{V}$$

AP 9.3 [a] $\omega L = (10^4)(20 \times 10^{-3}) = 200\,\Omega$

[b] $Z_L = j\omega L = j200\,\Omega$

[c] $\mathbf{V}_L = \mathbf{I}Z_L = (10\underline{/30^\circ})(200\underline{/90^\circ}) \times 10^{-3} = 2\underline{/120^\circ}\,\text{V}$

[d] $v_L = 2\cos(10{,}000t + 120^\circ)\,\text{V}$

AP 9.4 [a] $X_C = \dfrac{-1}{\omega C} = \dfrac{-1}{4000(5 \times 10^{-6})} = -50\,\Omega$

[b] $Z_C = jX_C = -j50\,\Omega$

[c] $\mathbf{I} = \dfrac{\mathbf{V}}{Z_C} = \dfrac{30\underline{/25^\circ}}{50\underline{/-90^\circ}} = 0.6\underline{/115^\circ}\,\text{A}$

[d] $i = 0.6\cos(4000t + 115^\circ)\,\text{A}$

AP 9.5 $\mathbf{I}_1 = 100\underline{/25^\circ} = 90.63 + j42.26$

$\mathbf{I}_2 = 100\underline{/145^\circ} = -81.92 + j57.36$

$\mathbf{I}_3 = 100\underline{/-95^\circ} = -8.71 - j99.62$

$\mathbf{I}_4 = -(\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3) = (0 + j0)\,\text{A}, \qquad \text{therefore} \quad i_4 = 0\,\text{A}$

AP 9.6 [a] $\mathbf{I} = \dfrac{125\underline{/-60^\circ}}{|Z|\underline{/\theta_z}} = \dfrac{125}{|Z|}\underline{/(-60 - \theta_Z)^\circ}$

But $-60 - \theta_Z = -105^\circ \qquad \therefore\ \theta_Z = 45^\circ$

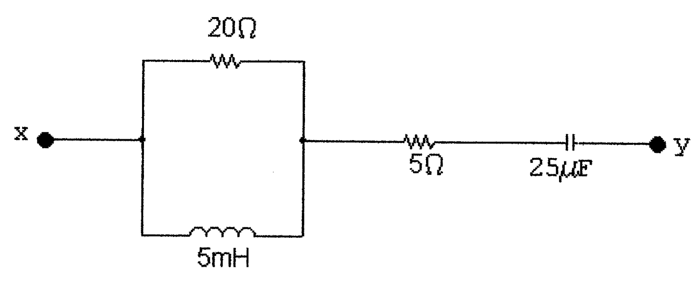
$Z = 90 + j160 + jX_C$

$\therefore\ X_C = -70\,\Omega; \qquad X_C = -\dfrac{1}{\omega C} = -70$

$\therefore\ C = \dfrac{1}{(70)(5000)} = 2.86\,\mu\text{F}$

[b] $\mathbf{I} = \dfrac{\mathbf{V}_s}{Z} = \dfrac{125\underline{/-60^\circ}}{(90 + j90)} = 0.982\underline{/-105^\circ}A; \qquad \therefore\ |\mathbf{I}| = 0.982\,\text{A}$

AP 9.7 [a]



$\omega = 2000\,\text{rad/s}$

$\omega L = 10\,\Omega, \qquad \dfrac{-1}{\omega C} = -20\,\Omega$

$Z_{xy} = 20\|j10 + 5 + j20 = \dfrac{20(j10)}{(20 + j10)} + 5 - j20$

$= 4 + j8 + 5 - j20 = (9 - j12)\,\Omega$

[b] $\omega L = 40\,\Omega, \qquad \dfrac{-1}{\omega C} = -5\,\Omega$

$$Z_{xy} = 5 - j5 + 20\|j40 = 5 - j5 + \left[\frac{(20)(j40)}{20 + j40}\right]$$

$$= 5 - j5 + 16 + j8 = (21 + j3)\,\Omega$$

[c] $Z_{xy} = \left[\dfrac{20(j\omega L)}{20 + j\omega L}\right] + \left(5 - \dfrac{j10^6}{25\omega}\right)$

$$= \frac{20\omega^2 L^2}{400 + \omega^2 L^2} + \frac{j400\omega L}{400 + \omega^2 L^2} + 5 - \frac{j10^6}{25\omega}$$

The impedance will be purely resistive when the j terms cancel, i.e.,

$$\frac{400\omega L}{400 + \omega^2 L^2} = \frac{10^6}{25\omega}$$

Solving for ω yields $\omega = 4000\,\text{rad/s}$.

[d] $Z_{xy} = \dfrac{20\omega^2 L^2}{400 + \omega^2 L^2} + 5 = 10 + 5 = 15\,\Omega$

AP 9.8 The frequency 4000 rad/s was found to give $Z_{xy} = 15\,\Omega$ in Assessment Problem 9.7. Thus,

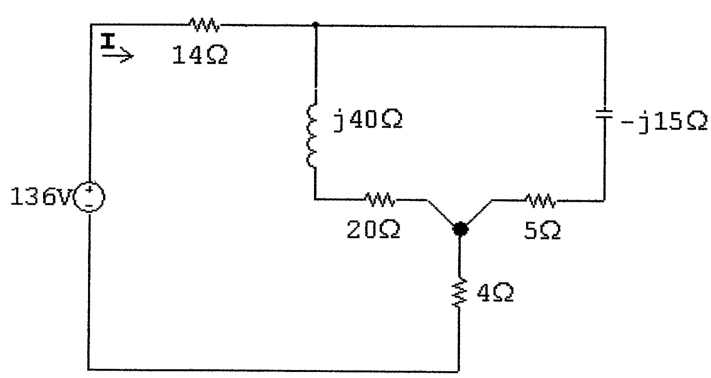
$$\mathbf{V} = 150\underline{/0^\circ}, \qquad \mathbf{I}_s = \frac{\mathbf{V}}{Z_{xy}} = \frac{150\underline{/0^\circ}}{15} = 10\underline{/0^\circ}\,\text{A}$$

Using current division,

$$\mathbf{I}_L = \frac{20}{20 + j20}(10) = 5 - j5 = 7.07\underline{/-45^\circ}\,\text{A}$$

$$i_L = 7.07\cos(4000t - 45^\circ)\,\text{A}, \qquad I_m = 7.07\,\text{A}$$

AP 9.9 After replacing the delta made up of the $50\,\Omega$, $40\,\Omega$, and $10\,\Omega$ resistors with its equivalent wye, the circuit becomes



The circuit is further simplified by combining the parallel branches,

$(20 + j40)||(5 - j15) = (12 - j16)\,\Omega$

Therefore $\mathbf{I} = \dfrac{136\underline{/0^\circ}}{14 + 12 - j16 + 4} = 4\underline{/28.07^\circ}\,\text{A}$

AP 9.10

$\mathbf{V}_1 = 240\underline{/53.13^\circ} = 144 + j192\,\text{V}$

$\mathbf{V}_2 = 96\underline{/-90^\circ} = -j96\,\text{V}$

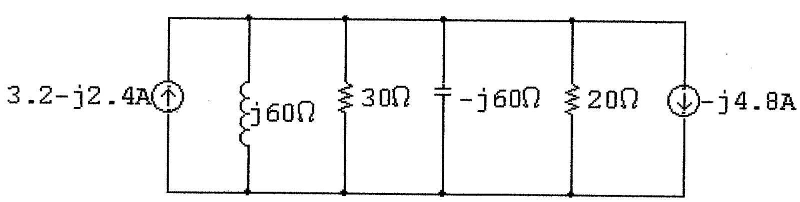
$j\omega L = j(4000)(15 \times 10^{-3}) = j60\,\Omega$

$\dfrac{1}{j\omega C} = -j\dfrac{6 \times 10^6}{(4000)(25)} = -j60\,\Omega$

Perform a source transformation:

$\dfrac{\mathbf{V}_1}{j60} = \dfrac{144 + j192}{j60} = 3.2 - j2.4\,\text{A}$

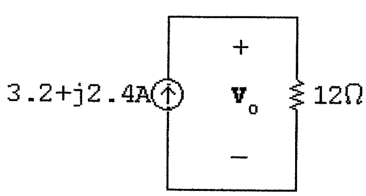
$\dfrac{\mathbf{V}_2}{20} = -j\dfrac{96}{20} = -j4.8\,\text{A}$



Combine the parallel impedances:

$Y = \dfrac{1}{j60} + \dfrac{1}{30} + \dfrac{1}{-j60} + \dfrac{1}{20} = \dfrac{j5}{j60} = \dfrac{1}{12}$

$Z = \dfrac{1}{Y} = 12\,\Omega$



$\mathbf{V}_o = 12(3.2 + j2.4) = 38.4 + j28.8\,\text{V} = 48\underline{/36.87^\circ}\,\text{V}$

$v_o = 48\cos(4000t + 36.87^\circ)\,\text{V}$

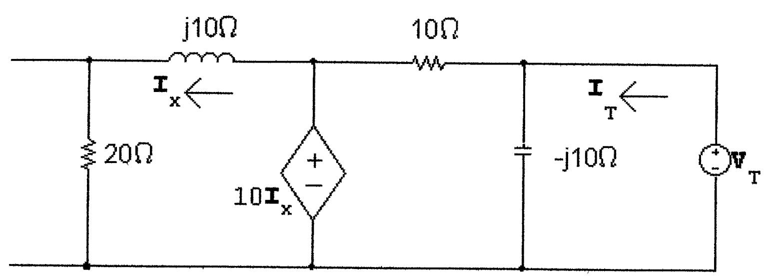
AP 9.11 Use the lower node as the reference node. Let $\mathbf{V}_1$ = node voltage across the $20\,\Omega$ resistor and $\mathbf{V}_{\rm Th}$ = node voltage across the capacitor. Writing the node voltage equations gives us

$$\frac{\mathbf{V}_1}{20} - 2\underline{/45^\circ} + \frac{\mathbf{V}_1 - 10\mathbf{I}_x}{j10} = 0 \quad \text{and} \quad \mathbf{V}_{\rm Th} = \frac{-j10}{10 - j10}(10\mathbf{I}_x)$$

We also have

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{20}$$

Solving these equations for $\mathbf{V}_{\rm Th}$ gives $\mathbf{V}_{\rm Th} = 10\underline{/45^\circ}V$. To find the Thévenin impedance, we remove the independent current source and apply a test voltage source at the terminals a, b. Thus



It follows from the circuit that

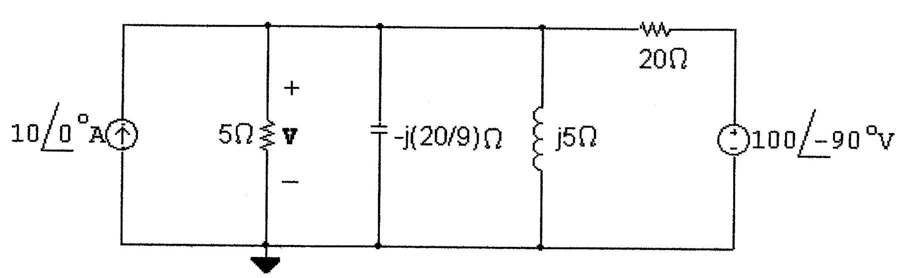
$$10\mathbf{I}_x = (20 + j10)\mathbf{I}_x$$

Therefore

$$\mathbf{I}_x = 0 \quad \text{and} \quad \mathbf{I}_T = \frac{\mathbf{V}_T}{-j10} + \frac{\mathbf{V}_T}{10}$$

$$Z_{\rm Th} = \frac{\mathbf{V}_T}{\mathbf{I}_T}, \quad \text{therefore} \quad Z_{\rm Th} = (5 - j5)\,\Omega$$

AP 9.12 The phasor domain circuit is as shown in the following diagram:



The node voltage equation is

$$-10 + \frac{\mathbf{V}}{5} + \frac{\mathbf{V}}{-j(20/9)} + \frac{\mathbf{V}}{j5} + \frac{\mathbf{V} - 100\underline{/-90^\circ}}{20} = 0$$

Therefore $\mathbf{V} = 10 - j30 = 31.62\underline{/-71.57^\circ}$

Therefore $v = 31.62\cos(50{,}000t - 71.57^\circ)$ V

AP 9.13 Let $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$ be the three clockwise mesh currents going from left to right. Summing the voltages around meshes a and b gives

$$33.8 = (1 + j2)\mathbf{I}_a + (3 - j5)(\mathbf{I}_a - \mathbf{I}_b)$$

and

$$0 = (3 - j5)(\mathbf{I}_b - \mathbf{I}_a) + 2(\mathbf{I}_b - \mathbf{I}_c).$$

But

$$\mathbf{V}_x = -j5(\mathbf{I}_a - \mathbf{I}_b),$$

therefore

$$\mathbf{I}_c = -0.75[-j5(\mathbf{I}_a - \mathbf{I}_b)].$$

Solving for $\mathbf{I} = \mathbf{I}_a = 29 + j2 = 29.07\underline{/3.95^\circ}$ A.

AP 9.14 [a] $M = 0.4\sqrt{0.0625} = 0.1$ H, $\quad \omega M = 80\,\Omega$

$$Z_{22} = 40 + j800(0.125) + 360 + j800(0.25) = (400 + j300)\,\Omega$$

Therefore $|Z_{22}| = 500\,\Omega$, $\quad Z_{22}^* = (400 - j300)\,\Omega$

$$Z_r = \left(\frac{80}{500}\right)^2 (400 - j300) = (10.24 - j7.68)\,\Omega$$

[b] $$\mathbf{I}_1 = \frac{245.20}{184 + 100 + j400 + Z_r} = 0.50\underline{/-53.13^\circ}\text{ A}$$

$$i_1 = 0.5\cos(800t - 53.13^\circ)\text{ A}$$

[c] $$\mathbf{I}_2 = \left(\frac{j\omega M}{Z_{22}}\right)\mathbf{I}_1 = \frac{j80}{500\underline{/36.87^\circ}}(0.5\underline{/-53.13^\circ}) = 0.08\underline{/0^\circ}\text{ A}$$

$$i_2 = 80\cos 800t\text{ mA}$$

AP 9.15

$$\mathbf{I}_1 = \frac{\mathbf{V}_s}{Z_1 + 2s^2 Z_2} = \frac{25 \times 10^3 \underline{/0^\circ}}{1500 + j6000 + (25)^2(4 - j14.4)}$$

$$= 4 + j3 = 5\underline{/36.87^\circ}\ \text{A}$$

$$\mathbf{V}_1 = \mathbf{V}_s - Z_1\mathbf{I}_1 = 25{,}000\underline{/0^\circ} - (4 + j3)(1500 + j6000)$$

$$= 37{,}000 - j28{,}500$$

$$\mathbf{V}_2 = -\frac{1}{25}\mathbf{V}_1 = -1480 + j1140 = 1868.15\underline{/142.39^\circ}\ \text{V}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{Z_2} = \frac{1868.15\underline{/142.39^\circ}}{4 - j14.4} = 125\underline{/216.87^\circ}\ \text{A}$$

Problems

P 9.1 [a] $\omega = 2\pi f = 240\pi\,\text{rad/s}, \qquad f = \frac{\omega}{2\pi} = 120\,\text{Hz}$

[b] $T = 1/f = 8.33\,\text{ms}$

[c] $V_m = 100\,\text{V}$

[d] $v(0) = 100\cos(45^\circ) = 70.71\,\text{V}$

[e] $\phi = 45^\circ; \qquad \phi = \frac{45^\circ(2\pi)}{360^\circ} = \frac{\pi}{4} = 0.7854\,\text{rad}$

[f] $V = 0$ when $240\pi t + 45^\circ = 90^\circ$. Now resolve the units:

$$(240\pi\,\text{rad/s})t = \frac{45^\circ}{57.3^\circ/\text{rad}} = \frac{\pi}{4}\,\text{rad}, \qquad t = 1.042\,\text{ms}$$

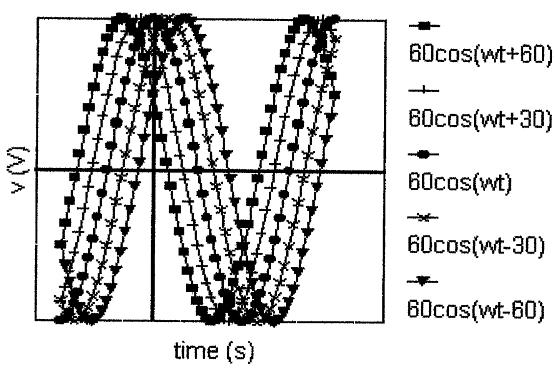
[g] $(dv/dt) = (-100)240\pi\sin(240\pi t + 45^\circ)$

$$(dv/dt) = 0 \quad \text{when} \quad 240\pi t + 45^\circ = 180^\circ$$

$$\text{or} \qquad 240\pi t = \frac{135^\circ}{57.3^\circ/\text{rad}} = \frac{3\pi}{4}\,\text{rad}$$

Therefore $t = 3.125\,\text{ms}$

P 9.2



[a] Left as ϕ becomes more positive

[b] Right

P 9.3 [a] $\frac{T}{2} = \frac{1250}{6} + \frac{250}{6} = 250\,\mu\text{s}; \qquad T = 500\,\mu\text{s}$

$$f = \frac{1}{T} = \frac{10^6}{500} = 2000\text{Hz}$$

[b] $v = V_m \sin(\omega t + \theta)$

$$\omega = 2\pi f = 4000\pi \text{ rad/s}$$

$$4000\pi\left(\frac{-250}{6} \times 10^{-6}\right) + \theta = 0; \qquad \therefore\ \theta = \frac{\pi}{6} \text{ rad} = 30^\circ$$

$$v = V_m \sin[4000\pi t + 30^\circ]$$

$$75 = V_m \sin 30^\circ; \qquad V_m = 150\,\text{V}$$

$$v = 150 \sin[4000\pi t + 30^\circ] = 150 \cos[4000\pi t - 60^\circ]\,\text{V}$$

P 9.4 [a] By hypothesis

$$i = 10\cos(\omega t + \theta)$$

$$\frac{di}{dt} = -10\omega \sin(\omega t + \theta)$$

$$\therefore\ 10\omega = 20{,}000\pi; \qquad \omega = 2000\pi\,\text{rad/s}$$

[b] $f = \frac{\omega}{2\pi} = 1000$ Hz; $\qquad T = \frac{1}{f} = 1$ ms $= 1000\,\mu$s

$$\frac{150}{1000} = \frac{3}{20}, \qquad \therefore\ \theta = -90 - \frac{3}{20}(360) = -144^\circ$$

$$\therefore\ i = 10\cos(2000\pi t - 144^\circ)\,\text{A}$$

P 9.5 [a] 170 V

[b] $2\pi f = 120\pi; \qquad f = 60\text{Hz}$

[c] $\omega = 120\pi = 376.99$ rad/s

[d] $\theta(\text{rad}) = \frac{-\pi}{180}(60) = \frac{-\pi}{3} = -1.05$ rad

[e] $\theta = -60^\circ$

[f] $T = \frac{1}{f} = \frac{1}{60} = 16.67\,\text{ms}$

[g] $120\pi t - \frac{\pi}{3} = 0; \qquad \therefore\ t = \frac{1}{360} = 2.78\,\text{ms}$

[h] $v = 170\cos\left[120\pi\left(t + \frac{0.125}{18}\right) - \frac{\pi}{3}\right]$

$$= 170\cos[120\pi t + (15\pi/18) - (\pi/3)]$$

$$= 170\cos[120\pi t + (\pi/2)]$$

$$= -170 \sin 120\pi t\,\text{V}$$

[i] $120\pi(t-t_o)-(\pi/3)=120\pi t-(\pi/2)$

$$\therefore\quad 120\pi t_o=\frac{\pi}{6};\qquad t_o=\frac{25}{18}\text{ ms}$$

[j] $120\pi(t-t_o)-(\pi/3)=120\pi t$

$$\therefore\quad 120\pi t_o=\frac{\pi}{3};\qquad t_o=\frac{25}{9}\text{ ms}$$

P 9.6

$$u=\int_{t_o}^{t_o+T}V_m^2\cos^2(\omega t+\phi)\,dt$$
$$=V_m^2\int_{t_o}^{t_o+T}\frac{1}{2}+\frac{1}{2}\cos(2\omega t+2\phi)\,dt$$
$$=\frac{V_m^2}{2}\left\{\int_{t_o}^{t_o+T}dt+\int_{t_o}^{t_o+T}\cos(2\omega t+2\phi)\,dt\right\}$$
$$=\frac{V_m^2}{2}\left\{T+\frac{1}{2\omega}\left[\sin(2\omega t+2\phi)\Big|_{t_o}^{t_o+T}\right]\right\}$$
$$=\frac{V_m^2}{2}\left\{T+\frac{1}{2\omega}[\sin(2\omega t_o+4\pi+2\phi)-\sin(2\omega t_o+2\phi)]\right\}$$
$$=V_m^2\left(\frac{T}{2}\right)+\frac{1}{2\omega}(0)=V_m^2\left(\frac{T}{2}\right)$$

P 9.7 $V_m=\sqrt{2}V_{\text{rms}}=\sqrt{2}(120)=169.71\text{ V}$

P 9.8

$$V_{\text{rms}}=\sqrt{\frac{1}{T}\int_0^{T/2}V_m^2\sin^2\frac{2\pi}{T}t\,dt}$$

$$\int_0^{T/2}V_m^2\sin^2\left(\frac{2\pi}{T}t\right)dt=\frac{V_m^2}{2}\int_0^{T/2}\left(1-\cos\frac{4\pi}{T}t\right)dt=\frac{V_m^2T}{4}$$

Therefore $V_{\text{rms}}=\sqrt{\dfrac{1}{T}\dfrac{V_m^2T}{4}}=\dfrac{V_m}{2}$

P 9.9 [a] The numerical values of the terms in Eq. 9.8 are

$V_m=100,\qquad R/L=533.33,\qquad \omega L=30$

$\sqrt{R^2+\omega^2L^2}=50$

$\phi=60^\circ,\qquad \theta=\tan^{-1}30/40,\qquad \theta=36.87^\circ$

$i=\left[-1.84e^{-533.33t}+2\cos(400t+23.13^\circ)\right]\text{ A},\qquad t\geq 0$

[b] Transient component $=-1.84e^{-533.33t}$ A
Steady-state component $=2\cos(400t+23.13^\circ)$ A

[c] By direct substitution into Eq 9.9, $i(1.875\text{ ms})=133.61\text{ mA}$

[d] 2 A, 400 rad/s, 23.13°

[e] The current lags the voltage by 36.87°.

P 9.10 [a] From Eq. 9.9 we have

$$L\frac{di}{dt} = \frac{V_m R\cos(\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}e^{-(R/L)t} - \frac{\omega L V_m \sin(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}$$

$$Ri = \frac{-V_m R\cos(\phi-\theta)e^{-(R/L)t}}{\sqrt{R^2+\omega^2L^2}} + \frac{V_m R\cos(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}$$

$$L\frac{di}{dt} + Ri = V_m\left[\frac{R\cos(\omega t+\phi-\theta) - \omega L\sin(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}\right]$$

But

$$\frac{R}{\sqrt{R^2+\omega^2L^2}} = \cos\theta \quad \text{and} \quad \frac{\omega L}{\sqrt{R^2+\omega^2L^2}} = \sin\theta$$

Therefore the right-hand side reduces to

$V_m\cos(\omega t+\phi)$

At $t=0$, Eq. 9.9 reduces to

$$i(0) = \frac{-V_m\cos(\phi-\theta)}{\sqrt{R^2-\omega^2L^2}} + \frac{V_m\cos(\phi-\theta)}{\sqrt{R^2+\omega^2L^2}} = 0$$

[b] $i_{ss} = \dfrac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos(\omega t+\phi-\theta)$

Therefore

$$L\frac{di_{ss}}{dt} = \frac{-\omega L V_m}{\sqrt{R^2+\omega^2L^2}}\sin(\omega t+\phi-\theta)$$

and

$$Ri_{ss} = \frac{V_m R}{\sqrt{R^2+\omega^2L^2}}\cos(\omega t+\phi-\theta)$$

$$L\frac{di_{ss}}{dt} + Ri_{ss} = V_m\left[\frac{R\cos(\omega t+\phi-\theta) - \omega L\sin(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}\right]$$
$$= V_m\cos(\omega t+\phi)$$

P 9.11 [a] $\mathbf{Y} = 100\underline{/45^\circ} + 500\underline{/-60^\circ} = 483.86\underline{/-48.48^\circ}$

$y = 483.86\cos(300t - 48.48^\circ)$

[b] $\mathbf{Y} = 250\underline{/30^\circ} - 150\underline{/50^\circ} = 120.51\underline{/4.8^\circ}$

$y = 120.51\cos(377t + 4.8^\circ)$

[c] $\mathbf{Y} = 60\underline{/60^\circ} - 120\underline{/-215^\circ} + 100\underline{/90^\circ} = 152.88\underline{/32.94^\circ}$

$y = 152.88\cos(100t + 32.94^\circ)$

[d] $\mathbf{Y} = 100\underline{/40^\circ} + 100\underline{/160^\circ} + 100\underline{/-80^\circ} = 0$

$y = 0$

P 9.12 [a] 50Hz

[b] $\theta_v = 0^\circ$

$$\mathbf{I} = \frac{340\underline{/0^\circ}}{j\omega L} = \frac{340}{\omega L}\underline{/-90^\circ} = 8.5\underline{/-90^\circ}; \qquad \theta_i = -90^\circ$$

[c] $\dfrac{340}{\omega L} = 8.5; \qquad \omega L = 40\,\Omega$

[d] $L = \dfrac{40}{100\pi} = \dfrac{400}{\pi}\,\text{mH} = 127.32\,\text{mH}$

[e] $Z_L = j\omega L = j40\,\Omega$

P 9.13 [a] $\omega = 2\pi f = 80\pi \times 10^3 = 251.33\,\text{krad/s} = 251{,}327.41\,\text{rad/s}$

[b] $\mathbf{I} = \dfrac{2.5 \times 10^{-3}\underline{/0^\circ}}{1/j\omega C} = j\omega C(2.5 \times 10^{-3})\underline{/0^\circ} = 2.5 \times 10^{-3}\omega C\underline{/90^\circ}$

$\therefore \quad \theta_i = 90^\circ$

[c] $125.66 \times 10^{-6} = 2.5 \times 10^{-3}\,\omega C$

$$\frac{1}{\omega C} = \frac{2.5 \times 10^{-3}}{125.66 \times 10^{-6}} = 19.89\,\Omega, \quad \therefore \quad X_C = -19.89\,\Omega$$

[d] $C = \dfrac{1}{19.89(\omega)} = \dfrac{1}{(19.89)(80\pi \times 10^3)}$

$C = 0.2 \times 10^{-6} = 0.2\,\mu\text{F}$

[e] $Z_c = j\left(\dfrac{-1}{\omega C}\right) = -j19.89\,\Omega$

P 9.14 [a] $\mathbf{V}_g = 150\underline{/20^\circ}; \qquad \mathbf{I}_g = 30\underline{/-52^\circ}$

$\therefore \quad Z = \dfrac{\mathbf{V}_g}{\mathbf{I}_g} = 5\underline{/72^\circ}\,\Omega$

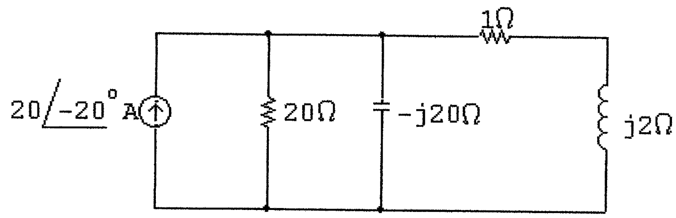
[b] i_g lags v_g by 72°:

$2\pi f = 8000\pi; \qquad f = 4000\,\text{Hz}; \qquad T = 1/f = 250\,\mu\text{s}$

$\therefore \quad i_g$ lags v_g by $\dfrac{72}{360}(250) = 50\,\mu\text{s}$

P 9.15 [a] $j\omega L = j(5\times10^4)(40\times10^{-6}) = j2\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{10^6}{5\times10^4} = -j20\,\Omega; \qquad \mathbf{I}_g = 20\underline{/-20^\circ}\,\text{A}$$



[b] $\mathbf{V}_o = 20\underline{/-20^\circ}Z_e$

$$Z_e = \frac{1}{Y_e}; \qquad Y_e = \frac{1}{20} + j\frac{1}{20} + \frac{1}{1+j2}$$

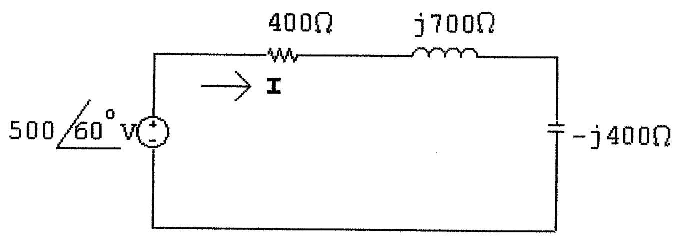
$$Y_e = 0.05 + j0.05 + 0.20 - j0.40 = 0.25 - j0.35\,\text{S}$$

$$Z_e = \frac{1}{0.25 - j0.35} = 2.32\underline{/54.46^\circ}\,\Omega$$

$$\mathbf{V}_o = (20\underline{/-20^\circ})(2.32\underline{/54.46^\circ}) = 46.4\underline{/34.46^\circ}\,\text{V}$$

[c] $v_o = 46.4\cos(5\times10^4 t + 34.46^\circ)\,\text{V}$

P 9.16 [a]



[b] $\mathbf{I} = \dfrac{500\underline{/60^\circ}}{400 + j700 - j400} = 1\underline{/23.13^\circ}\,\text{A}$

[c] $i = 1\cos(8000t + 23.13^\circ)\,\text{A}$

P 9.17 [a] $Z_1 = R_1 - j\dfrac{1}{\omega C_1}$

$$Z_2 = \frac{R_2/j\omega C_2}{R_2 + (1/j\omega C_2)} = \frac{R_2}{1 + j\omega R_2 C_2} = \frac{R_2 - j\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{R_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{and}$$

$$\frac{1}{\omega C_1} = \frac{\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{or} \quad C_1 = \frac{1 + \omega^2 R_2^2 C_2^2}{\omega^2 R_2^2 C_2}$$

[b] $R_1 = \dfrac{500}{1 + (64 \times 10^8)(25 \times 10^4)(625 \times 10^{-18})} = 250\,\Omega$

$$C_1 = \frac{2}{(64 \times 10^8)(25 \times 10^4)(25 \times 10^{-9})} = 50\,\text{nF}$$

P 9.18 [a] $Y_2 = \dfrac{1}{R_2} + j\omega C_2$

$$Y_1 = \frac{1}{R_1 + (1/j\omega C_1)} = \frac{j\omega C_1}{1 + j\omega R_1 C_1} = \frac{\omega^2 R_1 C_1^2 + j\omega C_1}{1 + \omega^2 R_1^2 C_1^2}$$

Therefore $\quad Y_1 = Y_2 \quad$ when

$$R_2 = \frac{1 + \omega^2 R_1^2 C_1}{\omega^2 R_1 C_1^2} \quad \text{and} \quad C_2 = \frac{C_1}{1 + \omega^2 R_1^2 C_1^2}$$

[b] $R_2 = \dfrac{1 + (4 \times 10^8)(4 \times 10^6)(2500 \times 10^{-18})}{(4 \times 10^8)(2 \times 10^3)(2500 \times 10^{-18})} = 2500 = 2.5\text{k}\Omega$

$$C_2 = \frac{50 \times 10^{-9}}{5} = 10\,\text{nF}$$

P 9.19 [a] $Z_1 = R_1 + j\omega L_1$

$$Z_2 = \frac{R_2(j\omega L_2)}{R_2 + j\omega L_2} = \frac{\omega^2 L_2^2 R_2 + j\omega L_2 R_2^2}{R_2^2 + \omega^2 L_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{\omega^2 L_2^2 R_2}{R_2^2 + \omega^2 L_2^2} \quad \text{and} \quad L_1 = \frac{R_2^2 L_2}{R_2^2 + \omega^2 L_2^2}$$

[b] $R_1 = \dfrac{(4 \times 10^8)(6.25)(5 \times 10^4)}{25 \times 10^8 + (4 \times 10^8)(6.25)} = 2.5 \times 10^4$

$$\therefore \quad R_1 = 25\,\text{k}\Omega$$

$$L_1 = \frac{(25 \times 10^8)2.5}{50 \times 10^8} = 1.25\,\text{H}$$

P 9.20 [a] $Y_2 = \dfrac{1}{R_2} - \dfrac{j}{\omega L_2}$

$$Y_1 = \frac{1}{R_1 + j\omega L_1} = \frac{R_1 - j\omega L_1}{R_1^2 + \omega^2 L_1^2}$$

Therefore $\quad Y_2 = Y_1 \quad$ when

$$R_2 = \frac{R_1^2 + \omega^2 L_1^2}{R_1} \quad \text{and} \quad L_2 = \frac{R_1^2 + \omega^2 L_1^2}{\omega^2 L_1}$$

[b] $R_2 = \dfrac{25 \times 10^6 + 10^8(0.25)}{5 \times 10^3} = 10 \times 10^3$

$\therefore \quad R_2 = 10\,\text{k}\Omega$

$L_2 = \dfrac{50 \times 10^6}{10^8(0.5)} = 1\,\text{H}$

P 9.21 [a] $Y = \dfrac{1}{4 - j3} + \dfrac{1}{16 + j12} + \dfrac{1}{-j100}$

$= 0.16 + j0.12 + 0.04 - j0.03 + j0.01$

$= 0.2 + j0.1 = 223.6\underline{/26.57^\circ}\,\text{mS}$

[b] $G = 200\,\text{mS}$

[c] $B = 100\,\text{mS}$

[d] $\mathbf{I} = 50\underline{/0^\circ}\,\text{A}, \qquad \mathbf{V} = \dfrac{\mathbf{I}}{Y} = \dfrac{50}{0.223\underline{/26.57^\circ}} = 223.61\underline{/-26.57^\circ}\,\text{V}$

$\mathbf{I}_C = \dfrac{\mathbf{V}}{Z_C} = \dfrac{223.6\underline{/-26.57^\circ}}{100\underline{/-90^\circ}} = 2.24\underline{/63.43^\circ}\,\text{A}$

$i_C = 2.24\cos(\omega t + 63.43^\circ)\,\text{A}, \qquad I_m = 2.24\,\text{A}$

P 9.22 [a] $Z_{\text{ab}} = j5\omega + \dfrac{(4000)(10^9/j\omega 625)}{4000 + (10^9/j625\omega)}$

$= j5\omega + \dfrac{4 \times 10^{12}}{25 \times 10^5 j\omega + 10^9}$

$= j5\omega + \dfrac{4 \times 10^7}{10^4 + j25\omega}$

$= j5\omega + \dfrac{4 \times 10^{11}}{10^8 + 625\omega^2} - j\dfrac{100 \times 10^7\omega}{10^8 + 625\omega^2}$

$\therefore \quad 5 = \dfrac{10^9}{10^8 + 625\omega^2}$

$5 \times 10^8 + 3125\omega^2 = 10^9$

$\omega = 4 \times 10^2 = 400\,\text{rad/s}$

[b] $Z_{\text{ab}}(400) = j2000 + \dfrac{(4000)(-j4000)}{4000 - j4000} = 2\,\text{k}\Omega$

P 9.23 $Z_1 = 10 - j40\,\Omega$

$Z_2 = \dfrac{(5 - j10)(10 + j30)}{15 + j20} = 10 - j10\,\Omega$

$$Z_3 = \frac{20(j20)}{20 + j20} = 10 + j10\,\Omega$$

$$\therefore \quad Z_{\text{ab}} = Z_1 + Z_2 + Z_3 = 30 - j40\,\Omega = 50\underline{/-53.13^\circ}\,\Omega$$

P 9.24 First find the admittance of the parallel branches

$$Y_p = \frac{1}{6 - j2} + \frac{1}{4 + j12} + \frac{1}{5} + \frac{1}{j10} = 0.375 - j0.125\,\text{S}$$

$$Z_p = \frac{1}{Y_p} = \frac{1}{0.375 - j0.125} = 2.4 + j0.8\,\Omega$$

$$Z_{\text{ab}} = -j12.8 + 2.4 + j0.8 + 13.6 = 16 - j12\,\Omega$$

$$Y_{\text{ab}} = \frac{1}{Z_{\text{ab}}} = \frac{1}{16 - j12} = 0.04 + j0.03\,\text{S}$$

$$= 40 + j30\,\text{mS} = 50\underline{/36.87^\circ}\,\text{mS}$$

P 9.25 $$Z = 400 + j(5)(40) - j\frac{1000}{(5)(0.4)} = 500\underline{/-36.87^\circ}\,\Omega$$

$$\mathbf{I}_o = \frac{750\underline{/0^\circ} \times 10^{-3}}{500\underline{/-36.87^\circ}} = 1.5\underline{/36.87^\circ}\,\text{mA}$$

$$i_o(t) = 1.5\cos(5000t + 36.87^\circ)\,\text{mA}$$

P 9.26 $\mathbf{V}_g = 50\underline{/-45^\circ}\,\text{V}; \qquad \mathbf{I}_g = 100\underline{/-8.13^\circ}\,\text{mA}$

$$Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = 500\underline{/-36.87^\circ}\,\Omega = 400 - j300\,\Omega$$

$$Z = 400 + j\left(0.04\omega - \frac{2.5 \times 10^6}{\omega}\right)$$

$$\therefore \quad 0.04\omega - \frac{2.5 \times 10^6}{\omega} = -300$$

$$\therefore \quad \omega^2 + 7500\omega - 62.5 \times 10^6 = 0$$

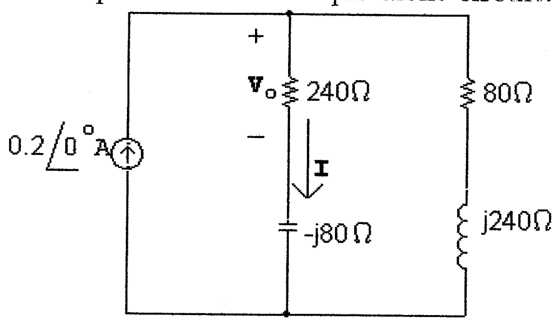
$$\therefore \quad \omega = -3750 \pm \sqrt{(3750)^2 + 62.5 \times 10^6} = -3750 \pm 8750$$

$$\omega > 0, \qquad \therefore \quad \omega = 5000\,\text{rad/s}$$

P 9.27 $Z_L = j(5000)(48 \times 10^{-3}) = j240\,\Omega$

$$Z_C = \frac{-j}{(5000)(2.5 \times 10^{-6})} = -j80\,\Omega$$

Construct the phasor domain equivalent circuit:



Using current division:

$$\mathbf{I} = \frac{(80 + j240)}{240 - j80 + 80 + j240}(0.2) = 0.1 + j0.1\,\text{A}$$

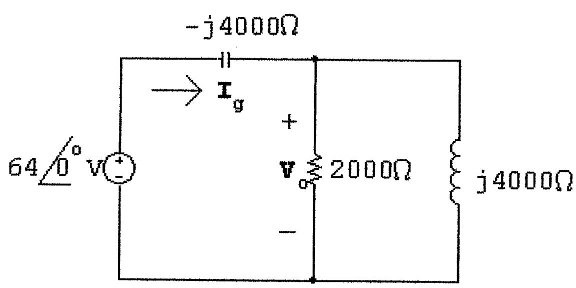
$$\mathbf{V}_o = 240\mathbf{I} = 24 + j24 = 33.94\underline{/45^\circ}$$

$$v_o = 33.94\cos(5000t + 45^\circ)\,\text{V}$$

P 9.28 $\dfrac{1}{j\omega C} = \dfrac{10^9}{(31.25)(8000)} = -j4000\,\Omega$

$$j\omega L = j8000(500)10^{-3} = j4000\,\Omega$$

$$\mathbf{V}_g = 64\underline{/0^\circ}\,\text{V}$$



$$Z_e = \frac{(2000)(j4000)}{2000 + j4000} = 1600 + j800\,\Omega$$

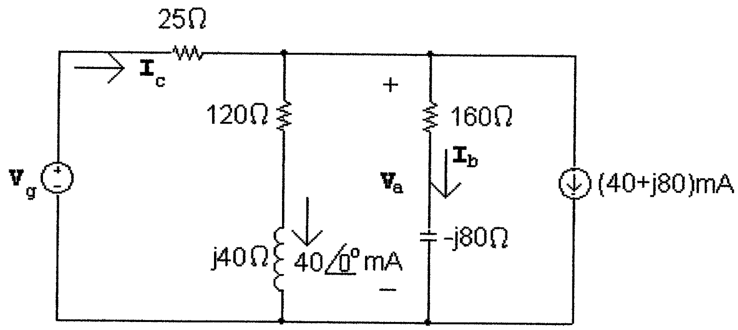
$$Z_T = 1600 + j800 - j4000 = 1600 - j3200\,\Omega$$

$$\mathbf{I}_g = \frac{64\underline{/0^\circ}}{1600 - j3200} = 8 + j16\,\text{mA}$$

$$\mathbf{V}_o = Z_e\mathbf{I}_g = (1600 + j800)(0.008 + j0.016) = j32 = 32\underline{/90^\circ}\,\text{V}$$

$$v_o = 32\cos(8000t + 90^\circ)\,\text{V}$$

P 9.29 [a]



$$\mathbf{V}_a = (120 + j40)(0.04\underline{/0^\circ}) = 4.8 + j1.6\,\text{V}$$

$$\mathbf{I}_b = \frac{4.8 + j1.6}{160 - j80} = 20 + j20\,\text{mA}$$

$$\mathbf{I}_c = 40\underline{/0^\circ} + (20 + j20) + (40 + j80)\,\text{mA} = 100 + j100\,\text{mA}$$

$$\mathbf{V}_g = 25\mathbf{I}_c + \mathbf{V}_a = 25(0.100 + j0.100) + 4.8 + j1.6 = 7.3 + j4.1\,\text{V}$$

[b] $i_b = 28.28\cos(800t + 45^\circ)\,\text{mA}$

$i_c = 141.42\cos(800t + 45^\circ)\,\text{mA}$

$v_g = 8.37\cos(800t + 29.32^\circ)\,\text{V}$

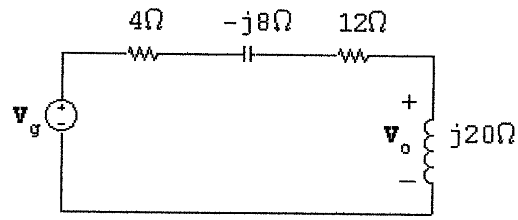
P 9.30 [a] $\dfrac{1}{j\omega C} = \dfrac{10^9}{j8 \times 10^5(125)} = -j10\,\Omega$

$$j\omega L = j8 \times 10^5(25 \times 10^{-6}) = j20\,\Omega$$

$$Z_e = \frac{(-j10)(20)}{20 - j10} = 4 - j8\,\Omega$$

$$\mathbf{I}_g = 5\underline{/0^\circ}$$

$$\mathbf{V}_g = \mathbf{I}_g Z_e = 5(4 - j8) = 20 - j40\,\text{V}$$



$$\mathbf{V}_o = \frac{(20 - j40)(j20)}{(16 + j12)} = 44 - j8 = 44.72\underline{/-10.30^\circ}\,\text{V}$$

$$v_o = 44.72\cos(8 \times 10^5 t - 10.30^\circ)\,\text{V}$$

[b] $\omega = 2\pi f = 8 \times 10^5; \qquad f = \dfrac{4 \times 10^5}{\pi}$

$$T = \frac{1}{f} = \frac{\pi}{4 \times 10^5} = 2.5\pi\ \mu\text{s}$$

$$\therefore\quad \frac{10.30}{360}(2.5\pi) = 224.82\,\text{ns}$$

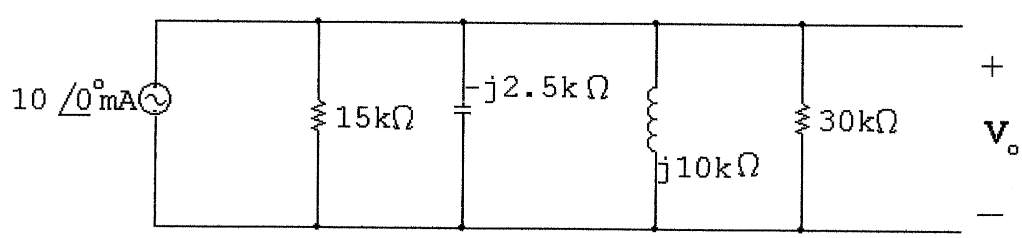
$\therefore\quad v_o$ lags i_g by 224.82 ns

P 9.31 $\mathbf{I}_s = 15\underline{/0^\circ}\,\text{mA}$

$$\frac{1}{j\omega C} = \frac{10^6}{j0.05(8000)} = -j2500\,\Omega$$

$$j\omega L = j8000(1.25) = j10{,}000\,\Omega$$

After two source transformations we have



$$15\,\text{k}\Omega \| 30\,\text{k}\Omega = 10\,\text{k}\Omega$$

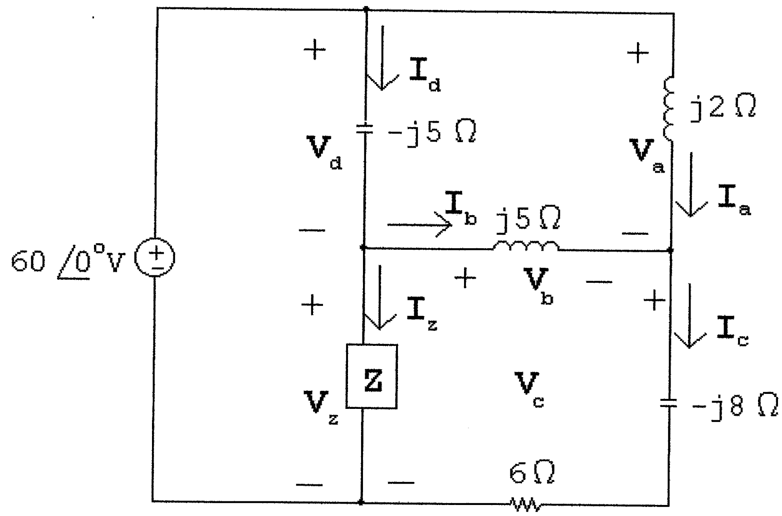
$$Y_o = \frac{10^{-3}}{10} + \frac{1}{-j2500} + \frac{1}{j10^4} = 10^{-4}(1 + j3)$$

$$Z_o = \frac{10^4}{1 + j3} = (1 - j3)\,\text{k}\Omega$$

$$\mathbf{V}_o = \mathbf{I}_g Z_o = (10)(1 - j3) = 10 - j30 = 31.62\underline{/-71.57^\circ}\,\text{V}$$

$$v_o = 31.62\cos(8000t - 71.57^\circ)\,\text{V}$$

P 9.32



$$\mathbf{V}_a = j2\mathbf{I}_a = j2(-j5) = 10\underline{/0^\circ}\text{ V}$$

$$\mathbf{V}_c = 60\underline{/0^\circ} - \mathbf{V}_a = 50\underline{/0^\circ}\text{ V}$$

$$\mathbf{I}_c = \frac{\mathbf{V}_c}{6 - j8} = \frac{50\underline{/0^\circ}}{10\underline{/-53.13^\circ}} = 5\underline{/53.13^\circ} = 3 + j4\text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_c - \mathbf{I}_a = 3 + j4 - (-j5) = 3 + j9\text{ A} = 9.49\underline{/71.57^\circ}\text{ A}$$

$$\mathbf{V}_b = \mathbf{I}_b(j5) = (3 + j9)(j5) = -45 + j15\text{ V}$$

$$\mathbf{V}_z = \mathbf{V}_b + \mathbf{V}_c = -45 + j15 + 50 + j0 = 5 + j15\text{ V}$$

$$\mathbf{V}_d + \mathbf{V}_z = 60\underline{/0^\circ}; \qquad \therefore\ \mathbf{V}_d = 60 - 5 - j15 = 55 - j15\text{ V}$$

$$\mathbf{I}_d = \frac{\mathbf{V}_d}{-j5} = 3 + j11\text{ A}$$

$$\mathbf{I}_z = \mathbf{I}_d - \mathbf{I}_b = 3 + j11 - 3 - j9 = j2\text{ A}$$

$$Z = \frac{\mathbf{V}_z}{\mathbf{I}_z} = \frac{5 + j15}{j2} = 7.5 - j2.5\,\Omega$$

P 9.33 $\mathbf{V}_2$ is the voltage across the $-j10\,\Omega$ impedance.

$$\frac{\mathbf{V}_1 - \mathbf{V}_g}{20} + \frac{\mathbf{V}_1}{j5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{Z} = 0$$

$$\frac{(40 + j30) - (100 - j50)}{20} + \frac{40 + j30}{j5} + \frac{(40 + j30) - \mathbf{V}_2}{Z} = 0$$

$\therefore\quad \mathbf{V}_2 = 40 + j30 + (3 - j4)Z$

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{Z} + \frac{\mathbf{V}_L}{-j10} - \mathbf{I}_g + \frac{\mathbf{V}_2 - \mathbf{V}_g}{3 + j1} = 0$$

$$\frac{\mathbf{V}_2 - (40 + j30)}{Z} + \frac{\mathbf{V}_2}{-j10} - (20 + j30) + \frac{\mathbf{V}_2 - (100 - j50)}{3 + j1} = 0$$

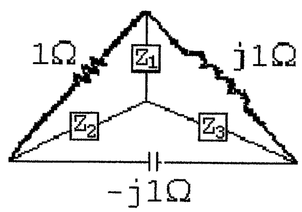
Substituting the expression for $\mathbf{V}_2$ found at the start and simplifying yields

$Z = 12 + j16\,\Omega$

P 9.34 Simplify the top triangle using series and parallel combinations:

$(1 + j1)\|(1 - j1) = 1\,\Omega$

Convert the lower left delta to a wye:

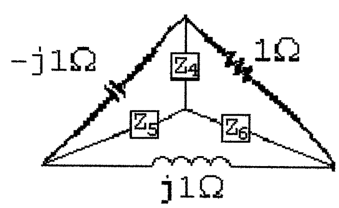


$$Z_1 = \frac{(j1)(1)}{1 + j1 - j1} = j1\,\Omega$$

$$Z_2 = \frac{(-j1)(1)}{1 + j1 - j1} = -j1\,\Omega$$

$$Z_3 = \frac{(j1)(-j1)}{1 + j1 - j1} = 1\,\Omega$$

Convert the lower right delta to a wye:

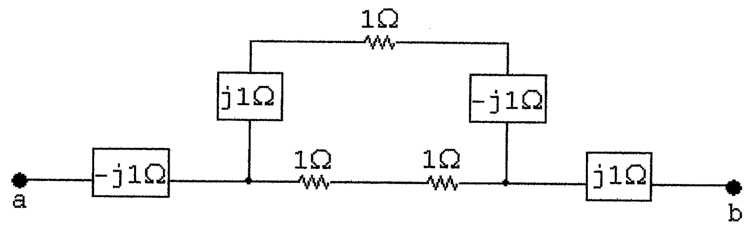


$$Z_4 = \frac{(-j1)(1)}{1 + j1 - j1} = -j1\,\Omega$$

$$Z_5 = \frac{(-j1)(j1)}{1 + j1 - j1} = 1\,\Omega$$

$$Z_6 = \frac{(j1)(1)}{1 + j1 - j1} = j1\,\Omega$$

The resulting circuit is shown below:



Simplify the middle portion of the circuit by making series and parallel combinations:

$$(1 + j1 - j1)\|(1 + 1) = 1\|2 = 2/3\,\Omega$$

$$Z_{ab} = -j1 + 2/3 + j1 = 2/3\,\Omega$$

P 9.35 [a] $Y_p = \dfrac{1}{10 + j2\omega} + j4 \times 10^{-3}\omega$

$$= \frac{10 - j2\omega}{100 + 4\omega^2} + j4 \times 10^{-3}\omega$$

$$= \frac{10}{100 + 4\omega^2} - \frac{j2\omega}{100 + 4\omega^2} + j4 \times 10^{-3}\omega$$

Y_p is real when

$$4 \times 10^{-3}\omega = \frac{2\omega}{100 + 4\omega^2}$$

or $\quad \omega^2 = 100; \qquad \omega = 10$ rad/s; $\qquad f = 5/\pi = 1.59$Hz

[b] $Y_p(10\,\text{rad/s}) = \dfrac{10}{500} = 20\,\text{mS}$

$$Z_p(10\,\text{rad/s}) = \frac{10^3}{20} = 50\,\Omega$$

$$Z(10\,\text{rad/s}) = 50 + 150 = 200\,\Omega$$

$$\mathbf{I}_o = \frac{\mathbf{V}_g}{200}\,\text{A} = \frac{10\underline{/0^\circ}}{200} = 50\underline{/0^\circ}\,\text{mA}$$

$$i_o = 50\cos 10t\,\text{mA}$$

P 9.36 [a] $Z_g = 4000 - j\frac{10^9}{25\omega} + \frac{10^4(j2\omega)}{10^4 + j2\omega}$

$= 4000 - j\frac{10^9}{25\omega} + \frac{2 \times 10^4 j\omega(10^4 - j2\omega)}{10^8 + 4\omega^2}$

$= 4000 - j\frac{10^9}{25\omega} + \frac{4 \times 10^4\omega^2}{10^8 + 4\omega^2} + j\frac{2 \times 10^8\omega}{10^8 + 4\omega^2}$

$\therefore \quad \frac{10^9}{25\omega} = \frac{0.2 \times 10^9\omega}{10^8 + 4\omega^2}$

$10^8 + 4\omega^2 = 5\omega^2$

$\omega^2 = 10^8; \qquad \omega = 10{,}000\,\text{rad/s}$

[b] When $\omega = 10{,}000\,\text{rad/s}$

$Z_g = 4000 + \frac{4 \times 10^4(10^4)^2}{10^8 + 4(10^4)^2} = 12{,}000\,\Omega$

$\therefore \quad \mathbf{I}_g = \frac{45\underline{/0^\circ}}{12{,}000} = 3.75\underline{/0^\circ}\,\text{mA}$

$\mathbf{V}_o = \mathbf{V}_g - \mathbf{I}_g Z_1$

$Z_1 = 4000 - j\frac{10^9}{25 \times 10^4} = 4000 - j4000\,\Omega$

$\mathbf{V}_o = 45\underline{/0^\circ} - (3.75 \times 10^{-3})(4000 - j4000) = 45 - (15 - j15)$

$= 30 + j15 = 33.54\underline{/26.57^\circ}\,\text{V}$

$v_o = 33.54\cos(10{,}000t + 26.57^\circ)\,\text{V}$

P 9.37 [a] $Y_1 = \frac{1}{5000} = 0.2 \times 10^{-3}\,\text{S}$

$Y_2 = \frac{1}{1200 + j0.2\omega}$

$= \frac{1200}{1.44 \times 10^6 + 0.04\omega^2} - j\frac{0.2\omega}{1.44 \times 10^6 + 0.04\omega^2}$

$Y_3 = j\omega 50 \times 10^{-9}$

$Y_T = Y_1 + Y_2 + Y_3$

For i_g and v_o to be in phase the j component of Y_T must be zero; thus,

$\omega 50 \times 10^{-9} = \frac{0.2\omega}{1.44 \times 10^6 + 0.04\omega^2}$

or

$$0.04\omega^2 + 1.44\times 10^6 = \frac{0.2\times 10^9}{50} = 4\times 10^6$$

$$\therefore\quad 0.04\omega^2 = 2.56\times 10^6 \qquad \therefore\quad \omega = 8000\,\text{rad/s} = 8\,\text{krad/s}$$

[b] $Y_T = 0.2\times 10^{-3} + \dfrac{1200}{1.44\times 10^6 + 0.04(64)\times 10^6} = 0.5\times 10^{-3}\,\text{S}$

$$\therefore\quad Z_T = 2000\,\Omega$$

$$\mathbf{V}_o = (2.5\times 10^{-3}\underline{/0^\circ})(2000) = 5\underline{/0^\circ}$$

$$v_o = 5\cos 8000t\,\text{V}$$

P 9.38 [a] $Z_p = \dfrac{\dfrac{R}{j\omega C}}{R + (1/j\omega C)} = \dfrac{R}{1 + j\omega RC}$

$$= \frac{12{,}500}{1 + j(1000)(12{,}500)C} = \frac{12{,}500}{1 + j12.5\times 10^6 C}$$

$$= \frac{12{,}500(1 - j12.5\times 10^6 C)}{1 + 156.25\times 10^{12}C^2}$$

$$= \frac{12{,}500}{1 + 156.25\times 10^{12}C^2} - j\frac{156.25\times 10^9 C}{1 + 156.25\times 10^{12}C^2}$$

$$j\omega L = j1000(5) = j5000$$

$$\therefore\quad 5000 = \frac{156.25\times 10^9 C}{1 + 156.25\times 10^{12}C^2}$$

$$\therefore\quad 781.25\times 10^{15}C^2 - 156.25\times 10^9 C + 5000 = 0$$

$$\therefore\quad C^2 - 20\times 10^{-8}C + 64\times 10^{-16} = 0$$

$$\therefore\quad C_{1,2} = 10\times 10^{-8} \pm \sqrt{100\times 10^{-16} - 64\times 10^{-16}}$$

$$C_1 = 10\times 10^{-8} + 6\times 10^{-8} = 16\times 10^{-8} = 0.16\,\mu\text{F}$$

$$C_2 = 10\times 10^{-8} - 6\times 10^{-8} = 4\times 10^{-8} = 0.04\,\mu\text{F}$$

[b] $R_e = \dfrac{12{,}500}{1 + 156.25\times 10^{12}C^2}$

When $C = 160\,\text{nF}$ $\quad R_e = 2500\,\Omega$;

$$\mathbf{I}_g = \frac{250\underline{/0^\circ}}{2500} = 0.1\underline{/0^\circ}\,\text{A}; \qquad i_g = 100\cos 1000t\,\text{mA}$$

When $C = 40\,\text{nF}$ $\quad R_e = 10{,}000\,\Omega$;

$$\mathbf{I}_g = \frac{250\underline{/0^\circ}}{10{,}000} = 0.025\underline{/0^\circ}\,\text{A}; \qquad i_g = 25\cos 1000t\,\text{mA}$$

P 9.39 [a] $Z_1 = 1600 - j\frac{10^9}{10^4(62.5)} = 1600 - j1600\,\Omega$

$$Z_1 = \frac{4000(j10^4L)}{4000 + j10^4L} = \frac{4 \times 10^5L^2 + j16 \times 10^4L}{16 + 100L^2}$$

$$Z_T = Z_1 + Z_2 = 1600 + \frac{4 \times 10^5L^2}{16 + 100L^2} - j1600 + j\frac{16 \times 10^4L}{16 + 100L^2}$$

Z_T is resistive when

$$\frac{16 \times 10^4L}{16 + 100L^2} = 1600 \qquad \text{or}$$

$$L^2 - L + 0.16 = 0$$

Solving, $L_1 = 0.8$ H and $L_2 = 0.2$ H.

[b] When $L = 0.8$ H:

$$Z_T = 1600 + \frac{4 \times 10^5(0.64)}{16 + 64} = 4800\,\Omega$$

$$\mathbf{I}_g = \frac{96\underline{/0^\circ}}{4.8} \times 10^{-3} = 20\underline{/0^\circ}\,\text{mA}$$

$$i_g = 20\cos 10{,}000t\,\text{mA}$$

When $L = 0.2$ H:

$$Z_T = 1600 + \frac{4 \times 10^5(0.04)}{16 + 4} = 2400\,\Omega$$

$$i_g = 40\cos 10{,}000t\,\text{mA}$$

P 9.40 Step 1 to Step 2:

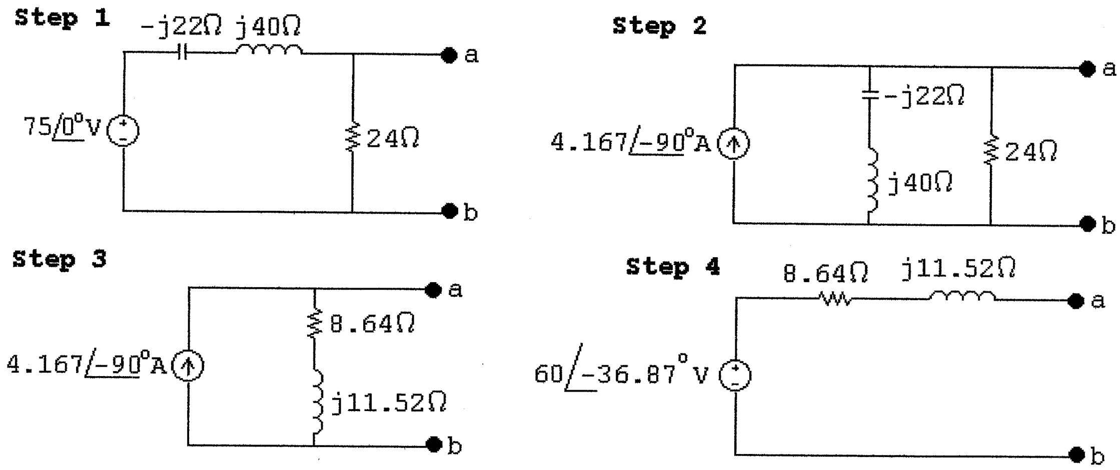
$$\frac{75\underline{/0^\circ}}{j18} = -j4.167 = 4.167\underline{/-90^\circ}\,\text{A}$$

Step 2 to Step 3:

$$(j18)\|24 = \frac{(j18)(24)}{24 + j18} = 8.64 + j11.52\,\Omega$$

Step 3 to Step 4:

$$(4.167\underline{/-90^\circ})(8.64 + j11.52) = 60\underline{/-36.87^\circ}\,\text{V}$$



P 9.41 Step 1 to Step 2:

$(16\underline{/0^\circ})(25) = 400\underline{/0^\circ}\,\text{V}$

Step 2 to Step 3:

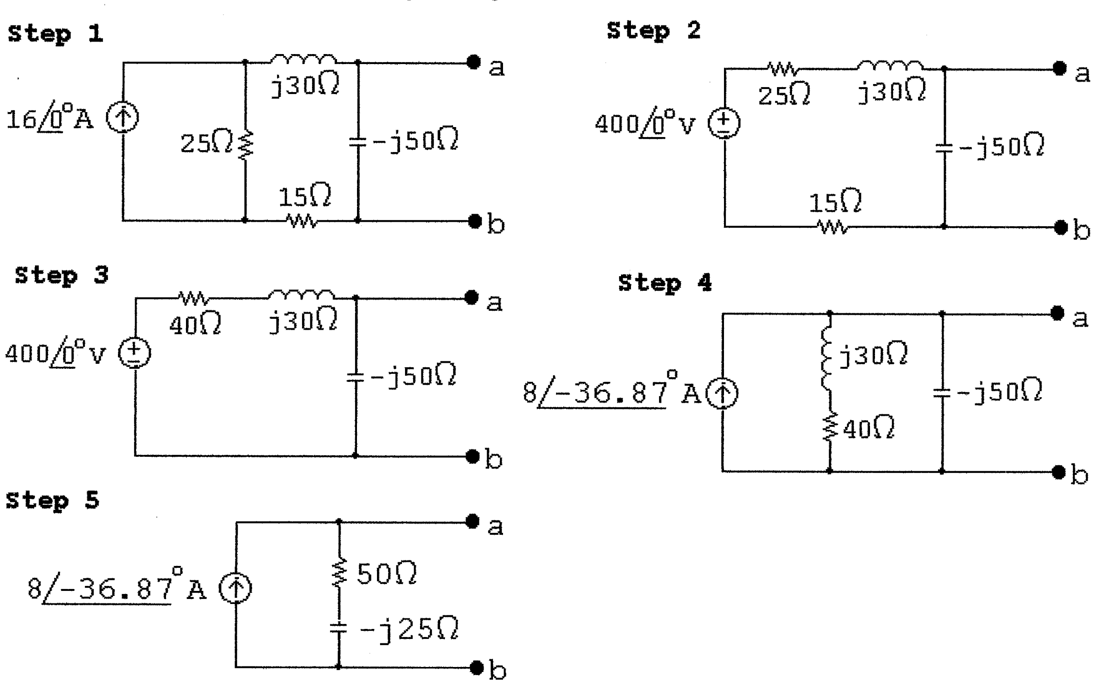
$25 + 15 + j30 = (40 + j30)\,\Omega$

Step 3 to Step 4:

$$\frac{400\underline{/0^\circ}}{(40 + j30)} = 8\underline{/-36.87^\circ}\,\text{A}$$

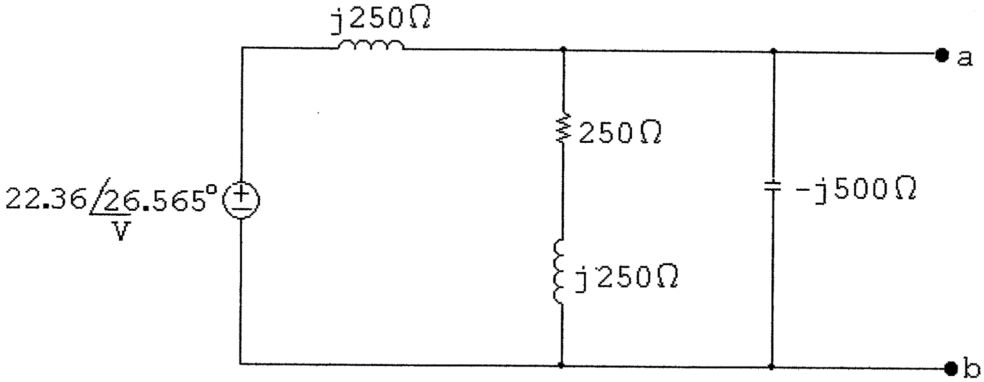
Step 4 to Step 5:

$$(40 + j30\| - j50 = \frac{(-j50)(40 + j30)}{40 + j30 - j50} = 50 - j25\,\Omega$$



P 9.42 [a] $j\omega L = j(5000)(50) \times 10^{-3} = j250\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{1}{(5000)(400 \times 10^{-9})} = -j500\,\Omega$$



Using voltage division,

$$\mathbf{V}_{\rm ab} = \frac{(250 + j250)\|(-j500)}{j250 + (250 + j250)\|(-j500)}(23.36\underline{/26.565^\circ}) = 20\underline{/0^\circ}$$

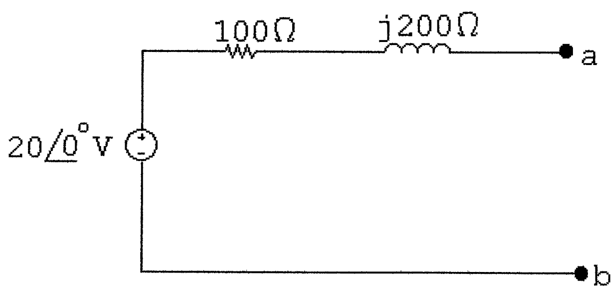
$$\mathbf{V}_{\rm Th} = \mathbf{V}_{\rm ab} = 20\underline{/0^\circ}\,\mathrm{V}$$

[b] Remove the voltage source and combine impedances in parallel to find $Z_{\rm Th} = Z_{\rm ab}$:

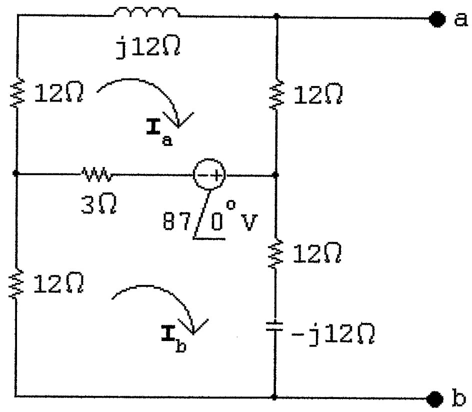
$$Y_{\rm ab} = \frac{1}{j250} + \frac{1}{250 + j250} + \frac{1}{-j500} = 2 - j4\ \mathrm{mS}$$

$$Z_{\rm Th} = Z_{\rm ab} = \frac{1}{Y_{\rm ab}} = 100 + j200\,\Omega$$

[c]



P 9.43



$$(27 + j12)\mathbf{I}_a - 3\mathbf{I}_b = -87\underline{/0^\circ}$$

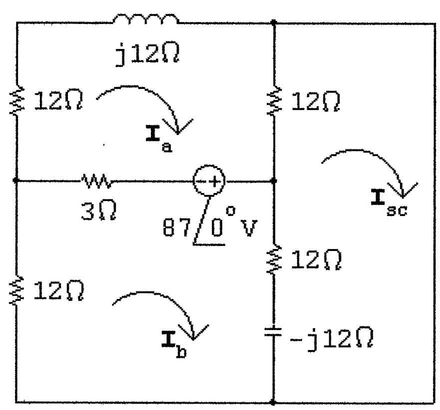
$$-3\mathbf{I}_a + (27 - j12)\mathbf{I}_b = 87\underline{/0^\circ}$$

Solving,

$$\mathbf{I}_a = -2.4167 + j1.21; \qquad \mathbf{I}_b = 2.4167 + j1.21$$

$$\mathbf{V}_{Th} = 12\mathbf{I}_a + (12 - j12)\mathbf{I}_b = 14.5\underline{/0^\circ}\text{ V}$$

Short Circuit Test:



$$(27 + j12)\mathbf{I}_a - 3\mathbf{I}_b - 12\mathbf{I}_{sc} = -87$$

$$-3\mathbf{I}_a + (27 - j12)\mathbf{I}_b - (12 - j12)\mathbf{I}_{sc} = 87$$

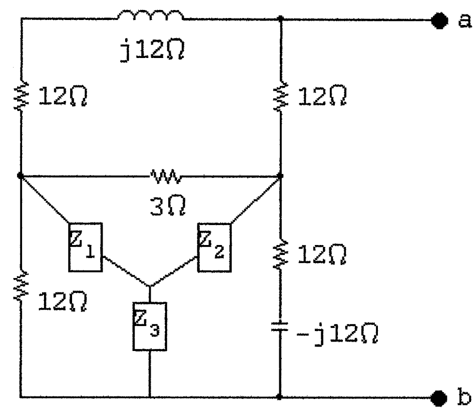
$$-12\mathbf{I}_a - (12 - j12)\mathbf{I}_b + (24 - j12)\mathbf{I}_{sc} = 0$$

Solving,

$$\mathbf{I}_{sc} = 1\underline{/0^\circ}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{14.5\underline{/0^\circ}}{1\underline{/0^\circ}} = 14.5\,\Omega$$

Alternate calculation for Z_{Th}:

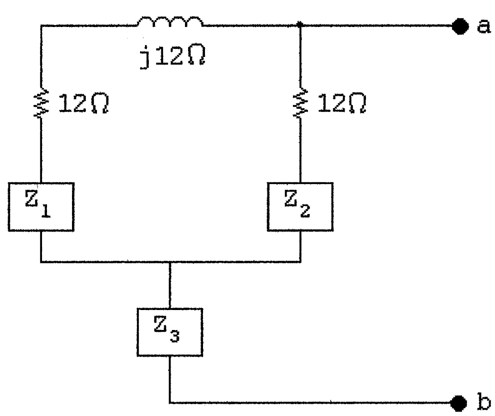


$$\sum Z = 12 + 3 + 12 - j12 = 27 - j12$$

$$Z_1 = \frac{36}{27 - j12} = \frac{12}{9 - j4}$$

$$Z_2 = \frac{36 - j36}{27 - j12} = \frac{12 - j12}{9 - j4}$$

$$Z_3 = \frac{12(12 - j12)}{27 - j12} = \frac{48 - j48}{9 - j4}$$



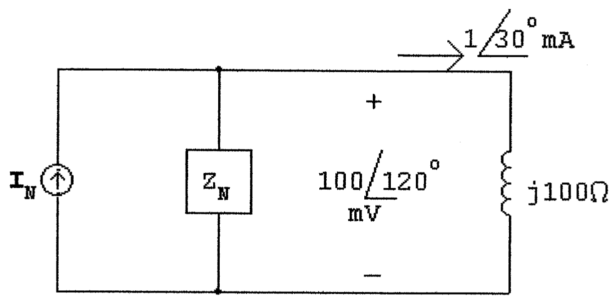
$$Z_{\text{a}} = 12 + j12 + \frac{12}{9 - j4} = \frac{12(14 + j5)}{9 - j4}$$

$$Z_{\text{b}} = 12 + \frac{12 - j12}{9 - j4} = \frac{12(10 - j5)}{9 - j4}$$

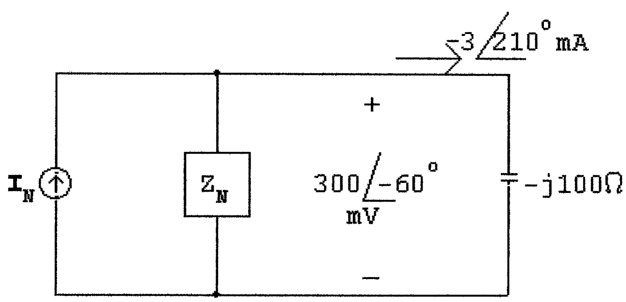
$$Z_a \| Z_b = \frac{165 - j20}{18 - j8}$$

$$Z_3 + Z_a \| Z_b = \frac{48 - j48}{9 - j4} + \frac{165 - j20}{18 - j8} = 14.5\,\Omega$$

P 9.44



$$\mathbf{I}_N = \frac{0.1\underline{/120^\circ}}{Z_N} + 1\underline{/30^\circ}\,\text{mA}, \quad Z_N \text{ in k}\Omega$$



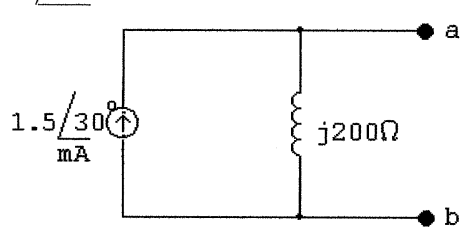
$$\mathbf{I}_N = \frac{0.3\underline{/-60^\circ}}{Z_N} + (-3\underline{/210^\circ})\,\text{mA}, \quad Z_N \text{ in k}\Omega$$

$$\frac{0.1\underline{/120^\circ}}{Z_N} + 1\underline{/30^\circ} = \frac{0.3\underline{/-60^\circ}}{Z_N} + (-3\underline{/210^\circ})$$

$$\frac{0.3\underline{/-60^\circ} - 0.1\underline{/120^\circ}}{Z_N} = 1\underline{/30^\circ} + 3\underline{/210^\circ}$$

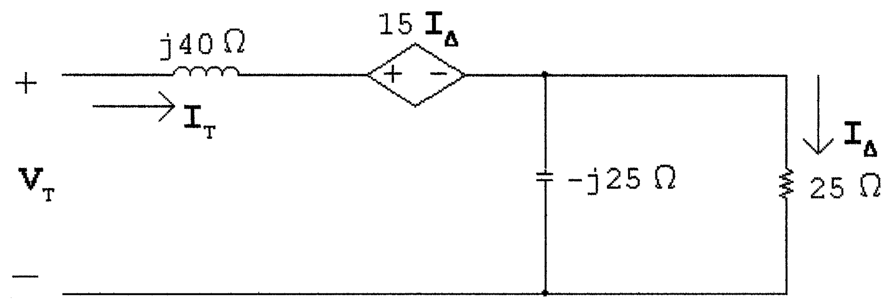
$$Z_N = \frac{0.3\underline{/-60^\circ} - 0.1\underline{/120^\circ}}{1\underline{/30^\circ} + 3\underline{/210^\circ}} = 0.2\underline{/90^\circ} = j0.2\,\text{k}\Omega$$

$$\mathbf{I}_N = \frac{0.1\underline{/120^\circ}}{0.2\underline{/90^\circ}} + 1\underline{/30^\circ} = 1.5\underline{/30^\circ}\,\text{mA}$$



P 9.45 $j\omega L = j1.6 \times 10^6(25 \times 10^{-6}) = j40\,\Omega$

$$\frac{1}{j\omega C} = \frac{10^{-6} \times 10^9}{j1.6(25)} = -j25\,\Omega$$



$$\mathbf{V}_T = j40\mathbf{I}_T + 15\mathbf{I}_\Delta + 25\mathbf{I}_\Delta$$

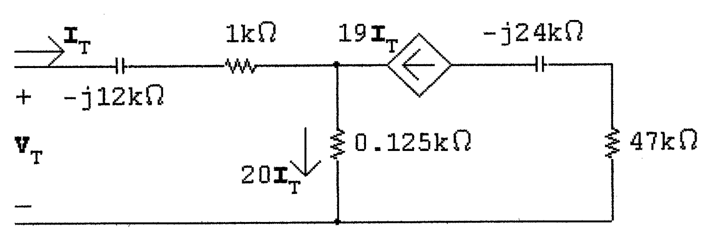
$$\mathbf{I}_\Delta = \frac{\mathbf{I}_T(-j25)}{25 - j25} = \frac{-j\mathbf{I}_T}{1 - j1}$$

$$\mathbf{V}_T = j40\mathbf{I}_T + 40\frac{(-j\mathbf{I}_T)}{1 - j1}$$

$$\frac{\mathbf{V}_T}{\mathbf{I}_T} = Z_{\text{ab}} = j40 + 20(-j)(1 + j) = 20 + j20\,\Omega = 28.28\underline{/45^\circ}\,\Omega$$

P 9.46 $\dfrac{1}{\omega C_1} = \dfrac{(10^{-3})(10^9)}{25(10/3)} = 12\,\text{k}\Omega$

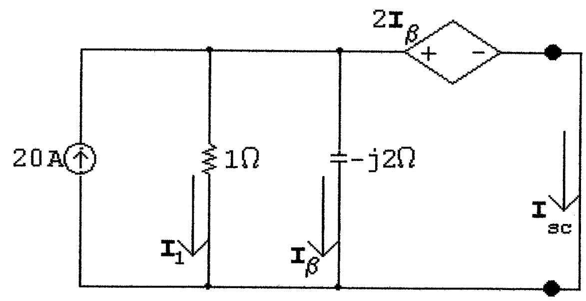
$$\frac{1}{\omega C_2} = \frac{(10^{-3})(10^9)}{25(5/3)} = 24\,\text{k}\Omega$$



$$\mathbf{V}_T = (1 - j12)\mathbf{I}_T + 20\mathbf{I}_T(0.125)$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = 3.5 - j12\,\text{k}\Omega$$

P 9.47 Short circuit current

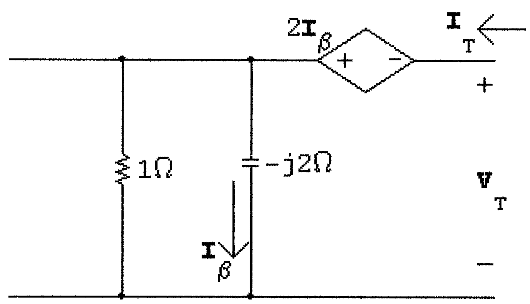


$$\mathbf{I}_\beta = \frac{2\mathbf{I}_\beta}{-j2}$$

$$-j2\mathbf{I}_\beta = 2\mathbf{I}_\beta; \qquad \therefore \quad \mathbf{I}_\beta = 0$$

$$\mathbf{I}_1 = 0; \qquad \therefore \quad \mathbf{I}_{sc} = 20\,\text{A} = \mathbf{I}_N$$

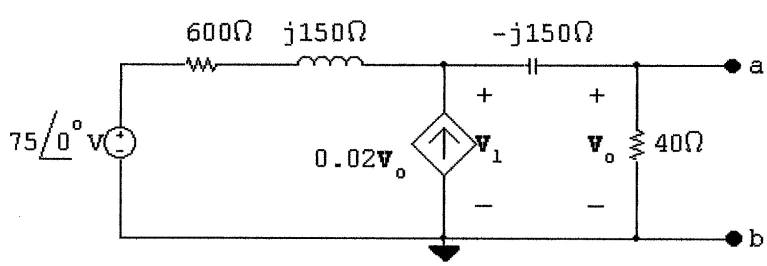
The Norton impedance is the same as the Thévenin impedance. Find it using a test source



$$\mathbf{V}_T = -2\mathbf{I}_\beta - j2\mathbf{I}_\beta = (-2-j2)\mathbf{I}_\beta, \qquad \mathbf{I}_\beta = \frac{1}{1-j2}\mathbf{I}_T$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{(-2-j2)\mathbf{I}_\beta}{[(1-j2)/1]\mathbf{I}_\beta} = \frac{-2-j2}{1-j2} = 0.4 - j1.2\,\Omega$$

P 9.48



$$\frac{\mathbf{V}_1 - 75}{150(4+j1)} - \frac{0.02\mathbf{V}_1(40)}{40-j150} + \frac{\mathbf{V}_1}{40-j150} = 0$$

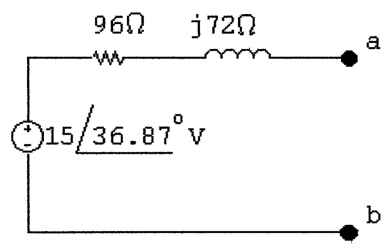
$$\therefore \quad \mathbf{V}_1 = \frac{75(4-j15)}{16-j12}$$

$$\mathbf{V}_{\text{Th}} = \frac{40\mathbf{V}_1}{40-j150} = \frac{4}{4-j15}\cdot\frac{75(4-j15)}{16-j12}$$

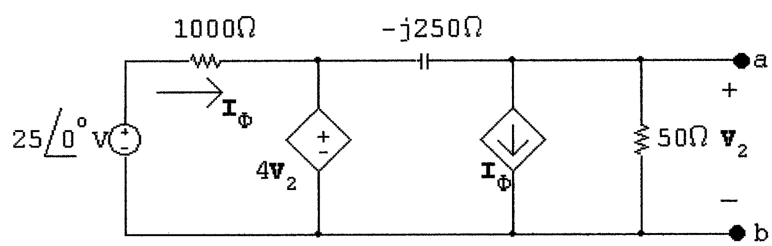
$$= \frac{75}{4-j3} = 15\underline{/36.87^\circ}\,\text{V}$$

$$\mathbf{I}_{\text{sc}} = \frac{75}{600} = \frac{1}{8}\,\text{A}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = 120\underline{/36.87^\circ} = 96 + j72\,\Omega$$



P 9.49



$$\frac{\mathbf{V}_2}{50} + \frac{25-4\mathbf{V}_2}{1000} + \frac{\mathbf{V}_2 - 4\mathbf{V}_2}{-j250} = 0$$

Solving,

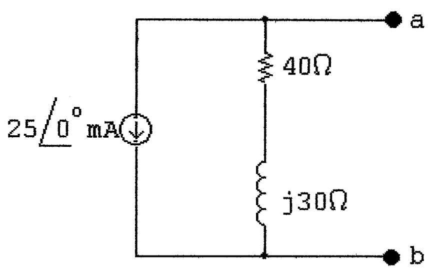
$$\mathbf{V}_2 = -1 - j0.75\,\text{V} = 1.25\underline{/216.87^\circ}\,\text{V}$$

$$\mathbf{I}_{sc} = -\mathbf{I}_\phi = \frac{-25\underline{/0^\circ}}{1000} = -25\underline{/0^\circ}\,\text{mA}$$

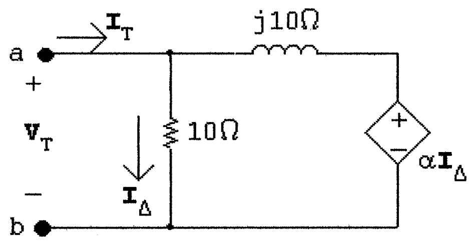
$$Z_{\text{Th}} = \frac{1.25\underline{/216.87^\circ}}{-25\times 10^{-3}\underline{/0^\circ}} = 50\underline{/36.87^\circ}\,\Omega = 40 + j30\,\Omega$$

$$\mathbf{I}_N = \mathbf{I}_{sc} = -25\underline{/0^\circ}\,\text{mA}$$

$Z_N = Z_{\text{Th}} = 50\underline{/36.87^\circ} = 40 + j30\,\Omega$



P 9.50 [a]



$$\mathbf{I}_T = \frac{\mathbf{V}_T}{10} + \frac{\mathbf{V}_T - \alpha\mathbf{V}_T/10}{j10}$$

$$\frac{\mathbf{I}_T}{\mathbf{V}_T} = \frac{1}{10} + \frac{(1-\alpha/10)}{j10} = \frac{(10-\alpha)+j10}{j100}$$

$$\therefore\quad Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{1000 + j100(10-\alpha)}{(10-\alpha)^2 + 100}$$

Z_{Th} is real when $\alpha = 10$.

[b] $Z_{\text{Th}} = \dfrac{1000}{100} = 10\,\Omega$

[c] $Z_{\text{Th}} = 5 + j5$

$$\frac{1000}{(10-\alpha)^2+100} = 5; \qquad (10-\alpha)^2 = 100$$

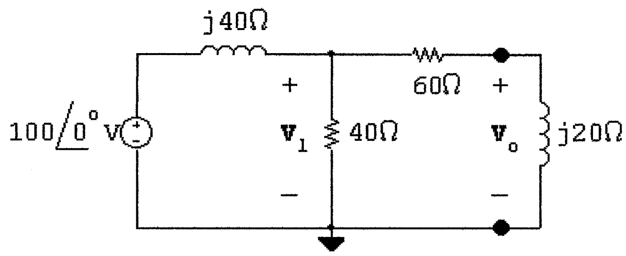
$$\therefore\quad 10 - \alpha = \pm 10; \qquad \alpha = 10 \mp 10$$

$$\alpha = 0; \qquad \alpha = 20$$

But the j term can only equal the real term with $\alpha = 0$. Thus, $\alpha = 0$.

[d] Z_{Th} will be inductive when $\alpha < 10$.

P 9.51



$$\frac{\mathbf{V}_1 - 100}{j40} + \frac{\mathbf{V}_1}{40} + \frac{\mathbf{V}_1}{60 + j20} = 0$$

Solving for $\mathbf{V}_1$ yields

$$\mathbf{V}_1 = 30 - j40\,\text{V}$$

$$\mathbf{V}_o = \frac{\mathbf{V}_1}{60 + j20}(j20) = \left(\frac{j}{3+j}\right)\mathbf{V}_1$$

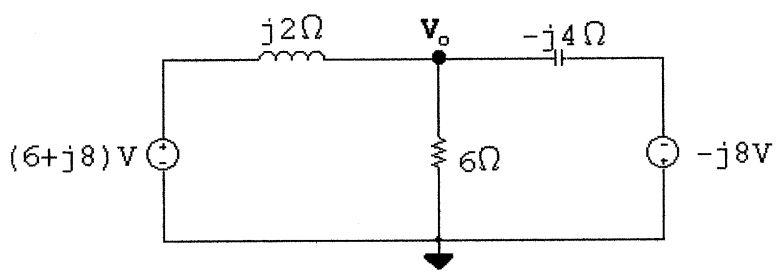
$$\mathbf{V}_o = 15 + j5\,\text{V} = 15.81\underline{/18.43^\circ}\,\text{V}$$

P 9.52 $j\omega L = j(5000)(0.4 \times 10^{-3}) = j2\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{10^6}{(5000)(50)} = -j4\,\Omega$$

$$\mathbf{V}_{g1} = 10\underline{/53.13^\circ} = 6 + j8\,\text{V}$$

$$\mathbf{V}_{g2} = 8\underline{/-90^\circ} = -j8\,\text{V}$$



$$\frac{\mathbf{V}_o - 6 - j8}{j2} + \frac{\mathbf{V}_o}{6} + \frac{\mathbf{V}_o + (-j8)}{-j4} = 0$$

Solving,

$$\mathbf{V}_o = 12\underline{/0^\circ}$$

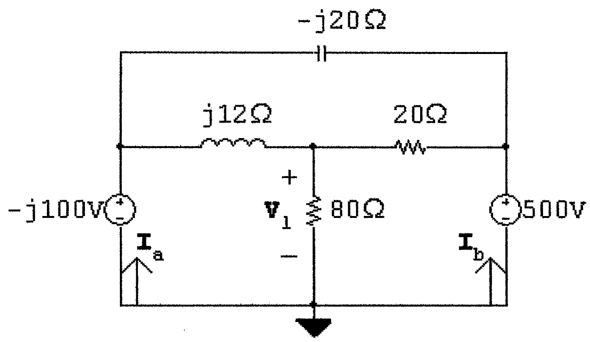
$$v_o(t) = 12\cos 5000t\,\text{V}$$

P 9.53 $j\omega L = j10^4(1.2 \times 10^{-3}) = j12\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j10^6}{5 \times 10^4} = -j20\,\Omega$$

$$\mathbf{V}_a = 100\underline{/-90^\circ} = -j100\,\text{V}$$

$$\mathbf{V}_b = 500\underline{/0^\circ} = 500\,\text{V}$$



$$\frac{\mathbf{V}_1}{80} + \frac{\mathbf{V}_1 - 500}{20} + \frac{\mathbf{V}_1 + j100}{j12} = 0$$

Solving,

$$\mathbf{V}_1 = 160\underline{/53.13^\circ}\,\text{V} = 96 + j128\,\text{V}$$

$$\mathbf{I}_a = \frac{-j100 - 96 - j128}{j12} + \frac{-j100 - 500}{-j20}$$

$$= -14 - j17 = 22.02\underline{/-129.47^\circ}\,\text{A}$$

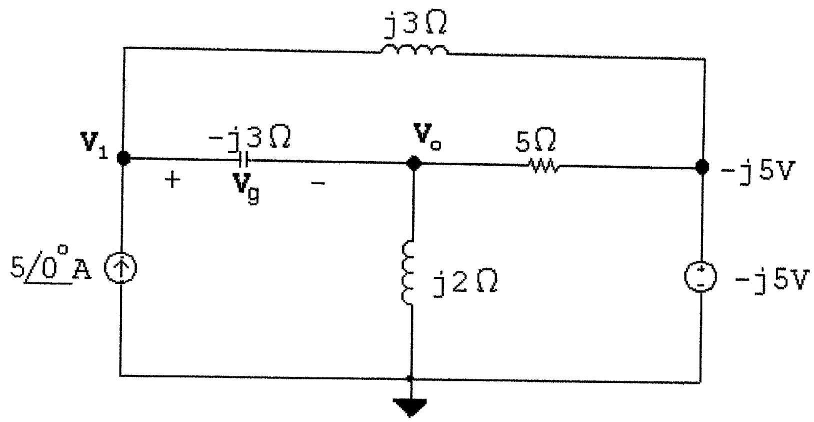
$$i_a = 22.02\cos(10{,}000t - 129.47^\circ)\,\text{A}$$

$$\mathbf{I}_b = \frac{500 - 96 - j128}{20} + \frac{500 + j100}{-j20}$$

$$= 15.2 + j18.6 = 24.02\underline{/50.74^\circ}\,\text{A}$$

$$i_b = 24.02\cos(10{,}000t + 50.74^\circ)\,\text{A}$$

P 9.54



$$\frac{\mathbf{V}_o}{j2} + \frac{\mathbf{V}_o + j5}{5} + \frac{\mathbf{V}_o - \mathbf{V}_1}{-j3} = 0$$

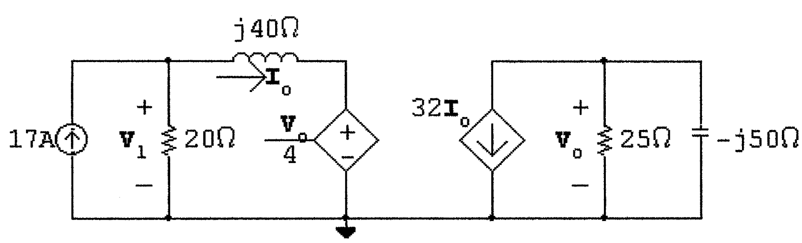
$$(5 + j6)\mathbf{V}_o + 10\mathbf{V}_1 = 30$$

$$-5 + \frac{\mathbf{V}_1 - \mathbf{V}_o}{-j3} + \frac{\mathbf{V}_1 + j5}{j3} = 0$$

$$\mathbf{V}_o = j10; \qquad \mathbf{V}_1 = 9 - j5$$

$$\mathbf{V}_g = \mathbf{V}_1 - \mathbf{V}_o = 9 - j5 - j10 = 9 - j15 = 17.49\underline{/-59.04^\circ}\text{ V}$$

P 9.55



$$\frac{\mathbf{V}_o}{25} + \frac{\mathbf{V}_o}{-j50} + 32\mathbf{I}_o = 0$$

$$(2 + j)\mathbf{V}_o = -1600\mathbf{I}_o$$

$$\mathbf{V}_o = (-640 + j320)\mathbf{I}_o$$

$$\mathbf{I}_o = \frac{\mathbf{V}_1 - (\mathbf{V}_o/4)}{j40}$$

$$\therefore \quad \mathbf{V}_1 = (-160 + j120)\mathbf{I}_o$$

$$17 = \frac{\mathbf{V}_1}{20} + \mathbf{I}_o = (-8 + j6)\mathbf{I}_o + \mathbf{I}_o = (-7 + j6)\mathbf{I}_o$$

$$\therefore \quad \mathbf{I}_o = \frac{17}{(-7 + j6)} = -1.4 - j1.2\,\text{A} = 1.84\underline{/-139.40^\circ}\,\text{A}$$

$$\mathbf{V}_o = (-640 + j320)\mathbf{I}_o = 1280 + j320 = 1319.39\underline{/14.04^\circ}\,\text{V}$$

P 9.56 $$-15\underline{/0^\circ} + \frac{\mathbf{V}_o}{8} + \frac{\mathbf{V}_o - 2.5\mathbf{I}_\Delta}{j5} + \frac{\mathbf{V}_o}{-j10} = 0$$

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j10}$$

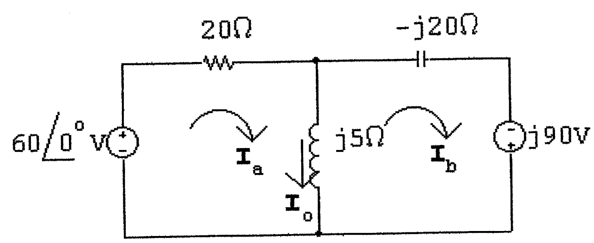
Solving,

$$\mathbf{V}_o = 72 + j96 = 120\underline{/53.13^\circ}\,\text{V}$$

P 9.57 $\mathbf{V}_a = 60\underline{/0^\circ}\,\text{V}$; $\qquad \mathbf{V}_b = 90\underline{/90^\circ}\,\text{V}$

$$j\omega L = j(4 \times 10^4)(125 \times 10^{-6}) = j5\,\Omega$$

$$\frac{-j}{\omega C} = \frac{-j10^6}{40{,}000(1.25)} = -j20\,\Omega$$



$$60 = (20 + j5)\mathbf{I}_a - j5\mathbf{I}_b$$

$$j90 = -j5\mathbf{I}_a - j15\mathbf{I}_b$$

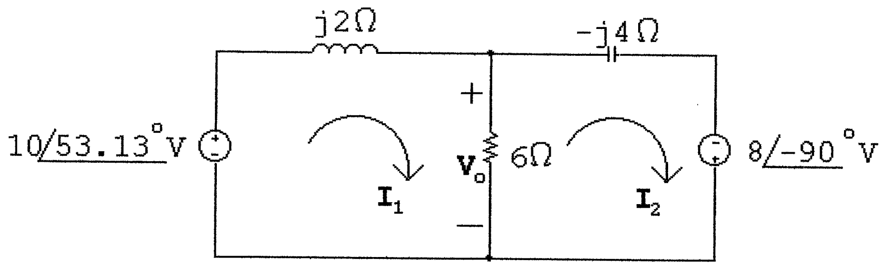
Solving,

$\mathbf{I}_a = 2.25 - j2.25\,\text{A}$; $\qquad \mathbf{I}_b = -6.75 + j0.75\,\text{A}$

$$\mathbf{I}_o = \mathbf{I}_a - \mathbf{I}_b = 9 - j3 = 9.49\underline{/-18.43^\circ}\,\text{A}$$

$$i_o(t) = 9.49\cos(40{,}000t - 18.43^\circ)\,\text{A}$$

P 9.58 From the solution to Problem 9.52 the phasor-domain circuit is



$$10\underline{/53.13^\circ} = (6 + j2)\mathbf{I}_1 - 6\mathbf{I}_2$$

$$8\underline{/-90^\circ} = -6\mathbf{I}_1 + (6 - j4)\mathbf{I}_2$$

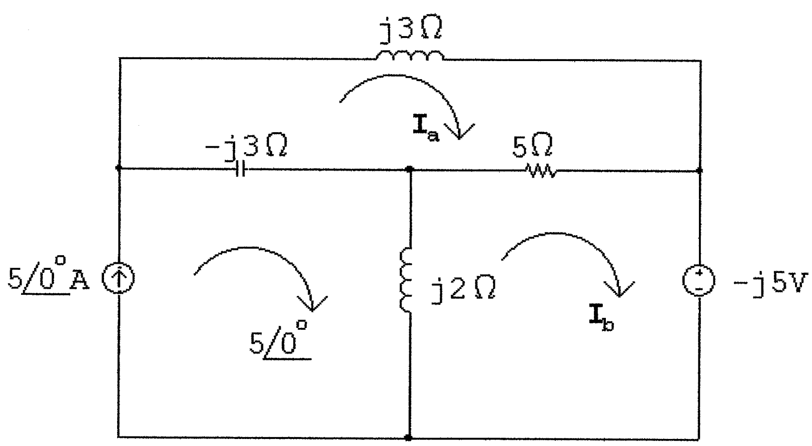
$$\mathbf{V}_o = (\mathbf{I}_1 - \mathbf{I}_2)6$$

Solving,

$$\mathbf{V}_o = 12\underline{/0^\circ}\text{ V}$$

$$v_o(t) = 12\cos 5000t\text{ V}$$

P 9.59



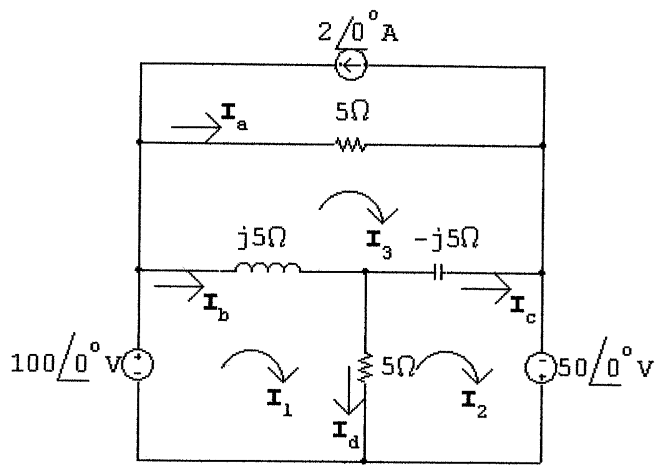
$$j3\mathbf{I}_a + 5(\mathbf{I}_a - \mathbf{I}_b) - j3(\mathbf{I}_a - 5) = 0$$

$$j2(\mathbf{I}_b - 5) + 5(\mathbf{I}_b - \mathbf{I}_a) - j5 = 0$$

Solving,

$$\mathbf{I}_a = -j3; \qquad \mathbf{I}_g = -j3 = 3\underline{/-90^\circ}\text{ A}$$

P 9.60



$$100\underline{/0^\circ} = (5 + j5)\mathbf{I}_1 - 5\mathbf{I}_2 - j5\mathbf{I}_3$$

$$50\underline{/0^\circ} = -5\mathbf{I}_1 + (5 - j5)\mathbf{I}_2 + j5\mathbf{I}_3$$

$$-10\underline{/0^\circ} = -j5\mathbf{I}_1 + j5\mathbf{I}_2 + 5\mathbf{I}_3$$

Solving,

$\mathbf{I}_1 = 58 - j20\,\text{A};\qquad \mathbf{I}_2 = 58 + j10\,\text{A};\qquad \mathbf{I}_3 = 28 + j0\,\text{A}$

$\mathbf{I}_a = \mathbf{I}_3 + 2 = 30 + j0\,\text{A}$

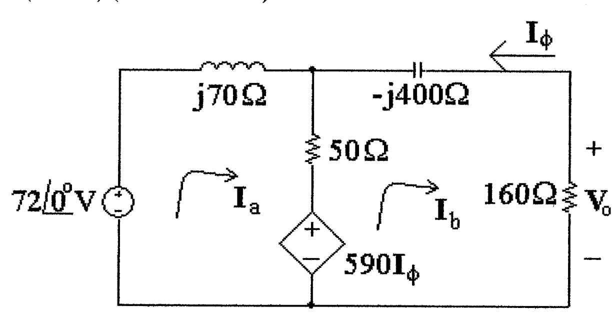
$\mathbf{I}_b = \mathbf{I}_1 - \mathbf{I}_3 = 58 - j20 - 28 = 30 - j20\,\text{A}$

$\mathbf{I}_c = \mathbf{I}_2 - \mathbf{I}_3 = 58 + j10 - 28 = 30 + j10\,\text{A}$

$\mathbf{I}_d = \mathbf{I}_1 - \mathbf{I}_2 = 58 - j20 - 58 - j10 = -j30\,\text{A}$

P 9.61 $j\omega L = j5000(14 \times 10^{-3}) = j70\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(5000)(0.5 \times 10^{-6})} = -j400\,\Omega$$



$$72\underline{/0^\circ} = (50 + j70)\mathbf{I}_a - 50\mathbf{I}_b + 590(-\mathbf{I}_b)$$

$0 = -50\mathbf{I}_a - 590(-\mathbf{I}_b) + (210 - j400)\mathbf{I}_b$

Solving,

$\mathbf{I}_b = (50 - j50)\,\text{mA}$

$\mathbf{V}_o = 160\mathbf{I}_b = 8 - j8 = 11.31\underline{/-45^\circ}$

$v_o = 11.31\cos(5000t - 45^\circ)\,\text{V}$

P 9.62 $Z_o = 600 - j\dfrac{10^6}{(5000)(0.25)} = 600 - j800\,\Omega$

$Z_T = 300 + j2000 + 600 - j800 = 900 + j1200\,\Omega = 1500\underline{/53.13^\circ}\,\Omega$

$$\mathbf{V}_o = \mathbf{V}_g\frac{Z_o}{Z_T} = \frac{(75\underline{/0^\circ})(1000\underline{/-53.13^\circ})}{1500\underline{/53.13^\circ}} = 50\underline{/-106.26^\circ}\,\text{V}$$

$v_o = 50\cos(5000t - 106.26^\circ)\,\text{V}$

P 9.63 $\dfrac{1}{j\omega C} = -j\dfrac{10^6}{10^4} = -j100\,\Omega$

$j\omega L = j(500)(1) = j500\,\Omega$

Let $Z_1 = 50 - j100\,\Omega$; $Z_2 = 250 + j500\,\Omega$

$\mathbf{I}_g = 125\underline{/0^\circ}\,\text{mA}$

$$\mathbf{I}_o = \frac{\mathbf{I}_g Z_1}{Z_1 + Z_2} = \frac{125\underline{/0^\circ}(50 - j100)}{(300 + j400)}$$

$$= -12.5 - j25\,\text{mA} = 27.95\underline{/-116.57^\circ}\,\text{mA}$$

$i_o = 27.95\cos(500t - 116.57^\circ)\,\text{mA}$

P 9.64 $\mathbf{V}_g = 1.2\underline{/0^\circ}\,\text{V}$; $\dfrac{1}{j\omega C} = \dfrac{10^6}{j100} = -j10\,\text{k}\Omega$

Let $\mathbf{V}_a$ = voltage across $1\,\mu$F capacitor, positive at upper terminal
Then:

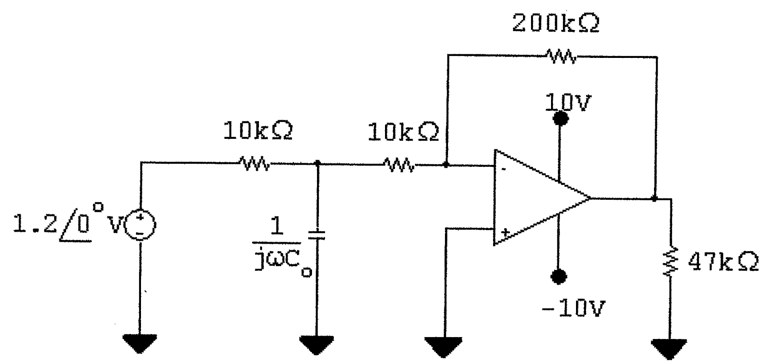
$$\frac{\mathbf{V}_a - 1.2\underline{/0^\circ}}{10} + \frac{\mathbf{V}_a}{-j10} + \frac{\mathbf{V}_a}{10} = 0; \qquad \therefore\ \mathbf{V}_a = (0.48 - j0.24)\,\text{V}$$

$$\frac{0 - \mathbf{V}_a}{10} + \frac{0 - \mathbf{V}_o}{200} = 0; \qquad \mathbf{V}_o = -20\mathbf{V}_a$$

$\therefore\ \mathbf{V}_o = -9.6 + j4.8 = 10.73\underline{/153.43^\circ}\,\text{V}$

$v_o = 10.73\cos(100t + 153.43^\circ)\,\text{V}$

P 9.65 [a]



$$\frac{\mathbf{V}_a - 1.2\angle 0^\circ}{10{,}000} + j\omega C_o \mathbf{V}_a + \frac{\mathbf{V}_a}{10{,}000} = 0$$

$$\mathbf{V}_a = \frac{1.2}{2 + j10^4 \omega C_o}$$

$\mathbf{V}_o = -20\mathbf{V}_a$ (see solution to Prob. 9.73)

$$\mathbf{V}_o = \frac{-24}{2 + j10^6 C_o} = \frac{24\angle 180^\circ}{2 + j10^6 C_o}$$

$\therefore$ denominator angle $= 60^\circ$

$\tan 60^\circ = \sqrt{3}$

$$\frac{10^6 C_o}{2} = \sqrt{3}$$

$$\text{or } C_o = \frac{2\sqrt{3}}{10^6} = 2\sqrt{3}\,\mu\text{F} = 3.46\,\mu\text{F}$$

[b] $$\mathbf{V}_o = \frac{24\angle 180^\circ}{2 + j2\sqrt{3}} = 6\angle 120^\circ\,\text{V}$$

$$v_o = 6\cos(100t + 120^\circ)\,\text{V}$$

P 9.66 [a] $\mathbf{V}_g = 2\angle 0^\circ\,\text{V}$

$$\mathbf{V}_p = \frac{80}{100}\mathbf{V}_g = 1.6\angle 0^\circ; \qquad \mathbf{V}_n = \mathbf{V}_p = 1.6\angle 0^\circ\,\text{V}$$

$$\frac{1.6}{160} + \frac{1.6 - \mathbf{V}_o}{Z_p} = 0$$

$$Z_p = \frac{(200)(1/j\omega C)}{200 + (1/j\omega C)}$$

$$\frac{1}{j\omega C} = \frac{10^9}{j10^5(0.1)} = -j10^5 = -j100\,\text{k}\Omega$$

$$Z_p = \frac{200(-j100)}{200-j100} = 40 - j80\,\text{k}\Omega$$

$$\mathbf{V}_o = 1.6 + \frac{Z_p}{100} = 2 - j0.8 = 2.15\underline{/-21.80^\circ}$$

$$v_o = 2.15\cos(10^5t - 21.80^\circ)\,\text{V}$$

[b] $\mathbf{V}_p = 0.8V_m\underline{/0^\circ}$; $\quad \mathbf{V}_n = \mathbf{V}_p = 0.8V_m\underline{/0^\circ}$

$$\frac{0.8V_m}{160} + \frac{0.8V_m - \mathbf{V}_o}{40-j80} = 0$$

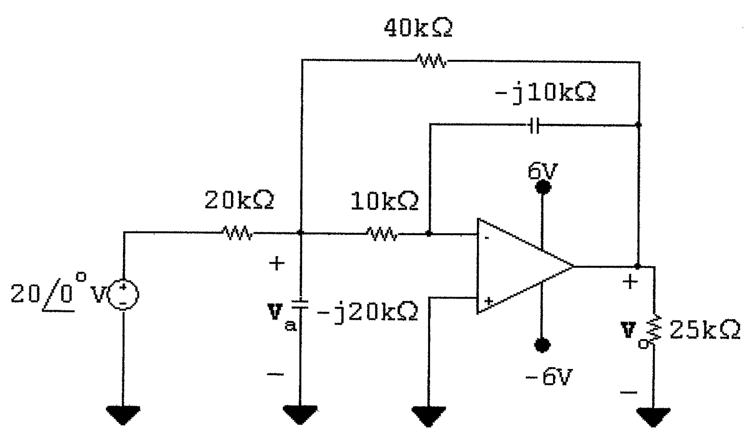
$$\therefore\quad \mathbf{V}_o = 0.8V_m + \frac{40-j80}{160}V_m(0.8) = 0.8V_m(1.25 - j0.5)$$

$$\therefore\quad |0.8V_m(1.25-j0.5)| \le 5$$

$$\therefore\quad V_m \le 4.64\,\text{V}$$

P 9.67 $\displaystyle \frac{1}{j\omega C_1} = \frac{10^{12}}{j10^6(100)} = -j10\,\text{k}\Omega$

$$\frac{1}{j\omega C_2} = \frac{10^{12}}{j(10^6)(50)} = -j20\,\text{k}\Omega$$



$$\frac{\mathbf{V}_a}{-j20} + \frac{\mathbf{V}_a - 20}{20} + \frac{\mathbf{V}_a - \mathbf{V}_o}{40} + \frac{\mathbf{V}_a}{10} = 0$$

$$\therefore\quad (-2 + j7)\mathbf{V}_a - j\mathbf{V}_o = j40$$

$$\frac{0 - \mathbf{V}_a}{10} + \frac{0 - \mathbf{V}_o}{-j10} = 0; \qquad \therefore\quad \mathbf{V}_a = -j\mathbf{V}_o$$

$$\therefore\quad (7 + j)\mathbf{V}_o = j40$$

$$\mathbf{V}_o = \frac{j40}{7+j} = 0.8 + j5.6 = 5.657\underline{/81.87^\circ}\,\text{V}$$

$$v_o(t) = 5.657\cos(10^6 t + 81.87^\circ)\,\text{V}$$

P 9.68 [a] $$\frac{1}{j\omega C} = \frac{-j10^9}{(2\times 10^5)(12.5)} = -j400\,\Omega$$

$$\frac{\mathbf{V}_\text{n}}{200} + \frac{\mathbf{V}_\text{n} - \mathbf{V}_o}{-j400} = 0$$

$$\frac{\mathbf{V}_o}{-j400} = \frac{\mathbf{V}_\text{n}}{200} + \frac{\mathbf{V}_\text{n}}{-j400}$$

$$\mathbf{V}_o = \mathbf{V}_\text{n} - j2\mathbf{V}_\text{n} = (1-j2)\mathbf{V}_\text{n}$$

$$\mathbf{V}_\text{p} = \frac{\mathbf{V}_g(1/j\omega C_o)}{500 + (1/j\omega C_o)} = \frac{\mathbf{V}_g}{1 + j(500)(2\times 10^5)C_o}$$

$$\mathbf{V}_g = 10\underline{/0^\circ}\,\text{V}$$

$$\mathbf{V}_\text{p} = \frac{10\underline{/0^\circ}}{1 + j10^8 C_o} = \mathbf{V}_\text{n}$$

$$\therefore\quad \mathbf{V}_o = \frac{(1-j2)10\underline{/0^\circ}}{1 + j10^8 C_o}$$

$$|\mathbf{V}_o| = \frac{\sqrt{5}(10)}{\sqrt{1 + 10^{16}C_o^2}} = 10$$

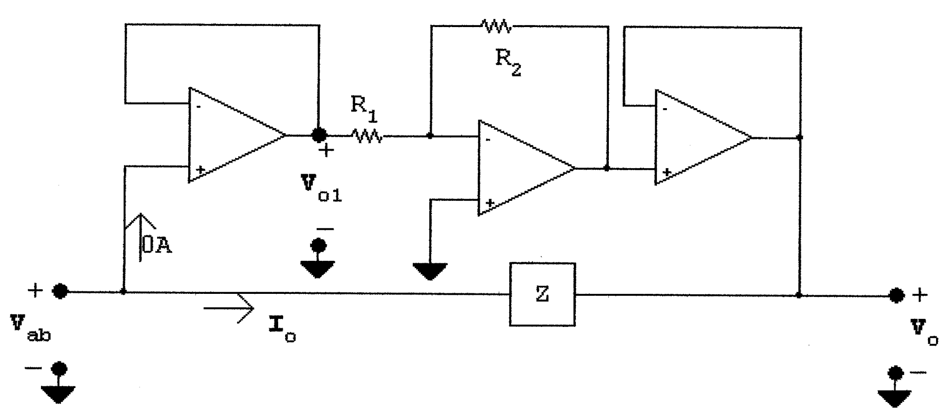
Solving,

$$C_o = 20\,\text{nF}$$

[b] $$\mathbf{V}_o = \frac{10(1-j2)}{1+j2} = 10\underline{/-126.87^\circ}$$

$$v_o = 10\cos(2\times 10^5 t - 126.87^\circ)\,\text{V}$$

P 9.69 [a]



Because the op-amps are ideal $\mathbf{I}_{\text{in}} = \mathbf{I}_o$, thus

$$Z_{\text{ab}} = \frac{\mathbf{V}_{\text{ab}}}{\mathbf{I}_{\text{in}}} = \frac{\mathbf{V}_{\text{ab}}}{\mathbf{I}_o}; \qquad \mathbf{I}_o = \frac{\mathbf{V}_{\text{ab}} - \mathbf{V}_o}{Z}$$

$$\mathbf{V}_{o1} = \mathbf{V}_{\text{ab}}; \qquad \mathbf{V}_{o2} = -\left(\frac{R_2}{R_1}\right)\mathbf{V}_{o1} = -K\mathbf{V}_{o1} = -K\mathbf{V}_{\text{ab}}$$

$$\mathbf{V}_o = \mathbf{V}_{o2} = -K\mathbf{V}_{\text{ab}}$$

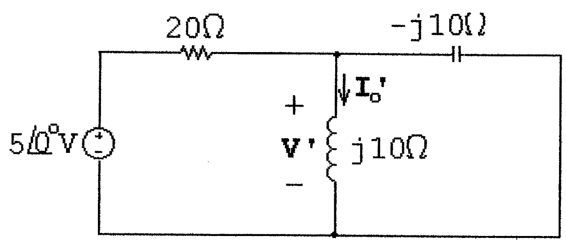
$$\therefore \quad \mathbf{I}_o = \frac{\mathbf{V}_{\text{ab}} - (-K\mathbf{V}_{\text{ab}})}{Z} = \frac{(1+K)\mathbf{V}_{\text{ab}}}{Z}$$

$$\therefore \quad Z_{\text{ab}} = \frac{\mathbf{V}_{\text{ab}}}{(1+K)\mathbf{V}_{\text{ab}}}Z = \frac{Z}{(1+K)}$$

[b] $Z = \dfrac{1}{j\omega C}; \qquad Z_{\text{ab}} = \dfrac{1}{j\omega C(1+K)}; \qquad \therefore \quad C_{\text{ab}} = C(1+K)$

P 9.70 [a] Superposition must be used because the frequencies of the two sources are different.

[b] For $\omega = 80{,}000$ rad/s:



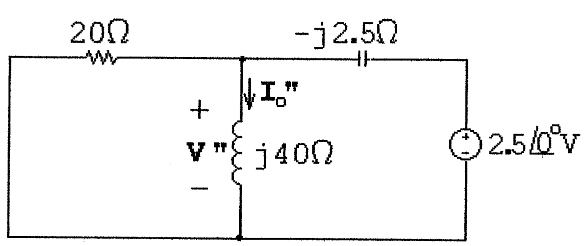
$$\frac{\mathbf{V}_o' - 5}{20} + \frac{\mathbf{V}_o'}{j10} + \frac{\mathbf{V}_o'}{-j10} = 0$$

$$\mathbf{V}_o'\left(\frac{1}{20} + \frac{1}{j10} + \frac{1}{-j10}\right) = \frac{5}{20}$$

$$\therefore \quad \mathbf{V}_o' = 5\underline{/0^\circ}\text{ V}$$

$$\mathbf{I}_o' = \frac{\mathbf{V}_o'}{j10} = -j0.5 = 500\underline{/-90^\circ}\text{ mA}$$

For $\omega = 320{,}000$ rad/s:



$$20\|j40 = 16 + j8\,\Omega$$

$$\mathbf{V}'' = \frac{16 + j8}{16 + j8 - j2.5}(2.5\underline{/0^\circ}) = 2.643\underline{/7.59^\circ}\text{ V}$$

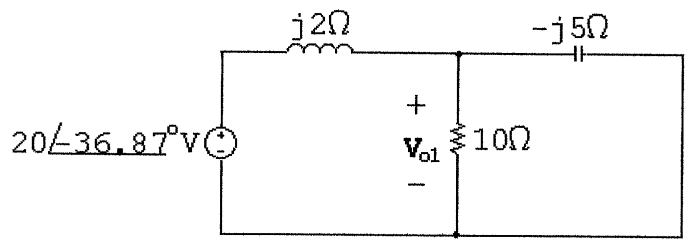
$$\therefore\ \mathbf{I}_o'' = \frac{\mathbf{V}''}{j40} = 66.08\underline{/-82.4^\circ}\text{ mA}$$

Thus,

$$i_o(t) = [500 \sin 80{,}000t + 66.08 \cos(320{,}000t - 82.4^\circ)]\text{ mA}, \quad t \geq 0$$

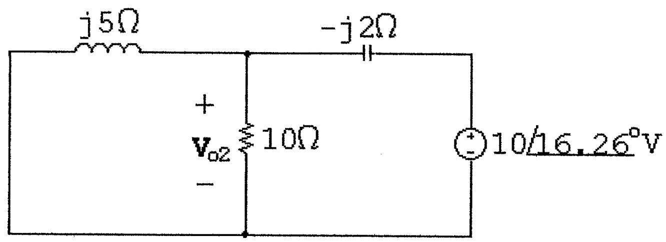
P 9.71 [a] Superposition must be used because the frequencies of the two sources are different.

[b] For $\omega = 2000$ rad/s:



$$10 \| -j5 = 2 - j4\,\Omega \quad \text{so} \quad \mathbf{V}_{o1} = \frac{2 - j4}{2 - j4 + j2}(20\underline{/-36.87^\circ} = 31.62\underline{/-55.3^\circ}\text{ V}$$

For $\omega = 5000$ rad/s:



$$j5 \| 10 = 2 + j4\,\Omega$$

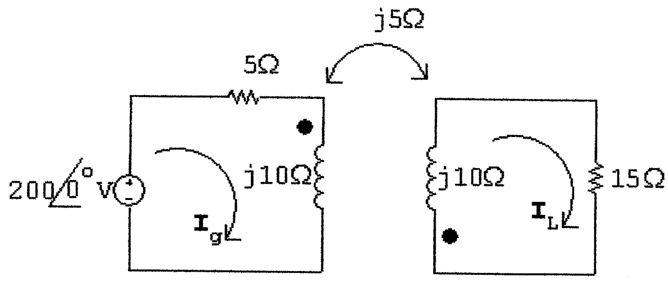
$$\mathbf{V}_{o2} = \frac{2 + j4}{2 + j4 - j2}(10\underline{/16.26^\circ}) = 15.81\underline{/34.69^\circ}\text{ V}$$

Thus,

$$v_o(t) = [31.62 \cos(2000t - 55.3^\circ) + 15.81 \cos(5000t + 34.69^\circ)]\text{ V}, \quad t \geq 0$$

P 9.72 [a] $j\omega L_1 = j\omega L_2 = j(10{,}000)(1 \times 10^{-3}) = j10\,\Omega$

$$j\omega M = j(10{,}000)(0.5 \times 10^{-3}) = j5\,\Omega$$



$$200 = (5 + j10)\mathbf{I}_g + j5\mathbf{I}_L$$

$$0 = j5\mathbf{I}_g + (15 + j10)\mathbf{I}_L$$

Solving,

$$\mathbf{I}_g = 10 - j15\,\text{A}; \qquad \mathbf{I}_L = -5\,\text{A}$$

$$i_g = 18.03\cos(10{,}000t - 56.31^\circ)\,\text{A}$$

$$i_L = 5\cos(10{,}000t - 180^\circ)\,\text{A}$$

[b] $k = \dfrac{M}{\sqrt{L_1L_2}} = \dfrac{0.5}{\sqrt{1}} = 0.5$

[c] When $t = 50\pi\ \mu$s,

$$10{,}000t = (10{,}000)(50\pi)\times 10^{-6} = 0.5\pi = \pi/2\,\text{rad} = 90^\circ$$

$$i_g(50\pi\mu s) = 18.03\cos(90 - 56.31^\circ) = 15\,\text{A}$$

$$i_L(50\pi\mu s) = 5\cos(90 + 180^\circ) = 0\,\text{A}$$

$$w = \frac{1}{2}L_1i_1^2 + \frac{1}{2}L_2i_2^2 + Mi_1i_2 = \frac{1}{2}(1\times 10^{-3})(15)^2 + 0 + 0 = 112.5\,\text{mJ}$$

When $t = 100\pi\ \mu$s,

$$10{,}000t = \pi\,\text{rad} = 180^\circ$$

$$i_g(100\pi\mu s) = -10\,\text{A}$$

$$i_L(100\pi\mu s) = 5\,\text{A}$$

$$w = \frac{1}{2}(1\times 10^{-3})(10)^2 + \frac{1}{2}(1\times 10^{-3})(5)^2 + 0.5\times 10^{-3}(-10)(5) = 37.5\,\text{mJ}$$

P 9.73 [a] $j\omega L_1 = j(50)(5) = j250\,\Omega$

$$j\omega L_2 = j(50)(20) = j1000\,\Omega$$

$$\frac{1}{j\omega C} = \frac{10^9}{j(50\times 10^3)(40)} = -j500\,\Omega$$

$$\therefore \quad Z_{22} = 75 + 300 + j1000 - j500 = 375 + j500\,\Omega$$

$$\therefore \quad Z_{22}^* = 375 - j500\,\Omega$$

$$M = k\sqrt{L_1 L_2} = 10k \times 10^{-3}$$

$$\omega M = (50)(10k) = 500k$$

$$Z_r = \left[\frac{500k}{625}\right]^2 (375 - j500) = k^2(240 - j320)\,\Omega$$

$$Z_{\text{in}} = 120 + j250 + 240k^2 - j320k^2$$

$$|Z_{\text{in}}| = [(120 + 240k^2)^2 + (250 - 320k^2)^2]^{\frac{1}{2}}$$

$$\frac{d|Z_{\text{in}}|}{dk} = \frac{1}{2}[(120 + 240k^2)^2 + (250 - 320k^2)^2]^{-\frac{1}{2}} \times$$

$$[2(120 + 240k^2)480k + 2(250 - 320k^2)(-640k)]$$

$$\frac{d|Z_{\text{in}}|}{dk} = 0 \text{ when}$$

$$960k(120 + 240k^2) - 1280k(250 - 320k^2) = 0$$

$$\therefore \quad k^2 = 0.32; \qquad \therefore \quad k = \sqrt{0.32} = 0.5657$$

[b] $Z_{\text{in}}\ (\text{min}) = 120 + 240(0.32) + j[250 - 0.32(320)]$

$$= 196.8 + j147.6 = 246\underline{/36.87^\circ}\,\Omega$$

$$\mathbf{I}_1\ (\text{max}) = \frac{369\underline{/0^\circ}}{246\underline{/36.87^\circ}} = 1.5\underline{/-36.87^\circ}\,\text{A}$$

$$\therefore \quad i_1\ (\text{peak}) = 1.5\,\text{A}$$

Note — You can test that the k value obtained from setting $d|Z_{\text{in}}|/dt = 0$ leads to a minimum by noting $0 \leq k \leq 1$. If $k = 1$,

$$Z_{\text{in}} = 360 - j70 = 366.74\underline{/-11^\circ}\,\Omega$$

Thus,

$$|Z_{\text{in}}|_{k=1} > |Z_{\text{in}}|_{k=\sqrt{0.32}}$$

If $k = 0$,

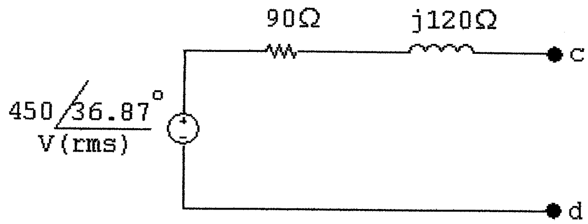
$$Z_{\text{in}} = 120 + j250 = 277.31\underline{/64.36^\circ}\,\Omega$$

Thus,

$$|Z_{\text{in}}|_{k=0} > |Z_{\text{in}}|_{k=\sqrt{0.32}}$$

P 9.74 $Z_{\text{Th}} = 30 + j200 + (50/25)^2(15 - j20) = 90 + j120\,\Omega$

$$\mathbf{V}_{\text{Th}} = \frac{225\underline{/0^\circ}}{15 + j20}(j50) = 450\underline{/36.87^\circ}\text{ V}$$



P 9.75 $j\omega L_1 = j(25\times 10^3)(3.2\times 10^{-3}) = j80\,\Omega$

$$j\omega L_2 = j(25\times 10^3)(12.8\times 10^{-3}) = j320\,\Omega$$

$$\frac{1}{j\omega C} = \frac{10^9}{j(25\times 10^3)(250)} = -j160\,\Omega$$

$$j\omega M = j(25\times 10^3)k\sqrt{(3.2)(12.8)}\times 10^{-3} = j160k\,\Omega$$

$$Z_{22} = 40 + j320 - j160 = 40 + j160\,\Omega$$

$$Z_{22}^* = 40 - j160\,\Omega$$

$$Z_r = \left[\frac{160k}{|40 + j160|}\right]^2(40 - j160) = 37.647k^2 - j150.588k^2$$

$$Z_{\text{ab}} = 10 + j80 + 37.647k^2 - j150.588k^2 = (10 + 37.647k^2) + j(80 - 150.588k^2)$$

Z_{ab} is resistive when

$$80 - 150.588k^2 = 0 \quad \text{or} \quad k^2 = 0.53125$$

$$\therefore\; Z_{\text{ab}} = 10 + (37.647)(0.53125) = 30\,\Omega$$

P 9.76 [a] $j\omega L_2 = j(500)10^3(500)10^{-6} = j250\,\Omega$

$$\frac{1}{j\omega C} = \frac{10^9}{j(500\times 10^3)(20)} = -j100\,\Omega$$

$$Z_{22} = 150 + 50 + j250 - j100 = 200 + j150\,\Omega$$

$$Z_{22}^* = 200 - j150\,\Omega$$

$$\omega M = (500\times 10^3)(100\times 10^{-6}) = 50\,\Omega$$

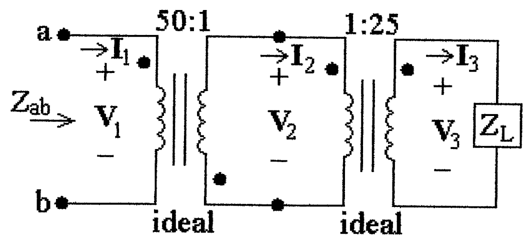
$$Z_r = \left(\frac{50}{250}\right)^2[200 - j150] = 8 - j6\,\Omega$$

[b] $Z_{ab} = R_1 + j\omega L_1 + 8 - j6$

$$j\omega L_1 = j(500 \times 10^3)(80 \times 10^{-6}) = j40\,\Omega$$

$$Z_{ab} = 20 + j34\,\Omega$$

P 9.77



$$Z_{ab} = \frac{\mathbf{V}_1}{\mathbf{I}_1}$$

$$\frac{\mathbf{V}_1}{50} = -\frac{\mathbf{V}_2}{1}; \qquad 50\mathbf{I}_1 = -\mathbf{I}_2$$

$$\therefore \quad Z_{ab} = \frac{-50\mathbf{V}_2}{-\mathbf{I}_2/50} = 2500\frac{\mathbf{V}_2}{\mathbf{I}_2}$$

$$\frac{\mathbf{V}_2}{1} = \frac{\mathbf{V}_3}{25}; \qquad \mathbf{I}_2 = 25\mathbf{I}_3$$

$$\therefore \quad Z_{ab} = 2500\frac{\mathbf{V}_3/25}{25\mathbf{I}_2} = \frac{2500}{625}\frac{\mathbf{V}_3}{\mathbf{I}_3}$$

$$= 4Z_L = 4(200 + j150) = (800 + j600)\,\Omega$$

P 9.78 In Eq. 9.69 replace $\omega^2 M^2$ with $k^2\omega^2 L_1 L_2$ and then write X_{ab} as

$$X_{ab} = \omega L_1 - \frac{k^2\omega^2 L_1 L_2(\omega L_2 + \omega L_L)}{R_{22}^2 + (\omega L_2 + \omega L_L)^2}$$

$$= \omega L_1 \left\{1 - \frac{k^2\omega L_2(\omega L_2 + \omega L_L)}{R_{22}^2 + (\omega L_2 + \omega L_L)^2}\right\}$$

For X_{ab} to be negative requires

$$R_{22}^2 + (\omega L_2 + \omega L_L)^2 < k^2\omega L_2(\omega L_2 + \omega L_L)$$

or

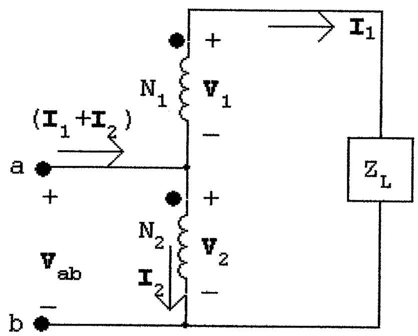
$$R_{22}^2 + (\omega L_2 + \omega L_L)^2 - k^2\omega L_2(\omega L_2 + \omega L_L) < 0$$

which reduces to

$$R_{22}^2 + \omega^2 L_2^2(1-k^2) + \omega L_2 \omega L_L(2-k^2) + \omega^2 L_L^2 < 0$$

But $k \leq 1$ hence it is impossible to satisfy the inequality. Therefore X_{ab} can never be negative if X_L is an inductive reactance.

P 9.79 [a]



$$Z_{\text{ab}} = \frac{\mathbf{V}_{\text{ab}}}{\mathbf{I}_1 + \mathbf{I}_2} = \frac{\mathbf{V}_2}{\mathbf{I}_1 + \mathbf{I}_2} = \frac{\mathbf{V}_2}{(1 + N_1/N_2)\mathbf{I}_1}$$

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{N_1}{N_2}\mathbf{I}_1$$

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1}{N_2}, \qquad \mathbf{V}_1 = \frac{N_1}{N_2}\mathbf{V}_2$$

$$\mathbf{V}_1 + \mathbf{V}_2 = Z_L\mathbf{I}_1 = \left(\frac{N_1}{N_2} + 1\right)\mathbf{V}_2$$

$$Z_{\text{ab}} = \frac{\mathbf{I}_1 Z_L}{(N_1/N_2 + 1)(1 + N_1/N_2)\mathbf{I}_1}$$

$$\therefore \quad Z_{\text{ab}} = \frac{Z_L}{[1 + (N_1/N_2)]^2} \quad \text{Q.E.D.}$$

[b] Assume dot on the N_2 coil is moved to the lower terminal. Then

$$\mathbf{V}_1 = -\frac{N_1}{N_2}\mathbf{V}_2 \quad \text{and} \quad \mathbf{I}_2 = -\frac{N_1}{N_2}\mathbf{I}_1$$

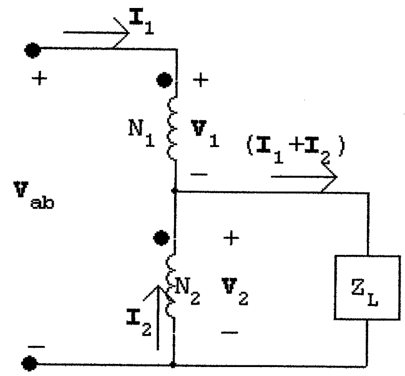
As before

$$Z_{\text{ab}} = \frac{\mathbf{V}_2}{\mathbf{I}_1 + \mathbf{I}_2} \quad \text{and} \quad \mathbf{V}_1 + \mathbf{V}_2 = Z_L\mathbf{I}_1$$

$$\therefore \quad Z_{\text{ab}} = \frac{\mathbf{V}_2}{(1 - N_1/N_2)\mathbf{I}_1} = \frac{Z_L\mathbf{I}_1}{[1 - (N_1/N_2)]^2\mathbf{I}_1}$$

$$Z_{\text{ab}} = \frac{Z_L}{[1 - (N_1/N_2)]^2} \quad \text{Q.E.D.}$$

P 9.80 [a]



$$Z_{ab} = \frac{\mathbf{V}_{ab}}{\mathbf{I}_1} = \frac{\mathbf{V}_1 + \mathbf{V}_2}{\mathbf{I}_1}$$

$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2}, \qquad \mathbf{V}_2 = \frac{N_2}{N_1}\mathbf{V}_1$$

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{N_1}{N_2}\mathbf{I}_1$$

$$\mathbf{V}_2 = (\mathbf{I}_1 + \mathbf{I}_2)Z_L = \mathbf{I}_1\left(1 + \frac{N_1}{N_2}\right)Z_L$$

$$\mathbf{V}_1 + \mathbf{V}_2 = \left(\frac{N_1}{N_2} + 1\right)\mathbf{V}_2 = \left(1 + \frac{N_1}{N_2}\right)^2 Z_L\mathbf{I}_1$$

$$\therefore \quad Z_{ab} = \frac{(1 + N_1/N_2)^2 Z_L\mathbf{I}_1}{\mathbf{I}_1}$$

$$Z_{ab} = \left(1 + \frac{N_1}{N_2}\right)^2 Z_L \quad \text{Q.E.D.}$$

[b] Assume dot on N_2 is moved to the lower terminal, then

$$\frac{\mathbf{V}_1}{N_1} = \frac{-\mathbf{V}_2}{N_2}, \qquad \mathbf{V}_1 = \frac{-N_1}{N_2}\mathbf{V}_2$$

$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{-N_1}{N_2}\mathbf{I}_1$$

As in part [a]

$$\mathbf{V}_2 = (\mathbf{I}_2 + \mathbf{I}_1)Z_L \quad \text{and} \quad Z_{ab} = \frac{\mathbf{V}_1 + \mathbf{V}_2}{\mathbf{I}_1}$$

$$Z_{ab} = \frac{(1 - N_1/N_2)\mathbf{V}_2}{\mathbf{I}_1} = \frac{(1 - N_1/N_2)(1 - N_1/N_2)Z_L\mathbf{I}_1}{\mathbf{I}_1}$$

$$Z_{ab} = [1 - (N_1/N_2)]^2\, Z_L \quad \text{Q.E.D.}$$

P 9.81 [a] $\mathbf{I} = \frac{240}{24} + \frac{240}{j32} = (10 - j7.5)\text{ A}$

$$\mathbf{V}_s = 240\underline{/0^\circ} + (0.1 + j0.8)(10 - j7.5) = 247 + j7.25 = 247.11\underline{/1.68^\circ}\text{ V}$$

[b] Use the capacitor to eliminate the j component of **I**, therefore

$$\mathbf{I}_c = j7.5\text{ A}, \qquad Z_c = \frac{240}{j7.5} = -j32\,\Omega$$

$$\mathbf{V}_s = 240 + (0.1 + j0.8)10 = 241 + j8 = 241.13\underline{/1.90^\circ}\text{ V}$$

[c] Let I_c denote the magnitude of the current in the capacitor branch. Then

$$\mathbf{I} = (10 - j7.5 + jI_c) = 10 + j(I_c - 7.5)\text{ A}$$

$$\mathbf{V}_s = 240\underline{/\alpha} = 240 + (0.1 + j0.8)[10 + j(I_c - 7.5)]$$
$$= (247 - 0.8I_c) + j(7.25 + 0.1I_c)$$

It follows that

$$240\cos\alpha = (247 - 0.8I_c) \quad\text{and}\quad 240\sin\alpha = (7.25 + 0.1I_c)$$

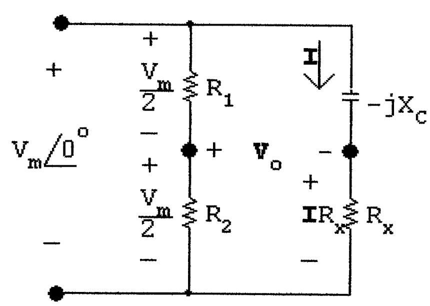
Now square each term and then add to generate the quadratic equation

$$I_c^2 - 605.77I_c + 5325.48 = 0; \qquad I_c = 302.88 \pm 293.96$$

Therefore

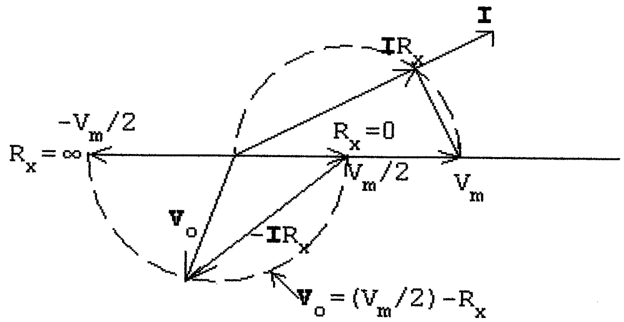
$$I_c = 8.92\text{ A (smallest value) and } Z_c = 240/j8.92 = -j26.90\,\Omega.$$

P 9.82 The phasor domain equivalent circuit is

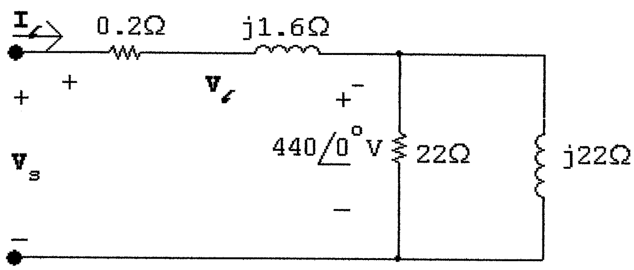


$$V_o = \frac{V_m}{2} - \mathbf{I}R_x; \qquad \mathbf{I} = \frac{V_m}{R_x - jX_C}$$

As R_x varies from 0 to ∞, the amplitude of v_o remains constant and its phase angle increases from 0° to -180°, as shown in the following phasor diagram:



P 9.83 [a]

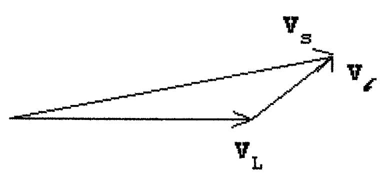


$$\mathbf{I}_\ell = \frac{440}{22} + \frac{440}{j22} = 20 - j20\ \mathrm{A}$$

$$\mathbf{V}_\ell = (0.2 + j1.6)(20 - j20) = 36 + j28 = 45.61\underline{/37.87^\circ}\ \mathrm{V(rms)}$$

$$\mathrm{V}_s = 440\underline{/0^\circ} + \mathbf{V}_\ell = 476 + j28 = 476.82\underline{/3.37^\circ}\ \mathrm{V}$$

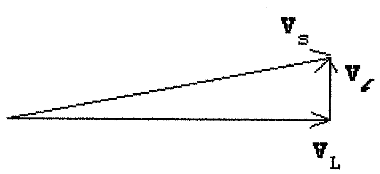
[b]



[c] $$\mathbf{I}_\ell = \frac{440}{22} + \frac{440}{j22} + \frac{440}{-j22} = 20 + j0\ \mathrm{A}$$

$$\mathbf{V}_\ell = (0.2 + j1.6)(20 + j0) = 4 + j32 = 32.25\underline{/82.87^\circ}$$

$$\mathbf{V}_s = 440 + \mathbf{V}_\ell = 444 + j32 = 445.15\underline{/4.12^\circ}$$



P 9.84 [a] $\mathbf{I}_1 = \frac{120}{24} + \frac{240}{8.4 + j6.3} = 23.29 - j13.71 = 27.02\underline{/-30.5^\circ}$ A

$$\mathbf{I}_2 = \frac{120}{12} - \frac{120}{24} = 5\underline{/0^\circ} \text{ A}$$

$$\mathbf{I}_3 = \frac{120}{12} + \frac{240}{8.4 + j6.3} = 28.29 - j13.71 = 31.44\underline{/-25.87^\circ} \text{ A}$$

$$\mathbf{I}_4 = \frac{120}{24} = 5\underline{/0^\circ} \text{ A}; \qquad \mathbf{I}_5 = \frac{120}{12} = 10\underline{/0^\circ} \text{ A}$$

$$\mathbf{I}_6 = \frac{240}{8.4 + j6.3} = 18.29 - j13.71 = 22.86\underline{/-36.87^\circ} \text{ A}$$

[b] When fuse A is interrupted,

$\mathbf{I}_1 = 0$ $\quad \mathbf{I}_3 = 15$ A $\quad \mathbf{I}_5 = 10$ A

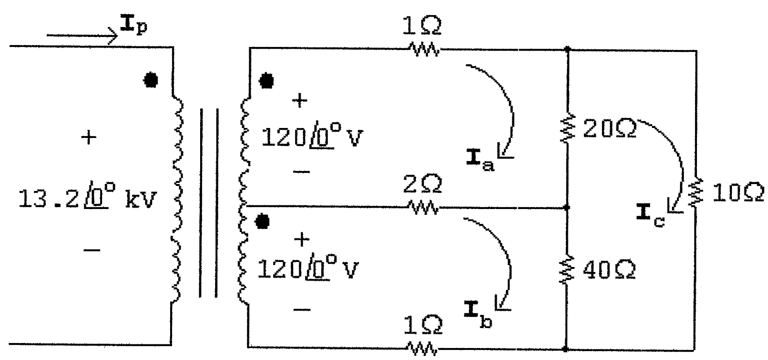
$\mathbf{I}_2 = 10 + 5 = 15$ A $\quad \mathbf{I}_4 = -5$ A $\quad \mathbf{I}_6 = 5$ A

[c] The clock and television set were fed from the uninterrupted side of the circuit, that is, the $12\,\Omega$ load includes the clock and the TV set.

[d] No, the motor current drops to 5 A, well below its normal running value of 22.86 A.

[e] After fuse A opens, the current in fuse B is only 15 A.

P 9.85 [a] The circuit is redrawn, with mesh currents identified:



The mesh current equations are:

$$120\underline{/0^\circ} = 23\mathbf{I}_a - 2\mathbf{I}_b - 20\mathbf{I}_c$$

$$120\underline{/0^\circ} = -2\mathbf{I}_a + 43\mathbf{I}_b - 40\mathbf{I}_c$$

$$0 = -20\mathbf{I}_a - 40\mathbf{I}_b + 70\mathbf{I}_c$$

Solving,

$\mathbf{I}_a = 24\underline{/0^\circ}$ A $\quad \mathbf{I}_b = 21.96\underline{/0^\circ}$ A $\quad \mathbf{I}_c = 19.40\underline{/0^\circ}$ A

The branch currents are:

$\mathbf{I}_1 = \mathbf{I}_a = 24\underline{/0^\circ}\text{ A}$

$\mathbf{I}_2 = \mathbf{I}_a - \mathbf{I}_b = 2.04\underline{/0^\circ}\text{ A}$

$\mathbf{I}_3 = \mathbf{I}_b = 21.96\underline{/0^\circ}\text{ A}$

$\mathbf{I}_4 = \mathbf{I}_c = 19.40\underline{/0^\circ}\text{ A}$

$\mathbf{I}_5 = \mathbf{I}_a - \mathbf{I}_c = 4.6\underline{/0^\circ}\text{ A}$

$\mathbf{I}_6 = \mathbf{I}_b - \mathbf{I}_c = 2.55\underline{/0^\circ}\text{ A}$

[b] Let N_1 be the number of turns on the primary winding; because the secondary winding is center-tapped, let $2N_2$ be the total turns on the secondary. From Fig. 9.58,

$$\frac{13{,}200}{N_1} = \frac{240}{2N_2} \qquad \text{or} \qquad \frac{N_2}{N_1} = \frac{1}{110}$$

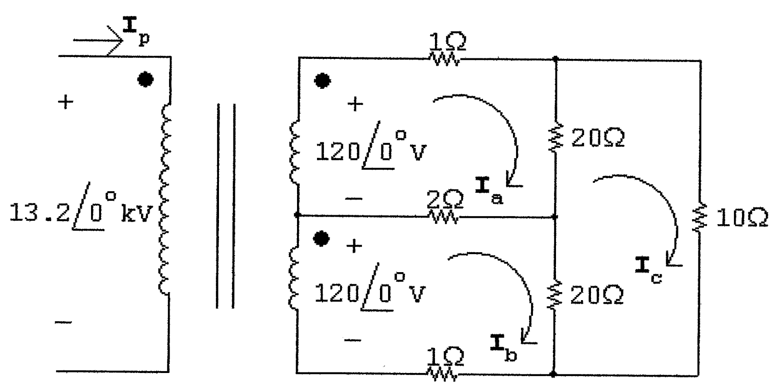
The ampere turn balance requires

$$N_1\mathbf{I}_p = N_2\mathbf{I}_1 + N_2\mathbf{I}_3$$

Therefore,

$$\mathbf{I}_p = \frac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_3) = \frac{1}{110}(24 + 21.96) = 0.42\underline{/0^\circ}\text{ A}$$

P 9.86 [a]



The three mesh current equations are

$120\underline{/0^\circ} = 23\mathbf{I}_a - 2\mathbf{I}_b - 20\mathbf{I}_c$

$120\underline{/0^\circ} = -2\mathbf{I}_a + 23\mathbf{I}_b - 20\mathbf{I}_c$

$0 = -20\mathbf{I}_a - 20\mathbf{I}_b + 50\mathbf{I}_c$

Solving,

$\mathbf{I}_a = 24\underline{/0^\circ}\text{ A}; \qquad \mathbf{I}_b = 24\underline{/0^\circ}\text{ A}; \qquad \mathbf{I}_c = 19.2\underline{/0^\circ}\text{ A}$

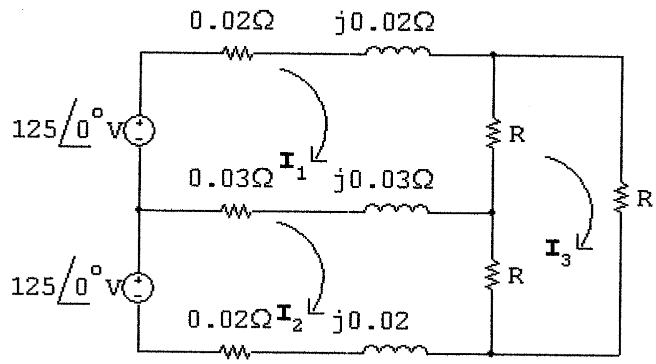
$\therefore\ \mathbf{I}_2 = \mathbf{I}_a - \mathbf{I}_b = 0\text{ A}$

[b] $\mathbf{I}_p = \frac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_3) = \frac{N_2}{N_1}(\mathbf{I}_a + \mathbf{I}_b$

$$= \frac{1}{110}(24 + 24) = 0.436\,\text{A}$$

[c] When the two loads are equal, more current is drawn from the primary.

P 9.87 [a]



$$125 = (R + 0.05 + j0.05)\mathbf{I}_1 - (0.03 + j0.03)\mathbf{I}_2 - R\mathbf{I}_3$$

$$125 = -(0.03 + j0.03)\mathbf{I}_1 + (R + 0.05 + j0.05)\mathbf{I}_2 - R\mathbf{I}_3$$

Subtracting the above two equations gives

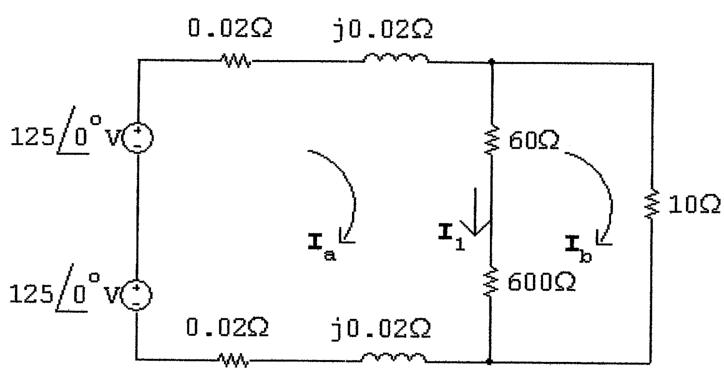
$$0 = (R + 0.08 + j0.08)\mathbf{I}_1 - (R + 0.08 + j0.08)\mathbf{I}_2$$

$$\therefore\quad \mathbf{I}_1 = \mathbf{I}_2 \quad \text{so} \quad \mathbf{I}_n = \mathbf{I}_1 - \mathbf{I}_2 = 0\,\text{A}$$

[b] $\mathbf{V}_1 = R(\mathbf{I}_1 - \mathbf{I}_3);\qquad \mathbf{V}_2 = R(\mathbf{I}_2 - \mathbf{I}_3)$

Since $\mathbf{I}_1 = \mathbf{I}_2$ (from part [a]) $\mathbf{V}_1 = \mathbf{V}_2$

[c]



$$250 = (660.04 + j0.04)\mathbf{I}_a - 660\mathbf{I}_b$$

$$0 = -660\mathbf{I}_a + 670\mathbf{I}_b$$

Solving,

$\mathbf{I}_a = 25.275945\underline{/-0.231714^\circ} = 25.275738 - j0.10222$ A

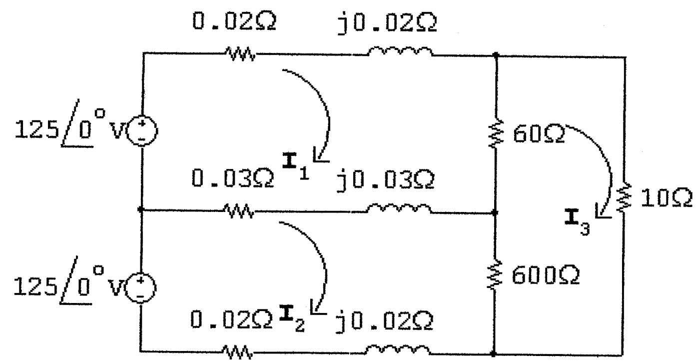
$\mathbf{I}_b = 24.898692\underline{/-0.231713^\circ} = 24.898488 - j0.100694$ A

$\mathbf{I}_1 = \mathbf{I}_a - \mathbf{I}_b = 0.37725 - j0.001526$ A

$\mathbf{V}_1 = 60\mathbf{I}_1 = 22.635 - j0.09156 = 22.635185\underline{/-0.231764^\circ}$ V

$\mathbf{V}_2 = 600\mathbf{I}_1 = 226.35 - j0.9156 = 226.35185\underline{/-0.231764^\circ}$ V

[d]



$125 = (60.05 + j0.05)\mathbf{I}_1 - (0.03 + j0.03)\mathbf{I}_2 - 60\mathbf{I}_3$

$125 = -(0.03 + j0.03)\mathbf{I}_1 + (600.05 + j0.05)\mathbf{I}_2 - 600\mathbf{I}_3$

$0 = -60\mathbf{I}_1 - 600\mathbf{I}_2 + 670\mathbf{I}_3$

Solving,

$\mathbf{I}_1 = 26.97\underline{/-0.24^\circ} = 26.97 - j0.113$ A

$\mathbf{I}_2 = 25.10\underline{/-0.24^\circ} = 25.10 - j0.104$ A

$\mathbf{I}_3 = 24.90\underline{/-0.24^\circ} = 24.90 - j0.104$ A

$\mathbf{V}_1 = 60(\mathbf{I}_1 - \mathbf{I}_3) = 124.4\underline{/-0.27^\circ}$ V

$\mathbf{V}_2 = 600(\mathbf{I}_2 - \mathbf{I}_3) = 124.6\underline{/-0.20^\circ}$ V

[e] Because an open neutral can result in severely unbalanced voltages across the 125 V loads.

P 9.88 [a] Let N_1 = primary winding turns and $2N_2$ = secondary winding turns. Then

$$\frac{14{,}000}{N_1} = \frac{250}{2N_2}; \qquad \therefore \quad \frac{N_2}{N_1} = \frac{1}{112} = a$$

In part c),

$\mathbf{I}_\mathrm{p} = 2a\mathbf{I}_\mathrm{a}$

$$\therefore\quad \mathbf{I}_\mathrm{p} = \frac{2N_2\mathbf{I}_\mathrm{a}}{N_1} = \tfrac{1}{56}\mathbf{I}_\mathrm{a}$$

$$= \frac{1}{56}(25.28 - j0.10)$$

$$\mathbf{I}_\mathrm{p} = 451.4 - j1.8\,\mathrm{mA}$$

In part d),

$\mathbf{I}_\mathrm{p}N_1 = \mathbf{I}_1N_2 + \mathbf{I}_2N_2$

$$\therefore\quad \mathbf{I}_\mathrm{p} = \frac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_2)$$

$$= \frac{1}{112}(26.97 - j0.11 + 25.10 - j0.10)$$

$$= \frac{1}{112}(52.07 - j0.22)$$

$$\mathbf{I}_\mathrm{p} = 464.9 - j1.9\,\mathrm{mA}$$

[b] Yes, because the neutral conductor carries non-zero current whenever the load is not balanced.