Solutions Manual

PREFACE

Here will be found a complete set of solutions to the exercises that appear at the end of sections and chapters in the Fourteenth Edition of *Introduction to Logic*. This (very sizable!) packet differs from the solutions manuals for earlier editions in the following important way: It is complete in the sense that it includes (as earlier manuals did not) the solutions that are also provided at the back of the book itself, in "Solutions to Selected Exercises."

The formal proofs of validity of deductive arguments provided here are (for many exercises) neither the only ones nor the shortest ones possible. For some proofs of invalidity not all truth value assignments that could serve the same purpose are included.

Many exercises cannot be said to have a single definitive solution. Where there are complications introduced by the possibility, or even the likelihood, of differing interpretations, we have given what seem to us to be the best answers. But our judgment will be disputable in some cases, and alternative analyses will often be plausible, sometimes perhaps superior. Ingenious students often surprise one with a variety of alternative answers for which some justifications can be supplied; it is surely proper to give credit for analyses and solutions that can be plausibly defended.

The responses to some exercises in Chapters 12 and 13, in addition to being subject to alternative interpretations, can be very lengthy, for which reason several models only are provided, in place of an extensive discursive response to each exercise.

It is nearly impossible, in work of this kind, to eliminate every flaw. We acknowledge with sincere thanks—and continue to welcome—improvements our readers may suggest. Especially valuable are corrections of errors, for which we are grateful. Readers are invited to send corrections and suggestions of every sort to: *ccohen@umich.edu*

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Chapter 1

Section 1.2 Identify premises and conclusions
Exercises on pages 9–11

1. Premise: A well-regulated militia is necessary for the security of a free state.

   Conclusion: The right of the people to keep and bear arms shall not be infringed.

2. Premises:

   (1) It's easier (than photocopying) to buy your friend a paperback copy of a book.

   (2) A paperback copy of the book is inexpensive.

   Conclusion: What stops many people from photocopying a book and giving it to a pal is not integrity but logistics.

3. Premise: Human intelligence is a gift from God.

   Conclusion: To apply human intelligence to understand the world is not an affront to God, but is pleasing to him.

4. Premise: Sir Edmund Hilary dedicated his life to helping build schools and hospitals for the Sherpas who helped him to climb Mount Everest.

   Conclusion: He is, for that reason, a hero.

5. Premises:

   (1) Standardized tests have a disparate racial impact, as illustrated by the difference in the average scores of different ethnic groups.

   (2) Ethnic differences arise on all kinds of tests, at all levels.

   Conclusion: If a racial gap is evidence of discrimination, then all tests discriminate.

6. Premise: Everybody thinks himself so abundantly provided with good sense that even those most difficult to please in all other matters do not commonly desire more of it than they already possess.

   Conclusion: Good sense is, of all things in the world, the most equally distributed.

7. Premise: Any words new to the United States are either stupid or foreign.

   Conclusion: There is no such thing as the American language; there's just bad English.

8. Premise: In New York State alone taxpayers spent more than $200 million in a failed death penalty experiment, with no one executed.

   Conclusion: The death penalty is too costly.

   Premise: [There has been] an epidemic of exonerations of death row inmates upon post-conviction investigation, including ten New York inmates freed in the last 18 months from long sentences being served for murders or rapes they did not commit.

   Conclusion: Capital punishment is unfair in its application, in addition to being too costly.

9. Premise: Houses are built to live in, not to look on.

   Conclusion: Use is to be preferred before [i.e., above] uniformity.

10. Premises:

    (1) A boycott, although not violent, can cause economic harm to many.

    (2) The greater the impact of a boycott, the more impressive is the statement it makes.

    (3) The economic consequences of a boycott are likely to be felt by innocent bystanders, who suffer loss of income because of it.

    Conclusion: The boycott weapon ought to be used sparingly.

11. Premises:

    (1) In the early part of the 20th century forced population shifts were not uncommon.

    (2) In that period multicultural empires crumbled and nationalism drove the formation of new, ethnically homogenous countries.

    Conclusion: Ethnic cleansing was viewed not so long ago as a legitimate tool of foreign policy.

12. Premises:

    (1) If a jury is sufficiently unhappy with the government's case or the government's conduct, it can simply refuse to convict.

    (2) This possibility puts powerful pressure on the state to behave properly.

    Conclusion: A jury is one of the most important protections of a democracy.

13. Premises:

    (1) Orangutans spend more than 95 percent of their time in the trees, which, along with vines and termites, provide more than 99 percent of their food.

    (2) Their only habitat is formed by the tropical rain forests of Borneo and Sumatra.

    Conclusion: Without forests, orangutans cannot survive.

14. Premise: If God is omniscient, he must already know how he is going to intervene to change the course of history using his omnipotence.

    Conclusion: God cannot change his mind about his intervention.

    Premise: God cannot change his mind about his intervention.

    Conclusion: If God is omniscient he is not omnipotent.

    Premise: If God is omniscient he is not omnipotent.

    Conclusion: Omniscient and omnipotence are mutually incompatible.

15. Premises:

    (1) Reason never comes to the aid of spiritual things.

    (2) More frequently than not, reason struggles against the divine Word, treating all that comes from God with contempt.

    Conclusion: Reason is the greatest enemy that faith has.

Section 1.4 Arguments and Explanations
Exercises on pages 20–24

1. This is essentially an explanation. *What* is being explained is the fact that humans have varying skin colors. The explanation is that different skin colors evolved as humans came to live at different distances from the Equator and hence needed different degrees of protection from the rays of the sun. One might interpret the passage as an argument whose conclusion is that skin color is not a permanent trait of all humans. Under this interpretation, all the propositions preceding the final sentence of the passage serve as premises.

2. This is an argument, whose conclusion is that the victories of American labor through the passage of ostensibly neutral laws regulating labor, were seriously adverse to the interests of blacks, and resulted in the now longstanding gap between black and white unemployment rates. One might interpret the passage simply as an explanation, in

which what is being explained is that gap, but this interpretation leaves aside the many ramifications of the argument.

3. This is an explanation. *What* is being explained is why sex feels good. The explanation is that those animals in which it does feel good have more offspring, and therefore more evolutionary success, than those animals in which sex does not so effectively motivate. If we did not know that sex feels good, this might be considered an argument to show that it does; but since the pleasure of sex is a fact not in serious question here, the passage is best viewed as an explanation of that reality.

4. This is an argument. Its premises are that (1) changes are real and (2) changes are only possible in time. The conclusion is that time must be something real.

5. This may be interpreted either as an explanation or as an argument. Viewed as an explanation, *what* is being explained is the fact, not doubted here, that the nursing shortage has turned into a crisis. The explanation of that fact is a combination of observations, including the fact that fewer young people are going into nursing, that many older nurses are on the verge of retirement, that nurses often report high rates of job dissatisfaction and plan to leave the profession, and that hospitals routinely cancel or delay surgical cases because of a lack of nursing staff. Viewed as an argument, all these factors are premises supporting the conclusion that the shortage of nurses has indeed turned into a crisis.

6. This is an argument. Dewey is calling attention to the fact that to show what caused an event is not sufficient to justify it or to condemn it, because justification or condemnation comes (in his view) only through the consequences of the event, not its origin.

7. This passage is mainly an argument, whose conclusion is that a king cannot be subject to his own laws. Its premises are: (1) it is impossible to bind oneself in any matter which is the subject of one's own free exercise of will, and (2) the laws are no more than the product of the king's free will. The passage also serves as an explanation of the words commonly used in completing edicts and ordinances of a king: "for such is our good pleasure." This reinforces the argument above, since the king plainly cannot be bound by that which is determined only by his own good pleasure.

8. This is a bit of Oscar Wilde's humor that can be interpreted in various ways—as a sardonic argument attacking Wagner's music, perhaps, or as a lighthearted explanation of Wilde's hidden pleasure in that music. Or perhaps there is nothing seriously intended in the passage at all!

9. Although this could be viewed as an argument, it was very probably intended by the author as an explanation of the increased likelihood of cheating, that explanation consisting of the enumeration of several aspects of contemporary American society.

10. This is an explanation. *What* is explained is the fact that Cupid has been traditionally painted as blind. The explanation is that love, which Cupid represents, does not look with the eyes and therefore does not see.

11. This may be viewed either as an explanation or as an argument. If one takes the reported suggestion (that it is greater sexual selection pressure on women that accounts for their quantity of body hair) as true or known to be highly probable, then this passage is a more detailed explanation of how this came to pass. If, on the other hand, one takes the conclusion (that the lesser amount of body hair on women is due to sexual selection pressure) as in genuine doubt, then this passage may be interpreted as an argument in support of that conclusion. Of the two interpretations, the former seems the more plausible.

12. This is an argument whose conclusion is that the threat of nuclear war is useless against Iranian president Mahmoud Ahmadinejad. The premises are: (1) Iran's leaders do not care about killing their people in great numbers. (2) Ahmadinejad is a religious fanatic. (3) To such a fanatic, dying while fighting the enemy is a quick pass to heaven. (4) The mutually assured destruction that worked so well as a deterrent during the Cold War would instead be an inducement to war.

13. This is an argument whose conclusion is that interesting life can exist only in three dimensions. The premises are that (1) blood flow and large numbers of neural connections cannot exist in fewer than three dimensions; and (2) stable planetary orbits are not possible in more than three dimensions. [The argument makes the unstated assumption that the conditions described are necessary conditions for interesting life.]

14. This is an argument; but the first sentence in the passage is background material and not strictly a premise, although it is needed by the reader to understand the argument that follows immediately. After the conclusion ("we need them") appears the traditional Q.E.D.—which is the abbreviation for "quod erat demonstradum," meaning "what was to be demonstrated."

15. This is an argument. Its conclusion is that the Treasury Department has violated Section 504 of the Rehabilitation Act. Its premises are: (1) The Department has failed to design and issue paper currency that is readily distinguishable to blind and visually impaired individuals; and (2) [implied] this failure subjects blind and visually impaired persons to discrimination under an activity by an Executive agency.

16. This is an argument, whose conclusion is that acting in ways that fulfill one's duty never guarantees the moral goodness of the actor. The premise is that the act may be done from a motive that is indifferent or bad, and that the act may therefore be morally indifferent or bad.

17. This is an argument. Its conclusion is that belief in God is not beyond reason. Its premises are: (1) Only the supreme mind of God could create immutable and eternal laws. (2) Human reason can grasp some immutable and eternal laws, such as the circle or the square or the laws of physics. (3) In having that capacity, human reason must possess an innate particle of the mind of God.

18. This is an explanation. What is explained is the author's unhesitating celebration of religious holidays, although he is an atheist. The explanation is that many such rituals did not originate with Christian practices or beliefs, and that they really celebrate universal human goods and relationships.

19. This is an argument. Its conclusion is that ethnic movements are "two-edged swords"—that is, they can serve good and evil ends. The premises are (1) the fact that such movements are often necessary to repair injured collective psyches, and (2) the fact that such movements often end in tragedy, especially when they turn political, as in Germany.

20. This is an argument. Its conclusion is that it is false to say that all who are happy are equally happy. Its premises are: (1) happiness consists in the multiplicity of agreeable consciousness, and (2) a peasant does not have the capacity for happiness that a philosopher does (presumably because a philosopher will have a greater multiplicity of agreeable consciousness), and so cannot be equally *happy*, although the peasant can be equally *satisfied*.

Chapter 2

Section 2.1
Exercises on pages 36–38

1. Premise: The Detroit Pistons are an all-around better team than the San Antonio Spurs.

   Conclusion: The Pistons did not lose [the NBA finals, in 2005] because of lack of ability.

   Premise: The Pistons will beat the Spurs two out of every three times; and the Spurs will win one out of every three times.

   Premise: The Pistons had won the 5^{th} and 6^{th} games of the series—two in a row—so if they had won the final game they would have won three out of three.

   Conclusion: The Pistons lost because of the law of averages.

2. Premise: Universities have commonly been offering strange literary theories and assorted oddities, in place of the writing courses that ought to have been offered. Students have been shortchanged.

   Conclusion: Vast numbers of students cannot express themselves well in writing.

3. Premise: People divided on ethnic lines tend not to adopt programs that will give mutual support.

   Conclusion (and premise of the following argument): Therefore nations that are racially diverse tend to have lower levels of social support than nations that are racially homogenous.

   Conclusion: A welfare state with a racially diverse population is in tension, and the more racially diverse a community is, the more difficult it is to maintain comprehensive welfare programs.

4. Premise: If freedom were a natural part of the human condition we could expect to find free societies spread throughout human history.

   Premise: We do not find that, but instead find every sort of tyrannical government, from time immemorial.

   Conclusion: It is simply false to say (as Orlando Patterson does) that freedom is a natural part of the human condition.

5. Premise: If future scientists find a way to signal back in time, their signals would already have reached us.

Premise: No such signals have ever reached us.

Conclusion: Future scientists never will find a way to signal back in time.

6. Premise: Japanese and European whale-hunting countries have no need to eat whales; they can choose their diets.

   Premise: Eskimos live in an environment so harsh that their survival obliges them to eat whales; they have no choice in dietary matters.

   Conclusion: Permitting primitive Eskimos to kill some whales for survival, while at the same time demanding that modern societies cease to hunt whales, is fair and reasonable, not hypocritical.

7. Premise: The number of atoms in all of space is so huge that we can never count them or count the forces that drive them in all places.

   Conclusion: There must be other worlds, in other places, with different kinds of men and animals.

8. Premise: Where marriages are prearranged, divorce rates are often very low.

   Premise: Where marriages are formed on the basis of romantic love, divorce rates are very high.

   Premise: You can come to love a person you married without love.

   Premise: You can fall out of love with a person you married for love (or the marriage can fail).

   Conclusion (unstated): We ought not suppose that romantic love is a necessary precondition of successful marriage.

9. Premise (unstated): Our tax system depends upon the willingness of persons to pay the taxes they owe.

   Premise: That willingness depends, in turn, upon the widespread belief that almost everyone, including competitors and neighbors, are also paying the taxes they owe.

   Conclusion: If the Internal Revenue Service (the IRS) cannot assure us that this fairness is reasonable for us to suppose, the entire system of voluntary tax payments is seriously (and perhaps irremediably) threatened.

10. Premise: People and government are obsessed with racism and talk about it endlessly.

    Premise: But we don't listen and we don't see, and therefore we remain in a state of denial, thinking ourselves absolved of all complicity in racism.

    Conclusion: Invariably we conclude that it is the other guy who is in the wrong.

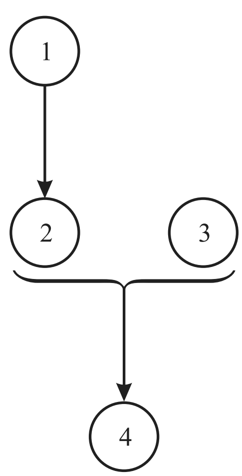
Section 2.2 – A
Exercises on pages 43–45

1. In a recent attack upon the evils of suburban sprawl, the authors argue as follows:

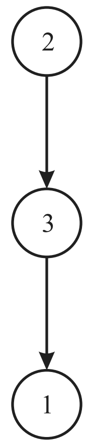
> The dominant characteristic of sprawl is that each component of a community—housing, shopping centers, office parks, and civic institutions—is segregated, physically separated from the others, causing the residents of suburbia to spend an inordinate amount of time and money moving from one place to the next. And since nearly everyone drives alone, even a sparsely populated area can generate the traffic of a much larger traditional town.

Solution

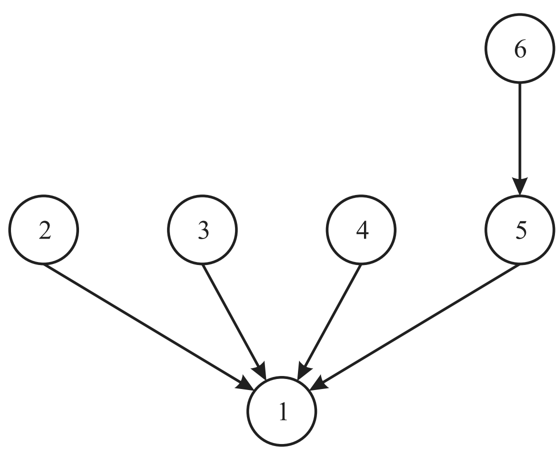
① The dominant characteristic of sprawl is that each component of a community—housing, shopping centers, office parks, and civic institutions—is segregated, physically separated from the others, causing ② the residents of suburbia to spend an ordinate amount of time and money moving from one place to the next. And since ③ nearly everyone drives alone, ④ even a sparsely populated area can generate the traffic of a much larger traditional town.



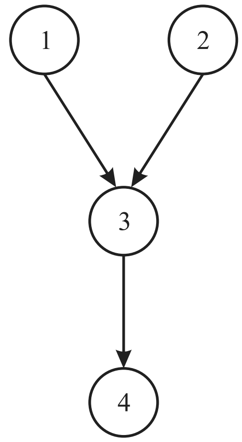
2. ① At any cost we must have filters on our Ypsilanti Township library computers. ② Pornography is a scourge on society at every level. ③ Our public library must not be used to channel this filth to the people of the area.



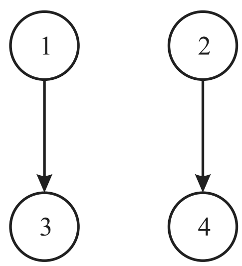
3. ① At his best, Lyndon Johnson was one of the greatest of all American presidents. ② He did more for racial justice than any president since Abraham Lincoln. ③ He built more social protections than anyone since Franklin Roosevelt. ④ He was probably the greatest legislative politician in American history. ⑤ He was also one of the most ambitious idealists. ⑥ Johnson sought power to use it to accomplish great things.



4. ① Married people are healthier and more economically stable than single people, and ② children of married people do better on a variety of indicators. ③ Marriage is thus a socially responsible act. ④ There ought to be some way of spreading the principle of support for marriage throughout the tax code.



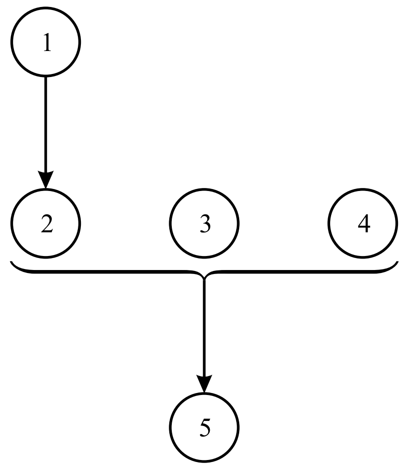
5. ① Vacuum cleaners to ensure clean houses are praiseworthy and essential in our standard of living. ② Street cleaners to ensure clean streets are an unfortunate expense. Partly as a result ③ our houses are generally clean and ④ our streets generally filthy.



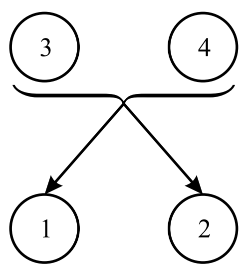
6. ① We are part of Europe. ② It affects us directly and deeply. Therefore ③ we should exercise leadership in order to change Europe in the direction we want.

7. ① California's "three strikes and you're out" law was enacted 10 years ago this month (March 2004). ② Between 1994 and 2002, California's prison population grew by 34,724, ③ while that of New York, a state without a "three strikes" law, grew by 315. ④ Yet during

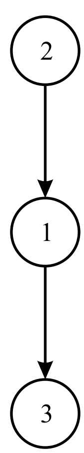
that time period New York's violent crime rate dropped 20 percent more than California's. ⑤ No better example exists of how the drop in crime cannot be attributed to draconian laws with catchy names.



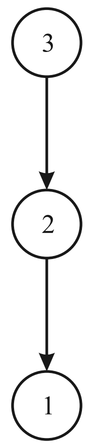
8. ① No one means all he says, and yet ② very few say all they mean, for ③ words are slippery and ④ thought is viscous.



9. ① The first impression becomes a self-fulfilling prophesy: ② we hear what we expect to hear. ③ The interview is hopelessly biased in favor of the nice.



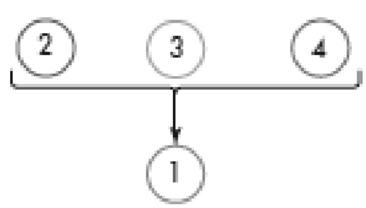
10. ① No government can ever guarantee that the small investor has an equal chance of winning. ② It is beyond dishonest to pretend that rules can be written to prevent future financial scandals. ③ No set of regulations can ensure fairness and transparency in the [securities] markets.



Section 2.2 – B
Exercises on pages 45–48

Solution

1. ① An outstanding advantage of nuclear over fossil fuel energy is how easy it is to deal with the waste it produces. ② Burning fossil fuels produces 27,000 million tons of carbon dioxide yearly, enough to make, if solidified, a mountain nearly one mile high with a base twelve miles in circumference. ③ The same quantity of energy produced from nuclear fission reactions would generate two million times less waste, and it would occupy a sixteen-meter cube. ④ All of the high-level waste produced in a year from a nuclear power station would occupy a space about a cubic meter in size and would fit safely in a concrete pit.



2. Premise: Economic inequality is correlated with political instability.

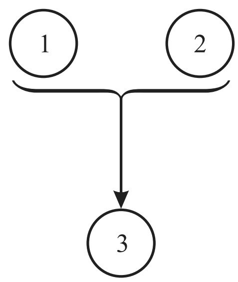
   Premise: Economic inequality is correlated with violent crime.

   Premise: Economic inequality is correlated with reduced life expectancy.

   Premise: Simple justice is offended when chief executives are paid hundreds of times more than is paid to ordinary employees.

   Conclusion: We should be gravely concerned about economic inequality—the wealth gap.

3. ① Genes and proteins are discovered, not invented. ② Inventions are patentable, discoveries are not. Thus, ③ protein patents are intrinsically flawed.



4. Premise: A growing number of Japanese don't want to eat whale meat.

   Conclusion: More and more Japanese consumers won't buy whale meat.

   Premise: If the Japanese won't buy whale meat, the Japanese whaling industry is in serious trouble and is probably doomed.

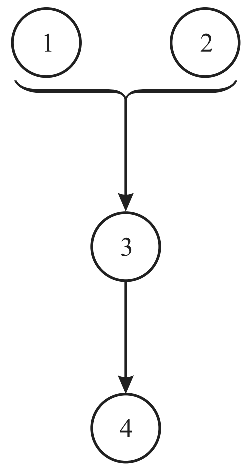
   Conclusion: The Japanese whaling industry is in serious trouble, and is probably doomed.

5. Premise: Without the memory of past horrors, there can be no justice for us. [*Sin memoria, no hay justicia*].

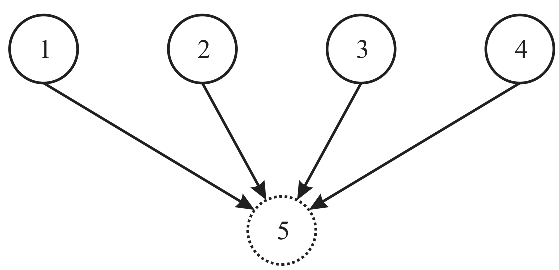
   Premise: Without justice , there will be no future for us. [*Sin justicia, no hay futuro*].

   Conclusion (unstated): If we do not remember the horrors of the past we will have no future. [*Sin memoria, no hay futuro*].

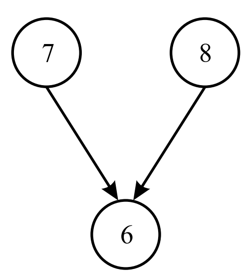
6. Since ① Grover Cleveland has a terrific public record, but a blemished private life, and since ② his opponent, James G. Blaine, has a storybook private life but a checkered public record, ③ it would be well to put both where they perform best. ④ Let's return Blaine to private life, and keep Cleveland in public life.



7. ① World War II solved problems called Nazi Germany and militaristic Japan, and created alliances with the nations we crushed. ② The Revolutionary War solved the problem of taxation without representation, and created the United States of America. ③ The Persian Gulf War solved the problem of the Iraqi invasion of Kuwait. ④ The Civil War solved the problem of slavery. ⑤ It is false to say that wars create problems but do not solve them.



⑥ These wars created a better world. ⑦ War is the only way to defeat evil enemies with whom there is no reasoning; it's either us or them. ⑧ What creates true peace is victory.



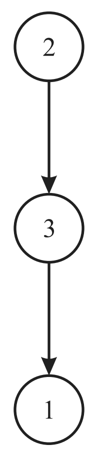
8. Premise: He who disobeys the laws is in effect disobeying his parents.

   Premise: He who disobeys the laws defies the authors of his education, to whom so very much is owed.

   Premise: He who disobeys the laws violates the agreement that he made, explicitly or tacitly, that he would obey the laws' commands.

   Conclusion: He who deliberately disobeys the laws is thrice wrong.

9. ① The reality is that money talks. ② Court officers, judges, and juries treat private lawyers and their clients differently from those who cannot pay for representation. ③ Just as better-dressed diners get prime tables at a restaurant, human nature dictates better results for those who appear to have money.



10. Premise: When Morton Grove, Illinois, passed a law *banning* guns, Kennesaw, Georgia, passed a law making gun ownership *mandatory*.

    Premise: Criminals would rather break into a house where they are not at risk of being shot.

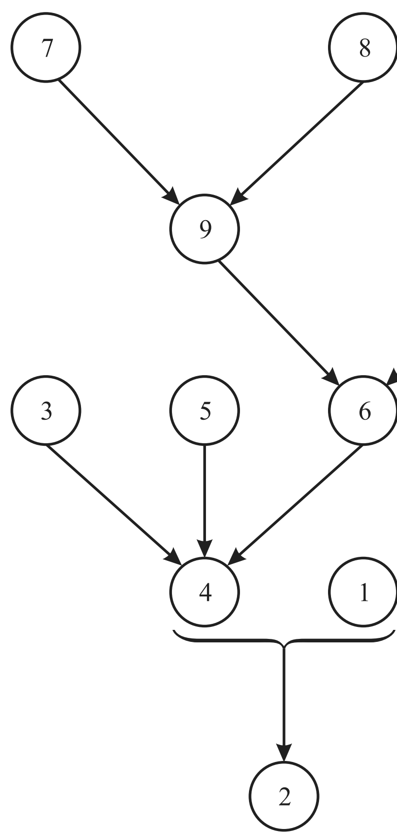
    Premise: Kennesaw's crime rate dropped sharply, but Morton Grove's did not.

    Conclusion:

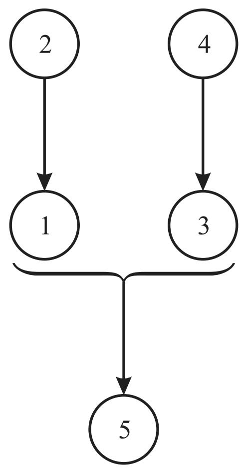
    A) Criminals will naturally believe that towns like Kennesaw, with such laws on their books, are very unsympathetic to them, and that if they plan to engage in crime they will be better off elsewhere. B) We are likely to see other communities adopting similar mandatory gun-ownership laws for self-protection.

Section 2.3
Exercises on pages 52–53

1. A question arises: whether it is better [for a prince] to be loved than feared or feared than loved? One should wish to be both, but, because ① it is difficult to unite them [being loved and being feared] in one person, ② it is much safer to be feared than loved, when, of the two, one must be dispensed with. Because ③ this is to be asserted in general of men, that they are ungrateful, fickle, false, cowards, covetous.... and ④ that prince who, relying entirely on their promises, has neglected other precautions, is ruined, because ⑤ friendships that are obtained by payments may indeed be earned but they are not secured, and in time of need cannot be relied upon. ⑥ Men have less scruple in offending one who is beloved than one who is feared, for ⑦ love is preserved by the link of obligation which, ⑧ owing to the baseness of men, ⑨ is broken at every opportunity for their advantage; ⑩ but fear preserves you by ⑪ a dread of punishment which never fails.



2. ① Democratic laws generally tend to promote the welfare of the greatest possible number; for ② they emanate from the majority of the citizens, who are subject to error, but who cannot have an interest opposed to their own advantage. ③ The laws of an aristocracy tend, on the contrary, to concentrate wealth and power in the hands of the minority; because ④ an aristocracy, by its very nature, constitutes a minority. It may therefore be asserted, as a general proposition, that ⑤ the purpose of a democracy in its legislation is more useful to humanity than that of an aristocracy.

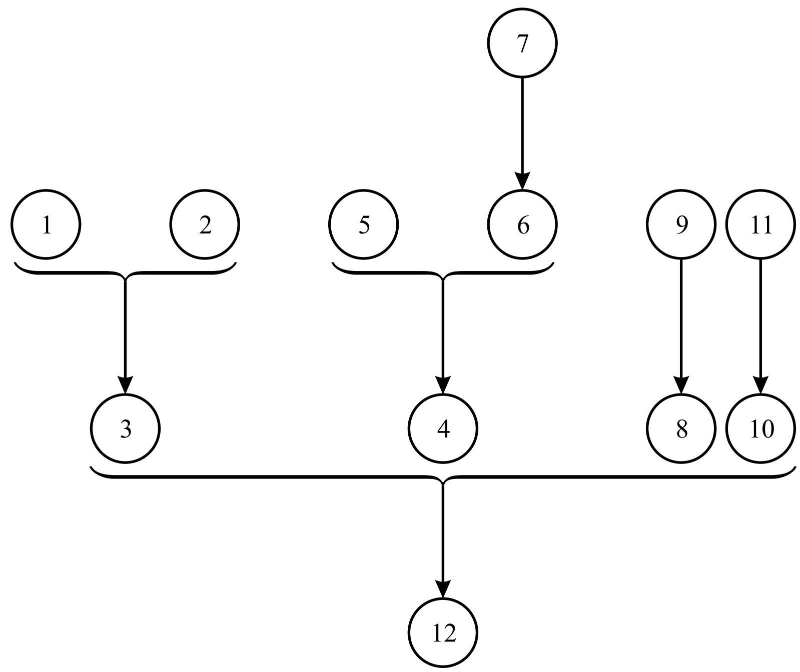


3. "…You appeared to be surprised when I told you, on our first meeting, that you had come from Afghanistan."

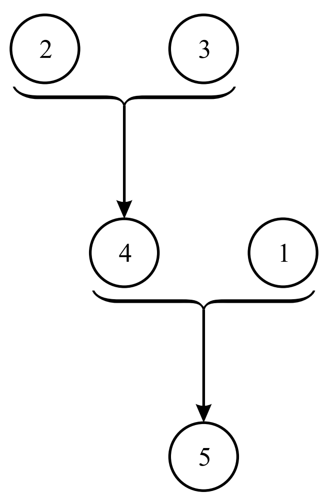
"You were told, no doubt."

"Nothing of the sort. I *knew* you came from Afghanistan. From long habit the train of thoughts ran so swiftly through my mind that I arrived at the conclusion without being conscious of intermediate steps. There were such steps, however. The train of reasoning ran, ① 'Here is a gentleman of medical type, but ② with the air of a military man. Clearly ③ an army doctor, then. ④ He has just come from the tropics, for ⑤ his face is dark, and ⑥ that is not the natural tint of his skin, for ⑦ his wrists are fair. ⑧ He has undergone hardship and sickness, as ⑨ his haggard face says clearly. ⑩ His left arm has been injured. ⑪ He holds it in a stiff and unnatural manner. ⑫ Where in the tropics could an English army doctor have seen much hardship and got his arm wounded? Clearly in Afghanistan.' The whole train of thought did not occupy a second. I then remarked that you came from Afghanistan, and you were astonished."

"It is simple enough as you explain it," I said, smiling.

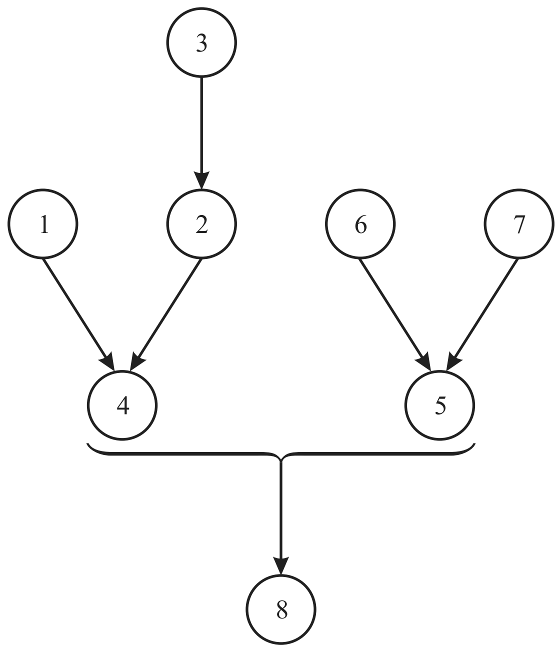


4. ① Nothing is demonstrable unless the contrary implies a contradiction. ② Nothing that is distinctly conceivable implies a contradiction. ③ Whatever we conceive as existent, we can also conceive as nonexistent. ④ There is no being whose nonexistence, therefore, implies a contradiction. Consequently ⑤ there is no being whose existence is demonstrable.



Challenge to the Reader: Prop 29, Book I, *Ethics Geometrically Demonstrated*, B. Spinoza

① Whatever is, is in God. ② But God cannot be called a contingent thing, for ③ He exists necessarily and not contingently. Moreover, ④ the modes of the divine nature [the creations which depend on, or have been created by, God immediately] have followed from it necessarily and not contingently.…But ⑤ God is the cause of these modes not only in so far as they simply exist, but also in so far as they are considered as determined to any action. ⑥ If they are not determined by God it is an impossibility and not a contingency that they should determine themselves; and, on the other hand, ⑦ if they are determined by God it is an impossibility and not a contingency that they should render themselves indeterminate. ⑧ Wherefore all things are determined from a necessity of the divine nature, not only to exist, but to exist and act in a certain manner, and there is nothing contingent.



Section 2.4
Exercises on pages 59–62

1. Only one.

   If the first native is a politician, then he lies and denies being a politician. If the first native is not a politician, then he tells the truth and denies being a politician. In either case, the first native denies being a politician. Since the second native reports that the first native denies being a politician, he tells the truth, and is, therefore, a nonpolitician. The third native asserts that the first native is a politician. If the first native is a politician, then the third native speaks the truth and is, therefore, a nonpolitician. If the first native is a non-politician, then the third native lies and is, therefore, a politician. Hence only one of the first and third natives is a politician, and since the second is a nonpolitician, there is only one politician among the three natives.

2. The blind prisoner must have had on a white hat, for if he had had on a red hat, one of the other prisoners would have known the color of the hat on his own head, and would have announced it. Call the prisoners A, B, and C. If A had seen a red hat on both B and C, he would have known and announced immediately that his hat must be white. But he did not announce and therefore did not know this; hence there must be a white hat on either B or C. B knew this, because he reasons well. Then, if B, having his opportunity to name the color of his hat, had seen a red hat on C, he would have concluded (from the fact that there is a white hat on either C or himself) that he himself must be wearing a white hat. He would have announced that, but he did not. Therefore B did not see a red hat on C. C realizes this because, although he is blind, he can reason well, and from the fact that B did not see a red hat on C, C concludes that his own hat must be white.

3. Mr. Jones is not the brakeman's next-door neighbor, for if he were then (by *e*) his earnings would be divisible by 3, but (by *c*) he earns exactly $40,000 a year, and that sum is not divisible by 3. Mr. Robinson is not the brakeman's next-door neighbor, for if he were then (by *b*) he would live halfway between Detroit and Chicago, but (by *a*) he lives in Detroit. Hence, Mr. Smith must be the brakeman's next-door neighbor.

   Neither Mr. Robinson (by *a*) nor Mr. Smith (by the preceding argument) lives in Chicago. Hence, Mr. Jones lives in Chicago and so (by *f*) Jones must be the brakeman.

   Smith (by *d*) is not the fireman, and Jones (by the preceding argument) is not the fireman either. Hence, Robinson is the fireman. Since (as has been shown) the brakeman is Jones, and the fireman is Robinson, the engineer's name must be Smith.

4. The manager has a grandson and is therefore neither Mr. Black, the bachelor, nor the twenty-two-year-old Mr. White, nor either Miss Ambrose or Miss Earnshaw who are unmarried, nor Mr. Kelly who is the manager's neighbor. Therefore the manager is Mrs. Coffee.

   The stenographer has a married child and is therefore neither Mr. Black nor Miss Ambrose nor Miss Earnshaw (who are unmarried), nor twenty-two-year-old Mr. White, nor Mrs. Coffee the manager. Therefore, Mr. Kelly is the stenographer.

   The cashier is a married man since he is a son-in-law, and is therefore neither Mr. Black, the bachelor, nor any of the females (Mrs. Coffee, Miss Ambrose, or Miss Earnshaw), nor is he Mr. Kelly, the stenographer. Therefore, Mr. White is the cashier.

   The assistant manager is a grandson, and therefore is none of the females, nor by the preceding arguments is he Mr. Kelly or Mr. White. Therefore, Mr. Black is the assistant manager.

   The teller is not her own stepsister and so is not Miss Ambrose. Nor is she any of the persons already identified: Mrs. Coffee, Mr. Kelly, Mr. White, or Mr. Black. Therefore, the teller is Miss Earnshaw. And so, by elimination, Miss Ambrose must be the clerk.

5. Since Lefty said that Spike did it, Spike's first and third statements are equivalent in meaning and therefore either both true or both false. Since only one statement is false, they are both true.

Dopey's third statement is, therefore, false, and so his first two are true. Therefore Butch's third statement is false and so his first two are true, of which the second reveals that Red is the guilty man.

(An alternative method of solving this problem is suggested by Peter M. Longley of the University of Alaska: All but Red both assert their innocence and accuse someone else. If their professions of innocence are false, so are their accusations of other persons. But no one makes two false statements, so their statements that they are innocent must be true. Hence Red is the guilty one. This solution, however, presupposes that only one of the men is guilty.)

(Still another method of solving this problem comes from James I. Campbell of Eisenhower College and Walter Charen of Rutgers College: If Dopey's second statement and Butch's third statement were false, Dopey's third statement would be true and Spike would be guilty. However, if Spike were guilty, his first and third statements would both be false, so he cannot be guilty and hence Dopey's second statement cannot be false. Therefore, Butch's third statement must be false, whence his second statement is true and Red is the guilty man.)

6. The following statements are known to be true:

(1) The best player's twin and the worst player are of opposite sex.

(2) The best player and the worst player are the same age.

The problem: Determine, by reasoning, who is the best player of the foursome.

We begin by focusing on the ages of the players. The best player and the worst player are the same age, by (2). The best player and the best player's twin must be the same age, from the meaning of the word "twin." The best player's twin and the worst player cannot be the same person, because, from (1), they are of opposite sex. Therefore, there are three players of the same age: the best player, the best player's twin, and the worst player.

The remaining player must be Mr. Short, since he must be older than his son and daughter.

So the three players of the same age must be Mr. Short's son, daughter, and sister. Therefore, the twins mentioned in (1) must be Mr. Short's son and daughter, and one of these two must be the best player.

But the best player cannot be Mr. Short's son, because if that were true, then none of the four could be the worst player! We can show this by assuming that the best player is Mr. Short's son. Then, by (1), his twin would be of the opposite sex from the worst player. Since his twin is Mr. Short's daughter, the worst player must be a male. Therefore, neither Mr. Short's sister nor Mr. Short's daughter can be the worst player. Nor can the worst player be Mr. Short's son, for we have assumed that he is the best player. This leaves Mr. Short himself. But Mr. Short cannot be the worst player, because by (2), the best player and the worst player are the same age, and a man cannot be the same age as his son.

Therefore, the best player must be Mr. Short's daughter.

7. Curly's four statements are the key to this problem.

   Otto accused the Kid. The Kid's first and fourth statements are equivalent and therefore either both true or both false. Since only one of his statements can be false, they must be both true. Therefore, Otto's third statement is false and the rest are true. The truth of his fourth statement entails that Mickey's third statement is false, and so Mickey's other statements are true. The truth of Mickey's fourth statement entails that Curly's fourth statement is true.

   The truth of Mickey's second statement entails that the Kid's second statement is true, and since the Kid's first and fourth statements have already been shown to be true, his third statement must be false, from which it follows that Curly's third statement is true.

   The truth of Otto's first statement entails that Slim's third statement is true. The statement of the problem shows that Slim's second and fourth statements are true also, so Slim's first statement must be false, from which it follows that Curly's second statement is true.

   Since Curly's fourth and third statements have already been shown to be true above, his first statement must be the false one. Hence we may know that Curly "dunnit."

8. First weighing: (R1 + G1) // (R2 + B1)

   There are three possible outcomes on this first weighing: (A) the two sides balance; (B) the left side goes down; and (C) the left side goes up. We will examine each outcome and show how all the balls can be identified in each case.

   (A) The two sides balance on the first weighing.

   We know that, of the pair R1 and R2, one ball is heavy and the other light. Since the two red balls are on opposite sides of the scale, we know that if the two sides balance there must be a heavy ball and a light ball on *each* side (because two heavies on one side would have to go down, and two lights on one side would have to go up). Therefore we know that either G1 is heavy and B1 is light, or G1 is light and B1 is heavy.

Having determined, on the first weighing, that G1 and B1 have different weights, we know that a second weighing of G1 // B1 cannot be balanced. Only two outcomes are possible:

(1) G1 goes down on the second weighing. In this case,

- G1 must be heavy (and therefore G2 must be light);
- B1 must be light (and therefore B2 must be heavy);
- R1 must be light (and therefore R2 must be heavy).

(2) G1 goes up on the second weighing. In this case,

- G1 must be light (and G2 must be heavy);
- B1 must be heavy (and B2 must be light);
- R1 must be heavy (and R2 must be light).

In each case, all the balls are identified. This exhausts all the possibilities for (A).

(B) The left side goes down on the first weighing.

We know that, in this case, R1 (the red ball on the side that goes down) must be heavy, because if R1 had been light, then R2 would have had to be heavy, and if R2 had been heavy, then (R1 + G1) could not have gone down. Knowing this, we can eliminate the possibility that G1 is light and B1 heavy, because in that case, (R1 + G1) could not have gone down. Therefore, one of the following three combinations must be the case:

(a) G1 is heavy and B1 is light.

(b) G1 is light and B1 is light.

(c) G1 is heavy and B1 is heavy.

We can now identify all the balls by choosing (R1 + R2) // (G1 + B1) for the second weighing. On this weighing, we know that the left side, (R1 + R2), has a heavy ball and a light ball, and therefore it may go down, go up, or balance the right side, (G1 + B1). We now show how all the balls can be identified in each case:

(1) The two sides balance on the second weighing.
In this case, G1 and B1 must be heavy and light, respectively. The combination must be pattern (a) above, and all the balls are identified (R1, G1, and B2 are heavy; R2, G2, and B1 are light).

(2) The left side goes down on the second weighing.
In this case, G1 and B1 must both be light, because a heavy and a light can outweigh only two lights. The combination must be pattern (b) above, and all the balls are identified (R1, G2, and B2 are heavy; R2, G1, and B1 are light).

(c) The left side goes up on the second weighing.
In this case, G1 and B1 must both be heavy, because a heavy and a light can be outweighed only by two heavies. The combination must be pattern (c) above, and all the balls are identified (R1, G1, and B1 are heavy; R2, G2, and B2 are light).

In each case, all the balls are identified. This exhausts all the possibilities for (B).

(C) The left side goes up on the first weighing.

In this case, the solution mirrors the steps described in (B), with the weights simply reversed.

9. Yes, the third native is a politician. This can be shown as follows:

   Call the natives A, B, and C. The first native, A, must be a politician because if he were a nonpolitician he would say that he is a nonpolitician, but instead he says that all three, including himself, are politicians. Therefore A is not telling the truth—it is not the case that all three are politicians. In other words, of the three natives, either two are politicians, or only one is a politician.

   However, it cannot be the case that only one of the three natives is a politician. If that were the case, then by the reasoning above the politician must be A. But in that case, B would have to be a nonpolitician and would tell the truth, which (under our assumption) is that there is only one politician. Instead, B says that there are two politicians, which would be false. Therefore, it is impossible that there is only one politician.

   Since there cannot be one politician only, there must be two politicians, of whom A must be one. B states that there are two politicians, which is true, and therefore B is a nonpolitician. And since we now know that there are two politicians among the three natives, the third native, C, must be a politician. (This is consistent with the fact that C accuses B of lying, and we know that B told the truth.)

10. It is not possible to distribute the strings so that no one triangle has all three sides (strings) of the same color; at least one triangle must have three sides of the same color.

    Consider any one nail; say the one on a wall we call A. From it stretch five strings, and among these five there must be a group of at least three strings of the same color, since only two colors (red and blue) are available.

Suppose that three of the strings from the nail in wall A are red, and that they go to the other three walls, B, C, and D. Now consider the triangle formed by the nails on these three other walls, B, C, and D. Its sides must not all be of the same color, so they cannot all be blue, so at least one of them must be red. But if any one of the strings connecting B, C, and D is red, it must complete a triangle of three red strings! (Suppose the string connecting B and D is the red one. Then there will be a triangle of three red strings connecting A, B, and D. The same problem arises if we try to connect B and C, or C and D.) No matter which nail we begin with, there is no way to avoid at least one triangle all of whose sides are strings of the same color.

Challenge to the Reader

NOTE: There are different solutions to this problem, and each solution has a mirror image. The second weighings may differ from those suggested below, but every correct solution must begin with four balls weighed against four. Every correct solution must also provide a proof that justifies the identification of the odd ball and shows why it is heavier or lighter than the others.

In the following discussion, we assume that the balls are uniquely numbered (1 through 12):

First weighing: (1 + 2 + 3 + 4) // (5 + 6 + 7 + 8)

There are three possible outcomes on this first weighing: (A) the two sides balance; (B) the left side goes down; and (C) the left side goes up. We will examine each outcome and show how all the balls can be identified in each case.

(A) The two sides balance on the first weighing.

In this case, we know that

- Balls 1–8 are all regular;
- The odd ball must be 9, 10, 11, or 12.

For a second weighing, we choose (9 + 10 + 1) // (11 + 2 + 3).
Three outcomes are possible:

(1) The two sides balance on the second weighing.
In this case, the odd ball must be 12. For the third weighing, we choose 12 // 1. The two balls cannot balance, because 1 is regular. Therefore, if 12 goes up, it is odd and light, and if 12 goes down, it is odd and heavy.

(2) The left side goes down on the second weighing.
In this case, the odd ball is either 9 or 10 (and heavy), or it is 11 (and light). For the third weighing, we choose 9 // 10. If one of those two balls goes down, it is odd and heavy. If the two balls balance, then the odd ball is 11, and it must be light.

(3) The left side goes up on the second weighing.
In this case, the odd ball is either 9 or 10 (and light), or it is 11 (and heavy). For the third weighing, we choose 9 // 10. If one of those balls goes up, it is odd and light. If they balance, the odd ball is 11, and it must be heavy.

In each case, the odd ball is identified. This exhausts all the possibilities for (A).

(B) The left side goes down on the first weighing.

In this case, either the odd ball is on the left side and it is heavy, or it is on the right side and it is light. Balls 9–12 are therefore known to be regular.

For the second weighing, we choose (1 + 2 + 5 + 9) // (3 + 4 + 6 + 10).
Three outcomes are possible, and in each case we can identify the odd ball, as follows:

(1) The two sides balance on the second weighing.
In this case, balls 1–6 must be regular, and since balls 9–12 were shown to be regular on the first weighing, the odd ball must be either 7 or 8, and it must be light. For the third weighing, we choose 7 // 8. The ball that goes up must be the odd ball, and it must be light.

(2) The left side goes down on the second weighing.
In this case, either 1 or 2 is odd (and heavy), or 6 is odd (and light). For the third weighing we choose 1 // 2. If either ball goes down, it must be the odd ball, and it is heavy. If the two balls balance, the odd ball must be 6, and it is light.

(3) The left side goes up on the second weighing.
In this case, either 3 or 4 is odd (and heavy), or 5 is odd (and light). For the third weighing we choose 3 // 4. If either ball goes down, it must be the odd ball, and it is heavy. If the two balls balance, the odd ball must be 5, and it is light.

In each case, the odd ball is identified. This exhausts all the possibilities for (B).

(C) The left side goes up on the first weighing.

In this case, as in (B), the odd ball can be identified. The pattern of second and third weighings will mirror the pattern described above for (B), with the weights reversed.

Chapter 3

Important note:

In evaluating the purpose and merit of particular passages or definitions taken from actual discourse, interpretation and judgment are required. For many such examples different responses may be justifiably given. Context is critical, and therefore students should be given latitude in these exercises, all of which are unavoidably taken out of context.

Section 3.1 – A
Exercises on pages 66–67

1. Directive
2. Expressive
3. The principal function of this passage is probably expressive; but in its context it also serves an informative function.
4. Informative
5. The expressive function is primary in this great poem, the poet's voice making his passion manifest. But an informative function may be supposed here as well, insofar as the poet may be describing his own life.
6. Directive. Of course it has an expressive function as well.
7. Directive.
8. Performative.
9. Chiefly expressive—but there is an informative function being served as well.
10. Informative. The report is correct, because a small portion of Alaska lies across the international date line.

Section 3.1 – B
Exercises on pages 67–69

1. The primary purpose of this passage is informative: to instruct all who read it that the Constitution of the United States permits no system of preference by class or caste. The passage also clearly expresses Justice Harlan's approval of this guarantee of equality under the law, and directs others to respect it—although his directive was ineffective in

this famous case, in which the doctrine of "separate but equal" for the races was applied and approved. It would not be overruled until 1954.

2. Directive: Let us learn how to rehabilitate criminals; OR: Don't let questions about rehabilitation of criminals into the legal issue of punishment for crimes and the protection of society from criminals.

   Expressive: To evoke a desire to learn how to rehabilitate criminals; OR: To evoke a sterner attitude toward criminals, who must be punished and from whom society must be protected; OR: To express exasperation with those who are concerned with the rehabilitation of criminals rather than with punishment, deterrence, and the protection of society.

   Informative: (an argument): No one knows how to rehabilitate criminals; therefore, judges do not know how to rehabilitate criminals.

3. Directive: Let us honor and reward farmers and farming.

   Expressive: To express and evoke admiration for farmers and farming.

   Informative: (an argument): When tillage begins, other arts follow. Therefore farmers are the founders of human civilization.

4. Directive: Let us do something about the evils that threaten us.

   Expressive: To evoke concern about threatening evils.

   Informative: If good men combat evil, evil cannot triumph.

5. The primary function in this passage of the novel is expressive, evoking the reader's antipathy towards lawyers. Because it is a utopian novel, many of its passages have a directive function also; here the direction is: Rid yourselves of lawyers! The passage may be said to inform the reader as well, explaining that lawyers, by profession, conceal and distort the facts.

6. Informative: Racial ghettos have been created and sustained by white society.

   Expressive: To express pained revulsion at historical racism and the attitudes that have condoned it.

   Directive: To urge whites to recognize their obligation to undo the racial injustices of the past.

7. Directive: Don't pay equal wages to all workers.

Expressive: To evoke a favorable attitude toward wage differentials.

Informative: It is the bad workmen who believe in the elimination of wage differentials.

8. Directive: Oppose war.

Expressive: To evoke a feeling of abhorrence for war.

Informative: War destroys religion, states, and families. It is the greatest plague that can afflict humanity, and any scourge is preferable to it.

9. Directive: Support and extend education.

Expressive: To evoke approval of education.

Informative: If education is not supported we shall suffer catastrophe.

10. The primary function of this passage is directive; Amiel wanted his readers not to delay decisions until perfect clearness is achieved. The passage may be said to inform also, teaching that perfect clearness is not required for wise decision making. And there is some expressive function here also, the author showing disapproval of those who demand perfect understanding before deciding.

11. Directive: Arm yourselves.

Expressive: To evoke a sense of alarm at the condition of being unarmed.

Informative: People despise those who are unarmed.

12. Directive: Make (or at least prepare for) war.

Expressive: To evoke hostility toward peace, and approval toward war.

Informative: Eternal peace is impossible. War develops the noblest virtues of man—courage and abnegation, dutifulness and self-sacrifice—and protects the world from materialism.

13. Directive: Watch your language!

Expressive: To evoke feelings of respect for language.

Informative: Language is essential to mental life. It embodies thought and is necessary for its development.

14. Directive: Strive for greater diversity in the scientific establishment.

Expressive: To express disdain for the institutional racism of the National Academy of Sciences.

Informative: To advise readers that the number of blacks elected to the National Academy of Sciences has been exceedingly small.

15. The primary function of this passage is probably informative. Bacon teaches that philosophy, studied in depth, brings one back to religion. There is a directive function also: the author thinks that his readers should be religious and that if they study philosophy, they should study it deeply, not superficially. To the extent that some contempt for atheism is implied (because atheism is presented as shallow), the passage also has an expressive function.

16. Directive: Work to eliminate patriotism.

    Expressive: To evoke antipathy toward patriotism.

    Informative: Patriotism is the basic cause of conflict.

17. Directive: Understand and acknowledge that there is a real difference between virtue and vice.

    Expressive: To evoke antipathy toward—and distrust of—the "ethical relativist."

    Informative: The "ethical relativist" is probably not to be trusted.

18. Directive: Practice eugenics.

    Expressive: To evoke disapproval of the casual selection of marriage partners.

    Informative: People try to improve the breed in mating domestic animals, but take no such care in human mating.

19. Directive: Don't believe the Bible.

    Expressive: To evoke feelings of amusement at the Bible, rather than belief in it.

    Informative: It would have been more nearly miraculous for Jonah to have swallowed the whale than vice versa.

20. The primary function of this passage is directive. A judgment is expressed in this passage concerning the function of the "notion of race," and the author's attitude toward these uses is expressed—but the plain purpose of the author is to cause his readers to attend less to race and more to the challenges of normal human interaction.

Section 3.1 – C
Exercises on pages 69–71

1. Asserts that the speaker will not accept the nomination and would not serve even if elected president. Intended to stop Republican politicians from working for his (Sherman's) nomination. Provides evidence that the speaker is not available as a candidate and is very forthright.

2. Asserts that the government's classification of ice as a "food product" implies that Antarctica is one of the world's foremost food producers.

   Intended to cause opposition to government bureaucrats' rulings and classifications.

   Provides evidence that the speaker (writer) has a sharp wit, and that he is opposed to (some) governmental intrusion into business.

3. Asserts that struggle (war) makes men strong, and that eternal peace would cause mankind to perish.

   Intended to cause people to support the nation's wars.

   Provides evidence that the speaker is warlike (and uninformed?).

4. Asserts two propositions, the first as premise and the second as conclusion:

   (1) Earth without music is like an incomplete and unoccupied house.

   (2) Therefore the earliest history of a nation begins with its music.

   Intended to cause people to take a greater interest in music and to have more respect for it.

   Provides evidence that the writer regards music as an important part of life and history.

5. Asserts that research requires continual reexamination of accepted beliefs; asserts further (as a conclusion) that research is critical of established practices.

   Intended to support and stimulate research, to stimulate a questioning attitude and a critical spirit, and to warn those who wish to enjoy the fruits of research that they must tolerate criticism of accepted doctrines and existing practices.

   Provides evidence that the speaker is committed to the continual reexamination of doctrines and axioms on which current thought and action are based, and is critical of existing practices.

6. Asserts that the speaker has refrained from emotional responses to men's actions, and has instead tried to understand them.

   Intended to promote acceptance of the writer's opinions as "objective" because they are unemotional.

   Provides evidence that the writer is persuasive and is more interested in explaining human actions than in mourning, condemning, or being entertained by them.

7. Asserts that political liberty is completely useless for the poor, and is valuable only to ambitious theorists and politicians.

   Intended to diminish the esteem in which political liberty is held and to produce hostility to those who praise it.

   Provides evidence that the speaker is more interested in economic issues than in political libertarian issues.

8. Asserts that the speaker identifies with the lower, criminal, and imprisoned segments of society.

   Intended to cause reappraisal of the worth of the lower classes.

   Provides evidence of the humanitarianism and the radical sentiments of the speaker.

9. Asserts that democracy is not a suitable form of government for men.

   Intended to diminish people's faith in the workability of democratic institutions.

   Provides evidence of the speaker's opposition to democratic institutions.

10. Asserts that there are the classes of citizens named, having the characteristics noted.

    Intended to cause hostility toward both the rich and poor, and to produce approval of the middle class.

    Provides some evidence that the speaker probably is not rich and almost certainly is not poor.

11. Asserts that turbulence and all other evil tempers of this evil age belong to the middle classes rather than to the lower classes.

    Intended to cause hostility toward the middle classes.

    Provides evidence that the speaker is hostile towards the middle classes, and is not (or at least does not regard himself as) a member of the middle class.

12. Asserts that war will always recur, as God's cure for ailing humanity.

    Intended to cause the acceptance of war (as holy and healing) and to diminish opposition to it.

    Provides evidence that the speaker is pro-war, religious, and persuaded that mankind is sick.

13. Asserts that the speaker would prefer to be qualified for, but not possessed of, the presidency rather than possessed of, but not qualified for, the presidency.

    Intended to recommend himself to the (Republican) party as candidate for president, and also to cast doubt on Abraham Lincoln's qualifications for the presidency.

    Provides evidence that the speaker is not president and also that the speaker is conceited.

14. Asserts that Disraeli achieved success by his own efforts, but that he is extremely conceited.

    Intended to cause laughter at, and scorn for, Disraeli.

    Provides evidence that the speaker scorns Disraeli and also that the speaker is witty.

15. Asserts that all who speak about constitutional rights, free speech, and the free press are Communists.

    Intended to cause hostility toward those who defend constitutional rights, free speech, and the free press, or who invoke such rights.

    Provides evidence that the speaker is of mixed mind about constitutional rights, free speech, and the free press, and is hostile toward Communists.

16. Asserts that wisdom is imputed to a silent man.

    Intended to cause people to stop chattering.

    Provides evidence that the speaker esteems silence (at least in others!) and is not always silent himself.

17. Asserts that well-chosen words are very valuable.

    Intended to cause people to choose their words well.

    Provides evidence that the speaker esteems eloquence and is himself eloquent.

18. Asserts that the speaker is hostile to tyranny.

    Intended to cause all others who oppose tyranny to give political support to the speaker.

    Provides evidence that the speaker is hostile to tyranny, eloquent, and religious.

19. Asserts that a free man does not think of death, and that a wise man thinks of life.

    Intended to cause people to stop worrying about death.

    Provides evidence that the speaker is more concerned with life than with death.

20. Asserts that the painting in question is overpriced and without merit.

    Intended to cause people to laugh, and especially to laugh at Whistler—and to refrain from buying or praising Whistler's paintings.

    Provides evidence that the speaker is hostile toward Whistler and his art and is witty and bombastic.

Section 3.2
Exercises on pages 73–75

1. Disagreement in belief regarding how a fool should be answered.

   Agreement in attitude (of contempt) toward fools.

2. Agreement in belief that our country is our country right or wrong, and belief that our country may be wrong on occasion. Possible disagreement in belief about what ought to be done on the latter occasions: Decatur says nothing about them, whereas Schurz says that when wrong our country ought to be put right.

   Agreement in patriotic attitude, with Decatur more vehement than Schurz.

   Disagreement in attitude, with Decatur more accepting of what the country does, whereas Schurz feels greater individual responsibility for the country's actions.

3. Disagreement in belief as to the relative values of peace and war: Tacitus says that some kinds of peace are worse than war; Erasmus says any kind of peace is better than any kind of war.

   Disagreement in attitude: Tacitus approves of some kinds of war and disapproves of some kinds of peace; Erasmus disapproves of all war and approves of every kind of peace.

4. Agreement in belief that some action is better than none.

   Disagreement in attitude: *a* seems to be more committed to timely action than *b* is.

5. Disagreement in belief as to how the physical separation of two persons affects their fondness or regard for one another.

Disagreement in attitude is suggested: *a* generally approves of separation, while *b* appears to be negative (or perhaps neutral) about it.

6. Disagreement in belief: *a* holds that the better qualified don't prevail over the less qualified, whereas *b* holds that they probably do (or will).

   Disagreement in attitude: *b* approves more of effort, preparation, and self-reliance than *a* does.

7. Disagreement in beliefs: Aristotle believes that slavery is necessary and expedient, and also in accord with innate differences in ability among humans. Rousseau denies that there are such innate differences among humans as would "justify" slavery, believing instead that slavishness is the result of corrupting those enslaved by force. By implication it is clear that Rousseau would deny that slavery is either necessary or expedient.

   Disagreement in attitude: Aristotle approves of slavery, admires the master, and despises the slave. Rousseau abominates slavery, despises the master, and pities the slave.

8. Disagreement in belief: Mussolini believes that war stimulates and ennobles those who have the courage to face it. Sumner believes that war diminishes justice, happiness, and whatever is noble ("Godlike") in people.

   Disagreement in attitude: Mussolini admires war, Sumner despises it.

9. Disagreement in belief about the importance and the consequences of education: Garfield believes it is next in importance to freedom and justice, for which it is necessary; Moore believes it is unimportant—or that its elimination is important, because education destroys artistic feeling, drives clerks to drink, and makes no contribution to learning.

   Disagreement in attitude: Garfield esteems education; Moore despises it.

10. Disagreement in belief is only implied or strongly suggested here: La Mettrie clearly believes in the truth of atheism, but that Smith disbelieves the atheist doctrine is suggested by his statement that atheists are scoundrels.

    Disagreement in attitude is expressed: La Mettrie approves of atheism and atheists, whereas Smith disapproves of atheists and—by implication—of atheism.

11. Agreement in belief that there is room and need for improvement in the practice of agriculture.

    Disagreement in attitude toward the agricultural life: Washington approves and Russell disapproves.

12. Disagreement in belief: Jefferson believes that under certain circumstances (the existence of uncultivated land and unemployed poor) the laws of private property conflict with natural rights; and Pope Leo XIII believes that the laws of private property are in full accord with natural rights, without exception.

    Disagreement in attitude: Under certain circumstances Jefferson disapproves of private property; under all circumstances Pope Leo XIII approves of it.

13. Disagreement in belief: Grant believes that there is a right to revolt under certain circumstances, and by implication, a right to incite to revolt under those circumstances; whereas Pope Leo XIII believes that there is no right to incite to revolt, and by implication, no right to revolt under any circumstances.

    Disagreement in attitude: Grant approves of revolution under some circumstances; Pope Leo XIII disapproves of revolution under all circumstances.

14. Disagreement in belief: Coleridge believes that language embodies past human achievements and is the means to our future conquests; Hawthorne believes that is not true, because human language is no better than animal sounds.

    Disagreement in attitude: Coleridge esteems language; Hawthorne does not.

15. Disagreement in belief as to the value of the American government: Thoreau believes it is disgraceful; Jefferson believes that though it is imperfect, it is better than any other up to that time.

    Disagreement in attitude: Thoreau disapproves of the American government; Jefferson approves.

Section 3.3 – B
Exercises on pages 76–79

1. An apparently verbal dispute that is really genuine. There is a verbal dispute here over the ambiguous phrase "greatest hitter," which is used by Daye to mean the one who gets the largest number of *hits* and by Knight to mean the one who hits the largest number of *home runs*. Beyond that, they really do disagree. They surely disagree in attitude about Rose and Bonds, since Daye holds Rose in highest esteem as a hitter, and Knight holds Bonds in highest esteem as a hitter. They probably also disagree in belief, defending different *criteria* for determining who is the greatest hitter.

2. An apparently verbal dispute that is really genuine. The ambiguous word "relevant" is used by Daye in the sense of dealing with eternally recurring problems and values, such as love

and sacrifice, the conflict of generations, life and death; and by Knight in the sense of dealing with the pressing and immediate issues of our time, such as inflation, unemployment, the population explosion, and the energy crisis. Behind the verbal dispute there is probably a disagreement in attitude, with Day esteeming the plays of Sophocles more highly than Knight does.

3. An obviously genuine dispute. Day and Knight have quite different criteria for excellence in fathers, but there is no evidence that any words are used by them in different senses. They obviously disagree in attitude.

4. An obviously genuine dispute about whether earnings are up or down. Daye and Knight evidently have different data upon which their statements are based. There may be a disagreement in attitude toward the company, but that is not clear.

5. A merely verbal dispute. The ambiguous phrase "business…good" is used by Daye in the sense of increased *sales*, and by Knight in the sense of increased *profit*. There *may* be disagreement in attitude toward the company in question, Daye approving and Knight disapproving, but this is not clear from their words.

6. An apparently verbal dispute that is really genuine. The ambiguous phrase "excellent student" is used by Daye in the sense of a student with a high level of interest and class participation and by Knight in the sense of a student who is punctual in turning in assignments. They disagree in attitude toward Ann, Daye approving and Knight disapproving.

7. Merely verbal. The ambiguous term "free will" is used by Daye when referring to actions not constrained by external pressure and accompanied by deliberation, but Knight uses it when referring to actions that are completely uncaused.

8. An apparently verbal dispute that is really genuine. The ambiguous phrase "productive scholar" is used by Daye in the sense of one who publishes extensively, and by Knight in the sense of one who produces new ideas or discoveries. They really disagree in attitude, Daye approving and Knight disapproving of Professor Graybeard.

9. Merely verbal. The ambiguous word "new" is used by Daye in the sense of different, and by Knight in the sense of not previously used. There does not seem to be any particular disagreement in attitude.

10. An obviously genuine dispute. Daye affirms and Knight denies that Dick bought himself a new car.

11. Merely verbal. The ambiguous phrase "long way" is used by Daye in the sense of taking nearly two hours to walk and by Knight in the sense of taking more than ten minutes to drive.

12. An apparently verbal dispute that is really genuine. The ambiguous phrase "liberal" is used by Daye in the sense of favoring progress or reform, and by Knight in the sense of giving freely or in ample measure. They really disagree in attitude toward Gray, Daye approving and Knight disapproving.

13. Obviously genuine. Can be regarded either as a disagreement in belief, with Daye affirming and Knight denying the proposition that the amount of emphasis given to athletics at the University of Winnemac is excessive, or as a disagreement in attitude, Day disapproving and Knight approving of the amount of emphasis placed on athletics at the University of Winnemac.

14. An apparently verbal dispute that is really genuine. The ambiguous phrase "bad taste" is used by Daye in the sense of indecorous, improper, or unseemly; and by Knight in the sense of being without flavor or unpleasant to eat. They really disagree in attitude toward the menu in question, Daye disapproving and Knight approving.

15. This is a tricky example, for which alternative analyses are plausible. One treatment is to regard it as an obviously genuine dispute, with Daye denying and Knight affirming the proposition that Knight should ask his wife. Another treatment is to regard the dispute as apparently verbal but really genuine. In this analysis, the phrase "your own judgment" (about it) is ambiguous, used by Daye in the sense of deciding about it without considering anyone else's opinion, and used by Knight in the (broader) sense of deciding everything about it by oneself, including the question of whether to consult the opinion of others. In this second analysis, there remains an underlying disagreement of belief as to whether Knight should consult his wife.

Section 3.4 – B
Exercises on page 86

Discussion:

What needs to be made more precise is the meaning of the phrase "carries a firearm." The better precising definition is the one that more nearly catches the sense intended by Congress when it increased the penalty imposed upon a person who, as he commits the drug-related offense, "uses or carries a firearm." The gravity of the offense is affected by the presence of a firearm (one might argue) only when there is some likelihood that the weapon may be used during the commission of the crime. On this view, the precising definition of Justice Ginsburg (joined by Justices Scalia and Souter) expressing a narrower sense of "carries" when the word appears in the phrase "carries a firearm" is the better. On this definition the gun locked in a trunk may be more accurately described as "transported" rather than "carried" in the sense Congress intended.

The Supreme Court held, in the case, that the phrase "carries a firearm" applies to a person who knowingly possesses and conveys firearms in a vehicle, *including* in the locked glove compartment or trunk of his car. They reasoned that the primary meaning of "carry" includes conveying in a vehicle, and there is no linguistic reason to think that Congress, in adopting this law, intended to limit the word to its secondary meaning. Moreover, the Court held, the statute's basic purpose—to combat the "dangerous combination of drugs and guns"—and its legislative history do not support limiting the scope of the word "carry" to carrying "on the person." Justice Breyer's view prevailed. [*Muscarello v. United States*, 524 U.S. 125 (1998)]

Three additional arguments were presented by the other side: (1) that this result would obliterate the distinction between "carry" and "transport"—a distinction used in other provisions of the "firearms" section of the United States Code; (2) that the wide reading of the statute would extend it to cover passengers on trains or buses who had placed a firearm in checked luggage; and (3) that the ambiguity of the statute should require the more lenient result, under what is called the "rule of lenity." These arguments were rejected by the majority of the Court as "unconvincing."

Section 3.5 – A
Exercises on page 89

1. Animal, vertebrate, mammal, feline, wildcat, lynx.
2. Beverage, alcoholic beverage, wine, white wine, fine white wine, champagne.
3. Athlete, ball player, baseball player, fielder, infielder, shortstop.
4. Dairy product, milk derivative, cheese, soft cheese, strong soft cheese, Limburger.
5. Number, real number, rational number, integer, positive integer, prime.

Section 3.5 – B
Exercises on page 89

1. Aquatic animal, fish, game fish, pike, muskellunge.
2. Domestic animal, beast of burden, horse, foal, filly.
3. Liquid, beverage, liquor, brandy, cognac.
4. Instrument, musical instrument, string instrument, violin, Stradivarius.
5. Polygon, quadrilateral, parallelogram, rectangle, square.

Section 3.5 – C
Exercises on page 91

One possible set of responses, as examples only:

1. John Gielgud, John Cleese, Lawrence Olivier
2. Joe Louis, Mohammed Ali, Mike Tyson
3. Bach, Beethoven, Brahms
4. Shakespeare, Marlowe, Ben Johnson
5. Fluorine, chlorine, iodine
6. Tulip, gladiolus, iris
7. Washington, Grant, Eisenhower
8. New York, San Francisco, Miami
9. Eli Whitney, Thomas Edison, Robert Fulton
10. Browning, Keats, Shelley

Section 3.5 – D
Problem heading on page 91; exercises on page 89

1. Britons
2. Heavyweight champions
3. Germans
4. Elizabethans
5. Halogens
6. Bulbs
7. United States presidents
8. American cities
9. Americans
10. Romantics

Section 3.5 – E
Exercises on page 94

1. Ridiculous
2. Clown
3. Graveyard
4. Autocrat
5. Vanity
6. Banquet
7. Attic
8. Hurry
9. Baby
10. Danger
11. Cows
12. Maze
13. Beggar
14. Tyro
15. Portent
16. Cure-all
17. Charlatan
18. Platform
19. Villain
20. Wigwam

Section 3.6 – A
Exercises on page 98

1. Very large meal
2. Young man
3. Male sibling
4. Young offspring
5. Young horse
6. Female offspring
7. Female sheep
8. Male parent
9. Very large person
10. Young woman
11. Young sheep
12. Female horse
13. Very small person
14. Female parent
15. Very small horse
16. Male sheep
17. Female sibling
18. Very small meal
19. Male offspring
20. Male horse

Section 3.6 – B
Exercises on pages 99–100

1. Both too broad and too narrow. Many persons with an innate capacity to affect the lives of others for good or evil are not geniuses; and there are some geniuses who do not affect the lives of others for good or evil. This definition violates Rule 3.

2. Too broad, because a casual opinion may just happen to be true. Rule 3.

3. Figurative language. Rule 4.

4. Circular. Rule 2.

5. Obscure; violates Rule 4. Also it fails to state the essence of alteration, which is "change over time," and thus it violates Rule 1.

6. Negative where it could be affirmative. Rule 5. Also too broad, since inanimate objects lack all intents. Rule 3.

7. Figurative language. Rule 4.

8. Obscure language: How can that which "dependeth not on the imagination" fill an "imagined place"? Rule 4.

9. Too narrow, because the pains of torture may be inflicted for other, very different, purposes as well. Rule 3.

10. Circular, since "produces" is synonymous with "causes." Violates Rule 2.

11. Too broad, because there are individual (private) acts of violence which would not be called "wars." Rule 3.

12. Too narrow, because there are rubber and treated-cloth raincoats. Rule 3.

13. Circular. Rule 2.

14. Too broad, because it also describes a snort, which is not a sneeze; and too narrow, because some sneezes are inaudible or come out through the mouth. Rule 3.

15. Figurative language; violates Rule 4.

16. Too narrow, because there are works of art that transmit feelings other than the highest and best, and abstract works that would seem not intended to transmit any feeling, and private or secret works which are not intended for transmission to others at all. Rule 3. It probably does not state the essence—if we accept theories that regard the essence of art as pleasure (objectified), beauty, unity-in-complexity, etc. Rule 1.

17. Not really a definition by genus and difference. "When" cannot play the role of a genus.

18. Too narrow, because some clouds are opaque, and some have textures that are not fleecy. Rule 3.

19. Circular. Rule 2.

20. This is a tricky example. The definition may be faulted for being both too narrow and too broad. It is too narrow in that it attends to well-being, but not to the normal physiological functions with which health is most commonly associated; it is too broad in that it introduces social circumstances not ordinarily viewed as within the ambit of health; thus it violates Rule 3 and Rule 1.

21. Circular. Rule 2.

22. This may be a satisfactory definition of "noise" as a technical term in acoustical engineering, but as a definition of the term "noise" in ordinary language, it is both too broad and too narrow. A conversation in which one is interested may be drowned out by a Mozart symphony, which is certainly not noise but would have to be considered noise, according to the definition, which is thus seen to be too broad. By the same token, during a demonstration of sound equipment, noise may be just what is wanted, but it would be excluded by the definition, which is thus seen to be too narrow. Rule 3 and Rule 1.

23. Figurative language. Rule 4.

24. Circular. Rule 2.

25. This definition is both too narrow and too broad. It is too narrow, in that "political correctness" may characterize a point of view that is absolutist as well as relativist. It is too broad, in that some dogmatic relativists may not be intolerant of believers in "traditional values." It violates Rule 3 and Rule 1.

Section 3.6 – C
Exercises on pages 100–103

1. Figurative language; Rule 4. It also fails to state the essence of faith, violating Rule 1.

2. Figurative language. Rule 4. Also too narrow, because one may have faith without knowing the truth or falsehood of that in which faith is put. Rule 3.

3. Figurative language. Rule 4. Also too narrow, because one may have faith in something probable. Rule 3.

4. Too narrow, because some poetry, due to its obscurity, is not "widely effective." Rule 3.

5. Too broad, since some prose records such moments; and too narrow, since some (great) poetry is tragic; violates Rule 3. It also may be criticized as being phrased in figurative language, violating Rule 4, although this is not altogether obvious.

6. Figurative language (with humorous intent). Rule 4.

7. Figurative language. Rule 4.

8. This is an excellent short definition, by genus and difference.

9. Too broad, because it may occasionally be very expedient to lie. Rule 3.

10. Too broad, since some persons with a very low opinion of themselves tend to behave this way; and too narrow, since some supremely conceited persons do not stoop to such vain-glory or social climbing; violates Rule 3. It also may be criticized for violating Rule 1 in not stating the essence, which is a trait of character rather than a tendency to overt behavior of the kinds specified.

11. Possibly circular. Rule 2. But possibly not, if the term "economic activities" has already been adequately defined.

12. Fails to state the essence of justice. Also, too narrow, because many acts of justice involve interference in other people's business; and too broad, because many instances of doing one's own business cannot be called acts of justice. Rules 1 and 3.

13. Hardly a definition at all, this is mainly an attempt at humor using figurative language. The definition may describe some university education, but it is too narrow. Rule 3.

14. Too narrow, because there are "useless" goods, and perhaps too broad in case there are useful evils. Rule 3.

15. Too narrow; not all political power is exercised "for the public good"—certainly not "only for the public good." Rule 3.

16. Figurative language. Rule 4.

17. Too narrow, because there may still be political power in a classless society—to keep the peace, deal with other societies, etc. Rule 3.

18. Too narrow, because a person may feel pity without imagining himself as a victim of the same fate. Rule 3.

19. Circular. Rule 2. Also too narrow, because justice is not a mere "kind of state of character"; Rule 3.

20. Too broad, violates Rule 3. In his *History of Western Philosophy*, Bertrand Russell criticized this definition on the grounds that "the dealings of a drill-sergeant with a crowd of recruits, or of a bricklayer with a heap of bricks…exactly fulfill Dewey's definition of 'inquiry.'" The definition also uses obscure language, in violation of Rule 4.

21. Figurative language. Rule 4.

22. Too broad. When thinking about a tragedy that did not materialize, one may feel pain at the thought of what might have happened, but this pain would not be called regret. Rule 3.

23. Obscure language. Rule 4.

24. Perhaps too narrow in its claim that the aim of tragedy is catharsis of pity and fear, which would violate Rule 3. But almost as many scholars would defend Aristotle's definition as would criticize it.

25. This definition fails to state the essential attributes of propaganda, and thus violates Rule 1. Propaganda essentially involves the promotion of ideas or doctrines to further one's own cause in opposition to some other cause—whether or not the conclusions encouraged are simplistic.

26. Although this is a very illuminating "definition," it is probably too narrow, because some insights of female intuition cannot only be examined syllogistically but can pass such examination. Rule 3.

27. This is a definition relying entirely upon the figurative use of language, and is therefore quite unsatisfactory, although it exhibits some genuine insight. Rule 4.

28. Obscure language. Rule 4.

29. Figurative language. Rule 4.

30. As it stands, this definition obviously is circular. It is followed in Wittgenstein's book, however, by "i.e.: if you want to understand the use of the word 'meaning,' look for what are called 'explanations of meaning.'" Thus emended, the definition is made consistent with Wittgenstein's tendency to identify meaning with use. Compare: "A spade is to dig."

Chapter 4

Important note:

It can hardly be emphasized too strongly that, in categorizing informal fallacies taken from actual discourse, interpretation and judgment are required. For many such examples different responses may be justifiably given. Context is critical, and therefore students should be given the widest latitude in assigning fallacy names to the passages in these exercises, all of which are unavoidably taken out of context.

Section 4.3 – A
Exercises on pages 121–124

1. Missing the point *(ignoratio elenchi)*: Mr. Chirac addresses the question of whether or not his remarks were on record—but the criticism he replies to was addressed to the substance of those remarks.
2. Argument *ad hominem* (abusive)
3. Appeal to emotion
4. Argument *ad hominem*
5. Appeal to emotion *(ad populum)*: The very sharp and emotionally colored language used in this attack upon a book is calculated to appeal to the feelings and attitudes of readers. Because some of this language is directed at the author himself, the passage also exhibits the demerits of an *ad hominem* argument.
6. Appeal to force (argument *ad baculum*)
7. Appeal to force (argument *ad baculum*)
8. Straw man
9. Straw man
10. Argument *ad hominem* (abusive). Socrates is being attacked by Thrasymachus as being immature, utterly naïve.
11. Argument *ad hominem*
12. Appeal to emotion
13. Missing the point

14. Argument *ad hominem* (circumstantial)

15. Appeal to emotion (*ad populum*): Showing the wanton cruelty of pouring out the water right in front of the thirsty, wounded soldier was a deliberate effort to build anger. The fat man, in the rear of the drawing, is a caricature of the Kaiser, drawn there to associate the cruelty of the nurse with the German government of the time.

Section 4.3 – B
Exercises on pages 124–126

1. Mr. Welch honestly believed that the attack on GE was based on a false premise, and his response may be taken as his emphatic way of insisting that it was false. But since his response is aimed at the speaker in her capacity as a nun, it is also a form of *ad hominem* argument, circumstantial.

2. The passage attacks gender feminism as a position that is not falsifiable because its advocates (according to the author of the passage) view those who reject its tenets as being "in thrall to the androcentric system." Thus, advocates of gender feminism are accused of committing an *ad hominem* attack—a "poisoning of the well" that holds opponents irrational simply because of their opposition. There may be some merit in this indictment of gender feminism, but the criticism itself may be used to poison the well by characterizing gender feminists as intrinsically intransigent.

3. This and other passages from *Common Sense* are plainly argument *ad populum*, and fallacious insofar as they rest on an appeal to the emotions of the readers. The passage is an argument *ad hominem* as well, in that Paine abuses those who seek reconciliation with the English Crown by calling them nasty names. There may, nevertheless, have been some truth in his characterization of his opponents, and we will agree that the cause of American patriotism is certainly a worthy one.

4. On the one hand, the argument is *ad hominem* circumstantial; Philo's view is being attacked as one flatly inconsistent with his everyday practices and needs. On the other hand it is not fallacious to call attention to the impracticality of some theoretical claim, or to the extent to which one is indeed unable to live in accord with the principles professed. And this is what the speaker (Cleanthes, in Hume's *Dialogues*) may be taken to be doing in this passage.

5. The attack is leveled against the NEA on the supposition that what is contained in the press release is no more than material designed to serve the interests of NEA members—an argument *ad hominem*, circumstantial. It is indeed wise practice to consider the interests of organizations that issue press releases, the better to interpret the

claims made; but it is unfair to suppose that the claims made are mistaken, or that the facts announced are false, just because they serve the purpose of the organizations issuing the press release.

6. This is plainly a threat, and is an appeal to emotion, to fear, if it is taken as a set of reasons for not altering the holy book. But it may also be viewed as no more than a prediction.

7. This is plainly a threat of the use of force, and to that extent it is an appeal to emotion. But it may also be viewed as a timely warning for the citizens of some states.

8. This is an appeal to emotion, in the form of a not very well-concealed threat. But it may be an effective argument in the dealings of strong national powers. Deterrence, a well-reputed view among many statesmen and diplomats, may be taken to be nothing more than a systematic appeal to the fears of the potential adversary; that is, a well-planned argument *ad baculum*.

9. Some will say that this is an appeal to inappropriate authority. Freud was a great thinker, whose understanding of the human psyche and its needs was penetrating and often wise. Whether his judgment regarding the plausibility of religious belief by enlightened moderns is truly authoritative is not at all clear. If not, then the appeal to Freud in this connection may indeed be fallacious.

10. Argument *ad hominem*, abusive.

Section 4.5
Exercises on pages 138–140

1. False cause. The fact that girls in the writer's classes did not become pregnant before marriage is not likely to have been causally related to the fact that condoms were not at that time distributed, or to the fact that contraception was not then discussed.

2. Complex question. Dubious claims are buried in the questions asked. When readers are asked if they "realize that *x* is the case," the truth of *x* is simply assumed.

3. False cause. What is noted is a *correlation*, not a causal relationship. A *correlation* of this sort certainly would not show that years of college education have a causal impact on the frequency of sexual activity.

4. Asserting the efficacy of what is highly unlikely to be efficacious is a variety of defective induction. However, one cannot totally discount the placebo effect.

5. The argument is circular, of course—a variety of begging the question. The thief keeps three pearls because he is the leader, and he is the leader because he keeps three pearls. That he is the leader is assumed by him when the division of the loot begins.

6. False cause, Mark Twain style.

7. False cause, although the argument was not likely to have been presented seriously.

8. An argument *ad ignorantiam*. Not knowing one's origins, on the view expressed in this passage, entitles one to dual citizenship everywhere, or everywhere in Africa.

9. On one interpretation this is an instance of hasty generalization; on another interpretation it is a variety of false cause.

10. False cause is the obvious fallacy here; neither the fact that dice are made from bone, nor the fact that bones heal, had any causal connection with Mr. Suzuki's gambling success.

Section 4.6 – A

Exercises on pages 148–149

1. Composition. It cannot be inferred from the fact that the parts have a specified shape that the whole has that same shape.

2. Amphiboly. "Lick with cornstalks" as first used is a figure of speech suggesting that even with the weakest of weapons the Confederates would thrash the Yankee troops. But after the Confederates were themselves thrashed, the expression is taken to mean "win in a battle in which the weapons of both sides are limited to cornstalks"—an absurd, and for that reason rather amusing, ambiguity.

3. Composition. Even if the imposition of an ordered wage structure on individual industries has a desirable outcome, it certainly does not follow that an analogous imposition on the economy as a whole will also have a desirable outcome.

4. Equivocation, of course. The passage is cute because of the play on the word "take." It is interesting to note that this word has so very many meanings and shades of meaning that the entry for "take" is one of the longest in the great *Oxford English Dictionary*.

5. This is only a joke, of course. The argument of the joke is that, since you need no instruction on how to play the concertina without success, you need no instruction on how to play the concertina at all. If one were thus to interpret the phrase "without success" as though it modified the phrase "how to play the concertina," when in fact it was intended

to modify "looked everywhere," this silly argument would commit the fallacy of amphiboly. Our recognition of the inadvertent amphiboly gives some amusement.

6. Composition. What constitutes a good for each member of an aggregate need not serve as a good in the same way for the aggregate as a whole.

7. Equivocation on the word "cries," which can mean "shouts" when selling turnips, but "weeps" when a parent dies.

8. Equivocation. "Bad" when applied to journalists refers to their lack of needed skills; but when applied to persons the adjective normally refers to their moral character. A woman who is morally bad may be a superb journalist, of course.

9. Amphiboly is the ground of the humor in this passage. The participial phrase "walking along the branch of a tree, singing, and in good view" was intended by the author of the passage to apply to the bird, but as written it seems to apply to Hazel Miller. The editor is, of course, making fun of the author of the amphibolous passage.

10. Composition—with sarcastic intent, of course.

Section 4.6 – B
Exercises on page 150

1. It may be argued that although the parts have functions, this does not permit the inference that the whole has functions. In this view, Aristotle here commits the fallacy of composition. On the other hand, many will argue that we may reasonably infer from the patterns found in some natural objects that similar patterns may be expected in other natural objects, in which case the passage would commit no fallacy.

2. Composition. What is true of all phenomena taken separately is not necessarily true of the universe itself, in which all phenomena arise.

3. This is a very famous instance of what is taken to be a huge mistake by a great philosopher, apparently misled by amphiboly. The grammatical constructions (adjectives ending in "-able" or "-ible") may sound alike, but they have very different force. When we say that some phenomenon is visible (or audible) we mean that it is possible for the phenomenon to be seen (or heard). But when we say that an outcome is desirable we do not simply mean that it is possible for that outcome to be desired. We mean that the outcome is worthy, or good—that it *should* be desired. "Desirable" refers to the thing itself, and not to the relations others may bear to it.

4. If Walt Whitman thought that, he most assuredly committed, in his thoughts, the fallacy of division!

5. It may be argued that the passage commits an *ad hominem* (circumstantial) fallacy in supposing that the competence of the school chancellor is suspect in view of the school placement of his own children. On the other hand, many will argue that in placing his own children in private schools, the chancellor does unavoidably undermine public confidence in his support of the public schools, and that this conclusion is not fallacious.

Section 4.6 – C
Exercises on pages 150–154

1. Equivocation, or alleged equivocation, is the nub of this dispute. If Justice Scalia is correct, the statute that increases the severity of punishment for "using" a firearm was not meant to impose that additional sanction on one who *traded* his firearm in the commission of the crime. Justice O'Connor, on the other hand, treats the term "using" in the statute very broadly, so that any role the firearm may have played would satisfy the condition of being "used." Justice Scalia insists that her argument commits an equivocation because it treats "use" as meaning "use in any way whatever," while statutes ought to be read so that their words carry ordinary meanings. There is no obvious resolution of the logical issue; the legal issue was resolved by a vote of the Court.

2. Equivocation. In the original passage, *genius* meant "extraordinary talent or capacity." In the decontextualized quote, the meaning of the word is likely to be taken to be "a person having such talent." In the first case, one has genius. In the second, one is a genius.

3. This is a delightful stew of fallacies, the primary ingredients being *petitio principii*, false cause, and sophistical *ignoratio elenchi*. But Miss Alabama was no doubt very charming.

4. This is, of course, a *petitio principii*. One may view such fallacious circles as being simply patterns of speech in which an important point is repeated for the sake of emphasis.

5. An argument *ad populum* is plainly involved here, insofar as it is believed that a conclusion may be held acceptable because it was so widely approved. But it is probable that the author (Croce) is doing no more than calling attention to widespread irrationality at the time of the Inquisition.

6. An *argumentum ad populum* of the baldest kind. As in much advertising, one is here urged to do something simply because "everybody" else is doing it.

7. An instance of the fallacy of division. In general, one cannot conclude that something must be true of the parts of a whole merely because that something is known to be true of the whole. (When additional information is available, however, the fallacy may only be

apparent. For example, if we take into account ordinary geometry, it becomes reasonable to state that a thing of limited size cannot have any parts of unlimited size.)

8. *Ad hominem* (abusive).

9. *Ad hominem* (abusive). As these two passages (8 and 9) indicate, the years of the Cold War offered many splendid examples of mutual abuse by the Soviet Union and the United States.

10. This passage plays with false cause, mixing with that fallacy an appeal to inappropriate authority. But the author, in jesting, is also ridiculing such an argument.

11. The fallacy in this passage is mainly one complex question, since the author strongly suggests, with the question asked, that there are risks for which there is apparently no evidence. One may also interpret the passage as another variety of argument *ad ignorantiam*.

12. The author of this passage attacks the defenders of the "multiverse theory" as committing the fallacy of the slippery slope, because of its suggestion of an infinite and ever extending regress into unbounded space; that accusation may be unfair. But the passage itself, although it may be mistaken in criticism, commits no fallacy.

13. The argument of Clavius (a great astronomer in his day, and one after whom one of the craters on the moon has been named) is plainly an appeal to inappropriate authority. "The philosophers" (referring to the long-venerated followers of Aristotle) were, of course, wildly mistaken about the simple motion of the Earth.

14. *Petitio principii*. Blunders of this sort are not rare.

15. This argument may be construed to contain no fallacy; or to contain a blatant argument *ad baculum*, a resort to the threat of force. If construed to mean that congregants ought to behave in certain ways lest they be severely punished by an angry God, the argument contains no fallacy—although its factual supposition may be questioned, of course. If construed to mean also that, because those punishments are so fearfully threatening, some propositions (having nothing directly to do with God's anger) are *true*, and should be believed, the argument is fallacious, since the threats would not be relevant to the truth or falsity of those propositions. Probably the argument was intended in both ways.

16. One can view this passage as an instance of false cause, or perhaps more accurately as a sophistical *ignoratio elenchi*. Religion is indeed terribly important. That religion is often touched by mysticism hardly justifies the conclusion that mysticism is one of the great forces of the world's history.

17. This is an appeal to ignorance. We cannot justify any important conclusions about an animal from the fact that we cannot prove that nothing is going on in its head! It is also a sophistical *ignoratio elenchi*, in that treating animals with the respect we accord ourselves (our obligation, according to the author of the passage) has no relation to our ignorance of their inner psychological states.

18. Fallacy of composition. Whether a state is brave cannot be inferred from the conduct of some (or even all) of its soldiers.

19. If this is taken to be an argument—that the question of our immortality is the most intelligible of all questions because it is the most important of all questions—it is plainly fallacious, a sophistical *ignoratio elenchi*, a great *non sequitur*. But the passage may not have been intended as an argument so much as an assertion that the question of immortality is both exceedingly important and perfectly intelligible.

20. A fallacy of false cause lies behind the humor in this passage. The answer to the query supposes, mistakenly, that the light in the daytime is caused by something other than the sun!

Chapter 5

Section 5.3
Exercises on page 170

1. **I:** *S* = historians;

   P = extremely gifted writers whose works read like first-rate novels.

   Particular affirmative.

2. **E:** *S* = athletes who have ever accepted pay for participating in sports;

   P = amateurs.

   Universal negative.

3. **E:** *S* = dogs that are without pedigrees;

   P = candidates for blue ribbons in official dog shows sponsored by the American Kennel Club.

   Universal negative.

4. **A:** *S* = satellites that are currently in orbits less than 10,000 miles high;

   P = very delicate devices that cost many thousands of dollars to manufacture.

   Universal affirmative.

5. **O:** *S* = members of families that are rich and famous;

   P = persons of either wealth or distinction.

   Particular negative.

6. **O:** *S* = paintings produced by artists who are universally recognized as masters;

   P = works of genuine merit that either are or deserve to be preserved in museums and made available to the public.

   Particular negative.

7. **A:** *S* = drivers of automobiles that are not safe;

   P = desperadoes who threaten the lives of their fellows.

   Universal affirmative.

8. **I:** *S* = politicians who could not be elected to the most minor positions;

   P = appointed officials in our government today.

   Particular affirmative.

9. **O:** *S* = drugs that are very effective when properly administered;

   P = safe remedies that all medicine cabinets should contain.

   Particular negative.

10. **E:** *S* = people who have not themselves done creative work in the arts;

    P = responsible critics on whose judgment we can rely.

    Universal negative.

Section 5.4
Exercises on pages 175–176

1. Quality: affirmative; quantity: particular; subject and predicate terms both undistributed.
2. Affirmative, universal. Subject term distributed, predicate term undistributed.
3. Negative, particular. Subject term undistributed, predicate term distributed.
4. Affirmative, particular. Subject and predicate terms both undistributed.
5. Quality: negative; quantity: universal; subject and predicate terms both distributed.
6. Affirmative, universal. Subject term distributed, predicate term undistributed.
7. Affirmative, particular. Subject and predicate terms both undistributed.
8. Negative, universal. Subject and predicate terms both distributed.
9. Negative, particular. Subject term undistributed, predicate term distributed.
10. Quality: affirmative; quantity: universal; subject term distributed, predicate term undistributed.

Section 5.5
Exercises on page 180

1. If we assume that (a) is true, then:

   (b), which is its contrary, is false, and

   (c), which is its subaltern, is true, and

   (d), which is its contradictory, is false.

   If we assume that (a) is false, then:

   (b), which is its contrary, is undetermined, and

   (c), which is its subaltern, is undetermined, and

   (d), which is its contradictory, is true.

2. If (a) is true: (b) is false, (c) is true, (d) is false; if (a) is false: (b) is true, (c) and (d) are undetermined.

3. If (a) is true: (b) and (c) are undetermined, (d) is false; if (a) is false: (b) is true, (c) is false, (d) is true.

4. If (a) is true: (b) is false, (c) and (d) are undetermined; if (a) is false: (b) is true, (c) is false, (d) is true.

Section 5.6 – A
Exercises on page 186

1. No reckless drivers who pay no attention to traffic regulations are people who are considerate of others. Equivalent.

2. All commissioned officers in the U.S. Army are graduates of West Point. Not in general equivalent.

3. Some overpriced and underpowered automobiles are European cars. Equivalent.

4. No warm-blooded animals are reptiles. Equivalent.

5. Some elderly persons who are incapable of doing an honest day's work are professional wrestlers. Equivalent.

Section 5.6 – B
Exercises on page 187

1. Some college athletes are not nonprofessionals. Equivalent.
2. All organic compounds are nonmetals. Equivalent.
3. Some clergy are nonabstainers. Equivalent.
4. All geniuses are nonconformists. Equivalent.
5. No objects suitable for boat anchors are objects that weigh less than fifteen pounds. Equivalent.

Section 5.6 – C
Exercises on page 187

1. All nonpessimists are nonjournalists. Equivalent.
2. Some nonofficers are not nonsoldiers. Equivalent.
3. All degenerates are nonscholars. Equivalent.
4. All objects more than four feet high are things weighing at least fifty pounds. Equivalent.
5. Some residents are not citizens. Equivalent.

Section 5.6 – D
Exercises on page 187

1. False
2. True
3. Undetermined
4. True
5. Undetermined
6. True
7. Undetermined
8. False
9. Undetermined
10. False

Section 5.6 – E
Exercises on pages 187–188

1. False
2. True
3. False
4. False
5. Undetermined
6. False
7. True
8. False
9. True
10. False

Section 5.6 – F
Exercises on page 188

1. Undetermined
2. False
3. True
4. False
5. False
6. Undetermined
7. True
8. False
9. Undetermined
10. Undetermined
11. Undetermined
12. Undetermined
13. Undetermined
14. Undetermined
15. True

Section 5.6 – G
Exercises on page 188

1. Undetermined
2. False
3. True
4. Undetermined
5. Undetermined
6. False
7. Undetermined
8. Undetermined
9. False
10. True

11. True
12. False
13. Undetermined
14. False
15. Undetermined

Section 5.7
Exercises on pages 196–197

A. Step (3) to (4) is invalid.

B. Step (3) to (4) is invalid (subalternation).

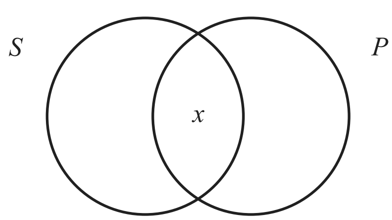
C. Step (2) to (3) is invalid (subalternation).

D. Step (1) to (2) is invalid (assumes A + E to be contraries).

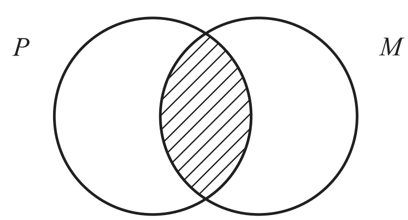
E. Step (1) to step (2) is invalid: (1) asserts the falsehood of an **I** proposition; (2) asserts the truth of its corresponding **O** proposition. In the traditional interpretation, corresponding **I** and **O** propositions are subcontraries and cannot both be false. Therefore, if the **I** proposition in (1) is false, the **O** proposition in (2) would have to be true, in *that* interpretation. But because both **I** and **O** propositions do have existential import, both *can* be false (in the Boolean interpretation) if the subject class is empty. The subject class is empty in this case, because there are no mermaids. Hence the inference from the falsehood of (1) to the truth of (2) is invalid. Corresponding **I** and **O** propositions are not subcontraries in the Boolean interpretation but the inference from (1) to (2) assumes that they are.

Section 5.8
Exercises on pages 202–203

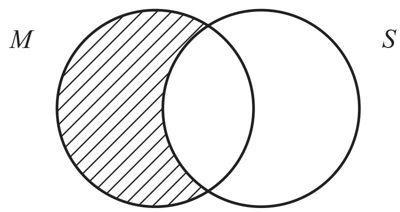
1. Some sculptors are painters
Solution
$SP \neq 0$



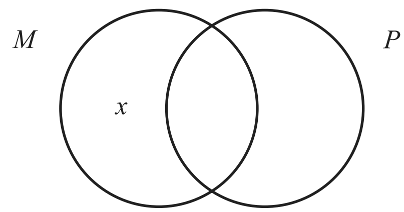
2. $PM = 0$



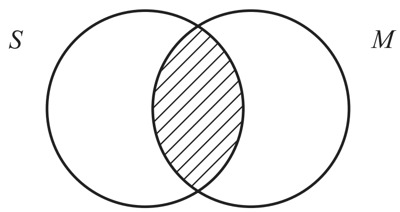
3. $M\overline{S} = 0$



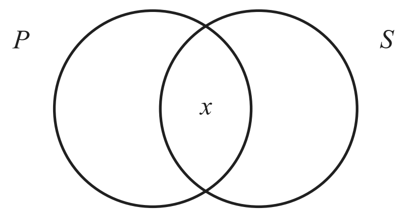
4. $M\overline{P} \neq 0$



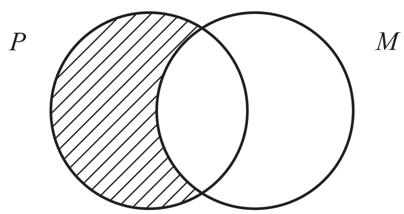
5. $SM = 0$



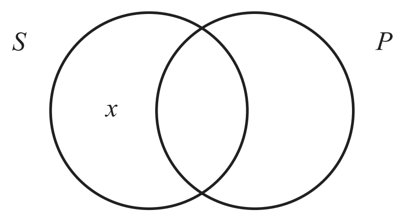
6. $PS \neq 0$



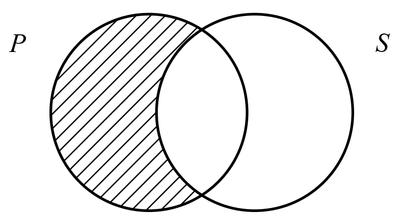
7. $P\overline{M} = 0$



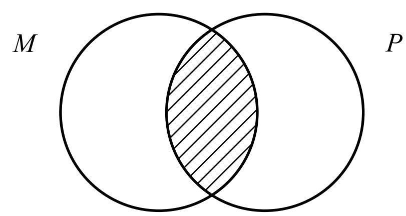
8. $S\overline{P} \neq 0$



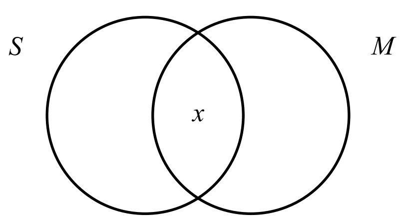
9. $P\overline{S} = 0$



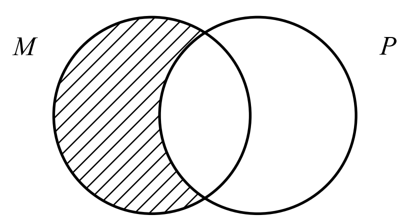
10. $MP = 0$



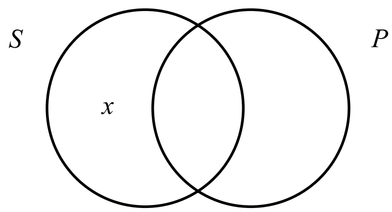
11. $SM \neq 0$



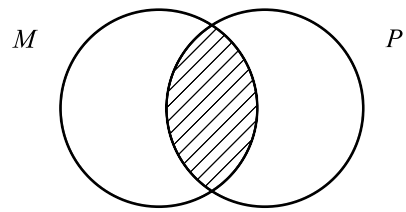
12. $M\overline{P} = 0$



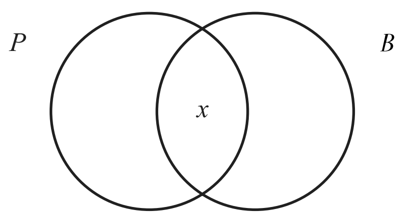
13. $S\overline{P} \neq 0$



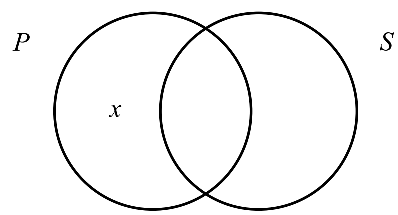
14. $MP = 0$



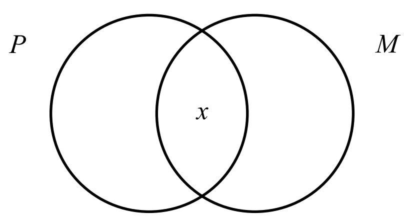
15. $PB \neq 0$



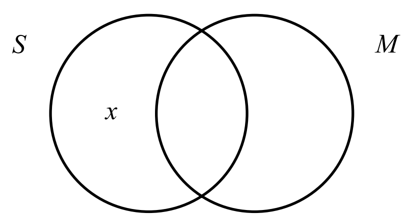
16. $P\overline{S} \neq 0$



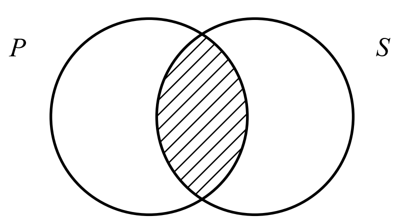
17. $PM \neq 0$



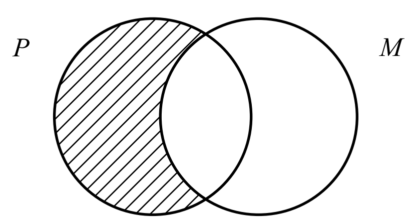
18. $S\overline{M} \neq 0$



19. $PS = 0$



20. $P\overline{M} = 0$



Chapter 6

Section 6.1
Exercises on pages 209–210

1. No nuclear-powered submarines are commercial vessels, so no warships are commercial vessels, since all nuclear-powered submarines are warships.

   Solution

   Step 1. The conclusion is "No warships are commercial vessels."

   Step 2. "Commercial vessels" is the predicate term of this conclusion and is therefore the major term of the syllogism.

   Step 3. The major premise, the premise that contains this term, is "No nuclear-powered submarines are commercial vessels."

   Step 4. The remaining premise, "All nuclear-powered submarines are warships," is indeed the major premise, because it does contain the subject term of the conclusion, "warships."

   Step 5. In standard form this syllogism is written thus:

   No nuclear-powered submarines are commercial vessels.

   All nuclear-powered submarines are warships.

   Therefore no warships are commercial vessels.

   Step 6. The three propositions in this syllogism are, in order, **E**, **A**, and **E**. The middle term, "nuclear-powered submarines," is the subject term of both premises, so the syllogism is in the *third* figure. The mood and figure of the syllogism therefore are **EAE–3**.

2. Some objects of worship are fir trees.

   All fir trees are evergreens.

   Therefore some evergreens are objects of worship.

   IAI–4.

3. Some artificial satellites are not U.S. inventions.

   All artificial satellites are important scientific achievements.

   Therefore some important scientific achievements are not U.S. inventions.

   OAO–3.

4. All certified public accounts are people of good business sense.

   No television stars are certified public accountants.

   Therefore no television stars are people of good business sense.

   AEE–1.

5. Step 1: The conclusion is: Some conservatives are not advocates of high tariff rates.

   Step 2: Major term: advocates of high tariff rates.

   Step 3: Major premise: All advocates of high tariff rates are Republicans.

   Step 4: Minor premise: Some Republicans are not conservatives.

   Step 5: This syllogism written in standard form:

   All advocates of high tariff rates are Republicans.

   Some Republicans are not conservatives.

   Therefore some conservatives are not advocates of high tariff rates.

   Step 6: The three propositions of this syllogism are, in order: **A, O, O**. The middle term, "Republicans," is the predicate term of the major premise and the subject term of the minor premise, so the syllogism is in the *fourth* figure. Thus its mood and figure are **AOO–4**.

6. No delicate mechanisms are suitable toys for children.

   All CD players are delicate mechanisms.

   Therefore no CD players are suitable toys for children.

   EAE–1.

7. Some juvenile delinquents are products of broken homes.

   All juvenile delinquents are maladjusted individuals.

   Therefore some maladjusted individuals are products of broken homes.

   IAI–3.

8. Some well-informed people are stubborn individuals who never admit a mistake.

   No stubborn individuals who never admit a mistake are good teachers.

   Therefore some good teachers are not well-informed people.

   IEO–4.

9. All proteins are organic compounds.

   All enzymes are organic compounds.

   Therefore all enzymes are proteins.

   AAA–2.

10. Step 1: The conclusion is: No sports cars are automobiles designed for family use.

    Step 2: Major term: Automobiles designed for family use.

    Step 3: Major premise: All automobiles designed for family use are vehicles intended to be driven at moderate speeds.

    Step 4: Minor premise: No sports cars are vehicles intended to be driven at moderate speeds.

    Step 5: The syllogism, written in standard form:

    All automobiles designed for family use are vehicles intended to be driven at moderate speeds.

    No sports cars are vehicles intended to be driven at moderate speeds.

    Therefore no sports cars are automobiles designed for family use.

    Step 6: The three propositions of this syllogism are, in order: **A**, **E**, **E**. The middle term, "vehicles intended to be driven at moderate speeds," is the predicate term of both the major and the minor premise, so the syllogism is in the second figure. Thus its mood and figure are: **AEE–2**.

Section 6.2
Exercises on pages 212–213

1. All business executives are active opponents of increased corporation taxes, for all active opponents of increased corporation taxes are members of the chamber of commerce, and all members of the chamber of commerce are business executives.

   Solution

   One possible refuting analogy is this: All bipeds are astronauts, for all astronauts are humans and all humans are bipeds.

2. Valid.

3. An example: No dogs are reptiles, so some reptiles are mammals, because some mammals are not dogs.

4. An example: No dogs are cats, but all cats are mammals, so no dogs are mammals.

5. One possible refuting analogy is this: All unicorns are mammals, so some mammals are not animals, because no animals are unicorns.

6. Valid.

7. An example: Some mammals are not house pets, so some dogs are not mammals, since some dogs are not house pets.

8. An example: No animals are cats, because no dogs are cats and some animals are dogs.

9. Valid.

10. One possible refuting analogy is this: All square circles are circles, and all square circles are squares; therefore some circles are squares.

Section 6.3 – A
Exercises on pages 222–223

1. **AEE–1**

   Solution

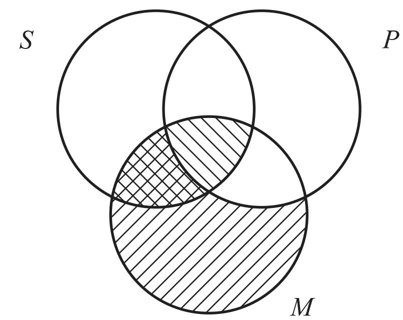
   We are told that this syllogism is in the first figure, and therefore the middle term, *M*, is the subject term of the major premise and the predicate term of the minor premise. The conclusion of the syllogism is an **E** proposition and therefore reads: No *S* is *P*. The first (major) premise (which contains the predicate term of the conclusion) is an **A** proposition, and therefore reads: All *M* is *P*. The second (minor) premise (which contains the subject term of the conclusion) is an **E** proposition and therefore reads: No *S* is *M*. This syllogism therefore reads as follows:

All *M* is *P*.

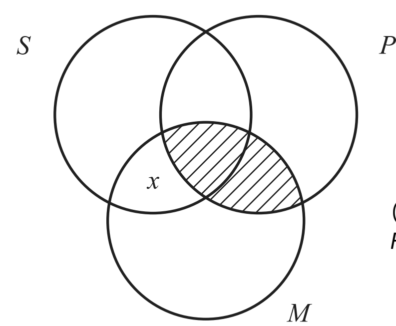
No *S* is *M*.

Therefore no *S* is *P*.

Tested by means of a Venn diagram this syllogism is shown to be invalid.

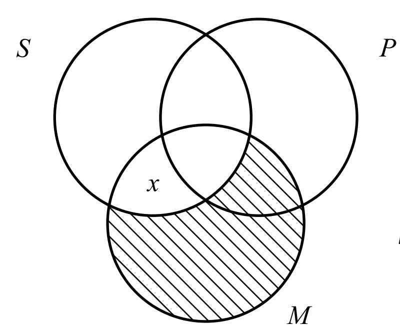


2. No *P* is *M*.
Some *S* is *M*.
∴ Some *S* is not *P*.



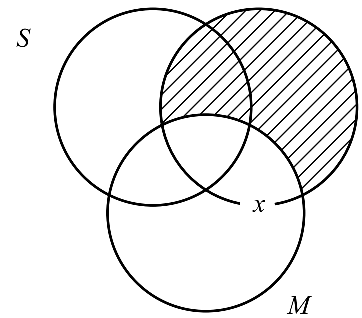
Valid
(**EIO–2**,
Festino)

3. Some *M* is not *P*.
All *M* is *S*.
∴ Some *S* is not *P*.



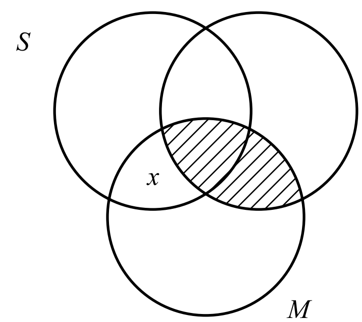
Valid
(**OAO–3**,
Bokardo)

4. All *P* is *M*.
Some *M* is not *S*.
∴ Some *S* is not *P*.



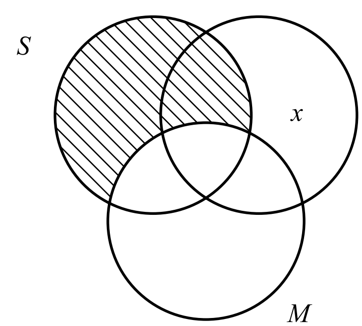
Invalid

5. No *P* is *M*.
Some *M* is *S*.
∴ Some *S* is not *P*.



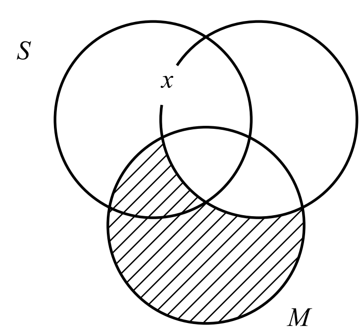
Valid
(**EIO–4**,
Fresison)

6. Some *P* is not *M*.
All *S* is *M*.
∴ Some *S* is not *P*.



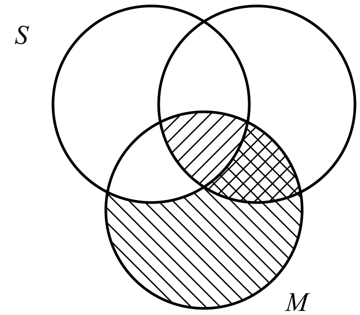
Invalid

7. All *M* is *P*.
Some *S* is not *M*.
∴ Some *S* is not *P*.



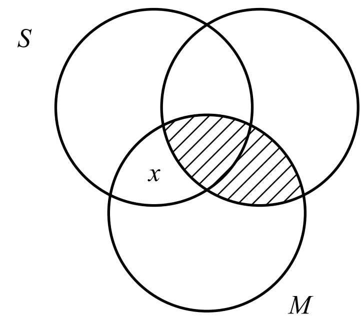
Invalid

8. No *M* is *P*.
All *M* is *S*.
∴ No *S* is *P*.



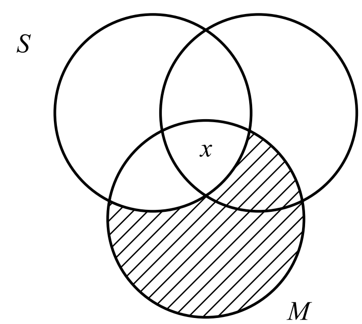
Invalid

9. No *M* is *P*.
Some *M* is *S*.
∴ Some *S* is not *P*.



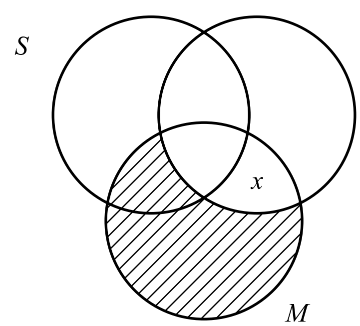
Valid
(**EIO–3**,
Ferison)

10. Some *P* is *M*.
All *M* is *S*.
∴ Some *S* is *P*.



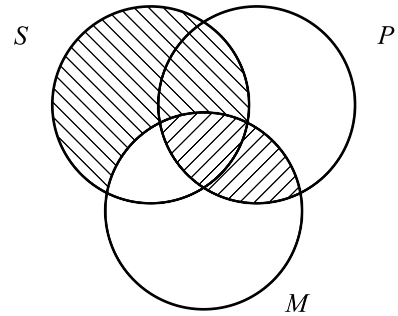
Valid
(**IAI–4**,
Dimaris)

11. All *M* is *P*.
Some *M* is not *S*.
∴ Some *S* is not *P*.



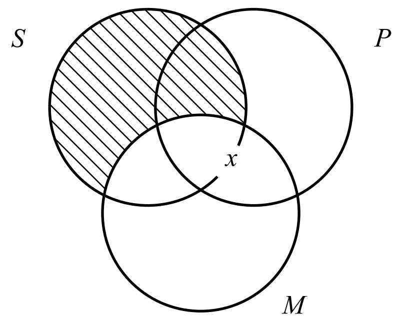
Invalid

12. No *M* is *P*.
All *S* is *M*.
∴ No *S* is *P*.



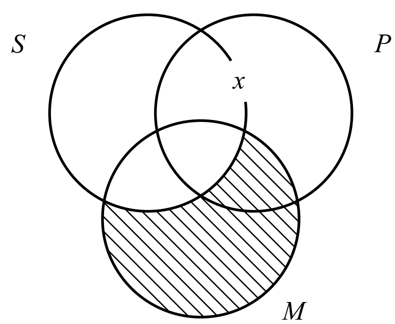
Valid
(**EAE–1**,
Celarent)

13. Some *M* is *P*.
All *S* is *M*.
∴ Some *S* is *P*.



Invalid

14. Some *P* is not *M*.
All *M* is *S*.
∴ Some *S* is not *P*.



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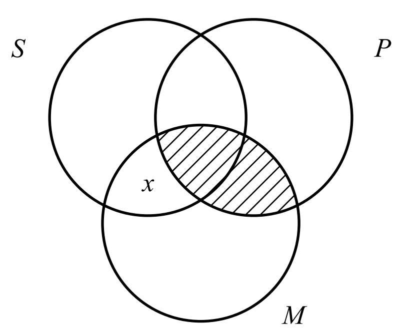
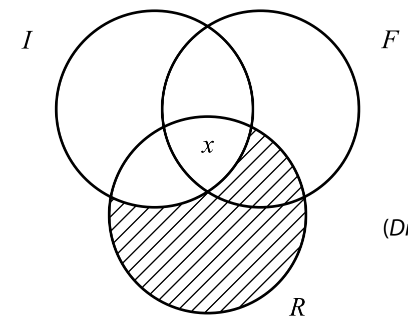
15. No *M* is *P*.
Some *S* is *M*.
∴ Some *S* is not *P*.



Valid
(**EIO–1**,
Ferio)

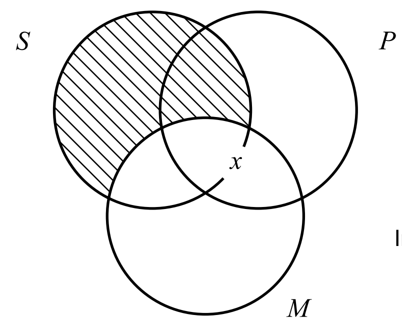
Section 6.3 – B
Exercises on pages 223–224

1. Some reformers are fanatics.
   All reformers are idealists.
   ∴ Some idealists are fanatics.



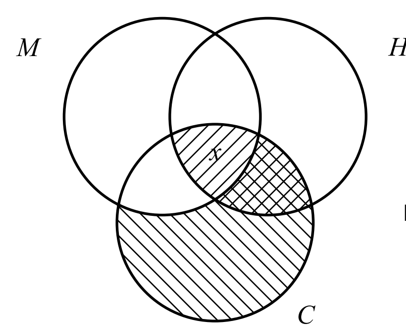
IAI–3
Valid
(*Disamis*)

2. Some philosophers are mathematicians.
   All scientists are mathematicians.
   ∴ Some scientists are philosophers.



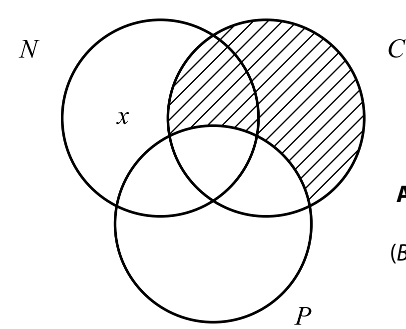
IAI–2
Invalid

3. No horses are centaurs.
   All centaurs are mammals.
   ∴ Some mammals are not horses.



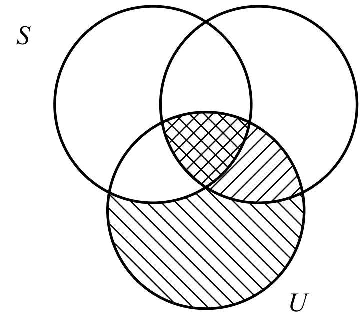
EAO–4
Invalid

4. All criminals are parasites.
   Some neurotics are not parasites.
   ∴ Some neurotics are not criminals.



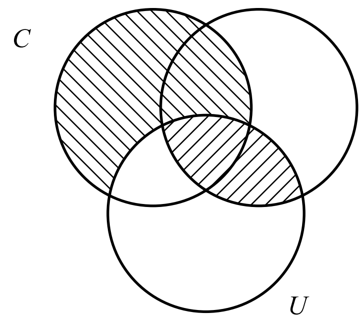
AOO–2
Valid
(*Baroko*)

5. No pleasure vessels are underwater craft.
All underwater craft are submarines.
∴ No submarines are pleasure vessels.



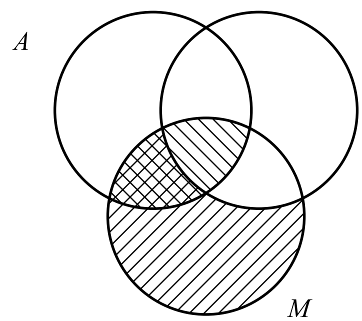
EAE–4
Invalid

6. No pioneers were unsavory persons.
All criminals are unsavory persons.
∴ No criminals were pioneers.



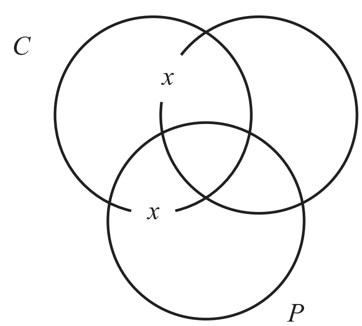
EAE–2
Valid
(*Cesare*)

7. All musicians are baseball fans.
No musicians are astronauts.
∴ No astronauts are baseball fans.



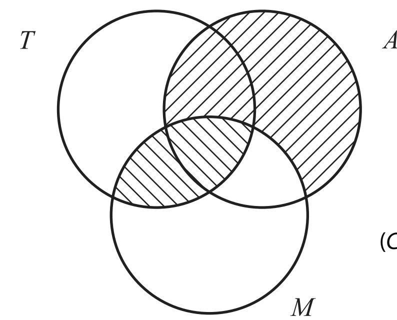
AEE–3
Invalid

8. Some Protestants are not Methodists.
Some Christians are not Protestants.
∴ Some Christians are not Methodists.



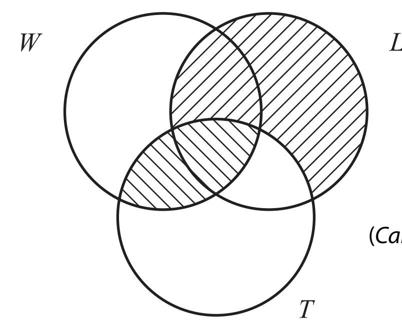
OOO–1
Invalid

9. All active politicians are people whose primary interest is in winning elections.
   No people whose primary interest is in winning elections are true liberals.
   ∴ No true liberals are active politicians.



AEE–4
Valid
(*Camenes*)

10. All labor leaders are true liberals.
    No weaklings are true liberals.
    ∴ No weaklings are labor leaders.



AEE–2
Valid
(*Camestres*)

Section 6.4 – A
Exercises on pages 231–232

1. **AAA–2**

   Solution

   Any syllogism in the second figure has the middle term as predicate of both the major and the minor premise. Thus any syllogism consisting of three **A** propositions, in the second figure, must read: All *P* is *M*; all *S* is *M;* therefore all *S* is *P*. But *M* is not distributed in either of the premises in that form, and therefore it cannot validly be inferred from such premises that all *S* is *P*. Thus every syllogism of the form **AAA–2** violates the rule that the middle term must be distributed in at least one premise, thereby committing the **fallacy of the undistributed middle**.

2. Affirmative conclusion from a negative premise. Breaks Rule 5.

3. Illicit major. Breaks Rule 3.

4. Exclusive premises. Breaks Rule 4.

5. Illicit minor. Breaks Rule 3.
6. Undistributed middle. Breaks Rule 2.
7. Illicit minor and affirmative conclusion from a negative premise. Breaks Rules 3 and 5.
8. Existential fallacy. Breaks Rule 6.
9. Affirmative conclusion from a negative premise. Breaks Rule 5.
10. Illicit major. Breaks Rule 3.
11. Existential fallacy. Breaks Rule 6.
12. Undistributed middle. Breaks Rule 2.
13. Exclusive premises. Breaks Rule 4.
14. Illicit major. Breaks Rule 3.
15. Illicit minor. Breaks Rule 3.

Section 6.4 – B
Exercises on pages 232–233

1. All textbooks are books intended for careful study.

   Some reference books are books intended for careful study.

   Therefore some reference books are textbooks.

   Solution

   In this syllogism, "textbooks" is the major term (the predicate of the conclusion) and "reference books" is the minor term (the subject of the conclusion). "Books intended for careful study" is therefore the middle term, and it appears as the predicate of both premises. But in neither of the premises is this middle term distributed, so the syllogism violates the rule that the middle term must be distributed in at least one premise, thereby committing the **fallacy of the undistributed middle**.

2. Four terms (equivocation on "criminal actions"). Breaks Rule 1.
3. Exclusive premises. Breaks Rule 4.
4. Illicit minor. Breaks Rule 3.
5. Existential fallacy. Breaks Rule 6.

6. Affirmative conclusion from a negative premise. Breaks Rule 5.

7. Illicit major. Breaks Rule 3.

8. Undistributed middle. Breaks Rule 2.

9. Four terms (equivocation on "most hungry," which is used to *mean most hungry before eating* in the major premises, and to mean *most hungry after eating* in the minor premise). Breaks Rule 1.

10. Illicit minor. Breaks Rule 3.

Section 6.4 – C
Exercises on pages 233–234

1. All chocolate éclairs are fattening foods, because all chocolate éclairs are rich desserts, and some fattening foods are not rich desserts.

   Solution

   In this syllogism the conclusion is affirmative ("all chocolate éclairs are fattening foods"), while one of the premises is negative ("some fattening foods are not rich desserts"). The syllogism therefore is invalid, violating the rule that if either premise is negative the conclusion must also be negative, thereby committing the **fallacy of affirmative conclusion from a negative premise**.

2. Undistributed middle. Breaks Rule 2.

3. Illicit major. Breaks Rule 3.

4. Existential fallacy. Breaks Rule 6.

5. Illicit minor. Breaks Rule 3.

6. Exclusive premises and affirmative conclusion from a negative premise. Breaks Rules 4 and 5.

7. Four terms ("democrats" and "Democrats" are two different terms). Breaks Rule 1.

8. Illicit minor. Breaks Rule 3.

9. Illicit major. Breaks Rule 3.

10. Four terms. (There is an equivocation on the term "people who like it," which has a very different meaning in the conclusion from the one it has in the premise.) Breaks Rule 1.

Section 6.5
Exercises on page 238

1. **IAI–3** (*Disamis*)

4. **AOO–2** (*Baroko*)

6. **EAE–2** (*Cesare*)

9. **AEE–4** (*Camenes*)

10. **AEE–2** (*Camestres*)

Section 6 – Appendix
Exercises on pages 242–243

1. Can any standard-form categorical syllogism be valid that contains exactly three terms, each of which is distributed in both of its occurrences?

   Solution

   No, such a syllogism cannot be valid. If each of the three terms were distributed in both of its occurrences, all three of the syllogism's propositions would have to be **E** propositions, and the mood of the syllogism would thus be **EEE**, which violates Rule 4, which forbids two negative premises.

2. **AA_** and **AE_** violate Rule 6; **AIO** violates Rule 3; **AOI** violates Rule 5; **AOO** violates Rule 3. **EA_** violates Rule 6; **EE_** violates Rule 4 and Rule 6; **EII** violates Rule 5; **EO_** violates Rule 4. **IA_** and **II_** violate Rule 2; **IEI** violates Rule 5; **IEO** violates Rule 3; **IOI** violates Rule 5; **IOO** violates Rule 3. **OA_**and **OI_** violate Rule 2; **OE_** and **OO_** violate Rule 4. Therefore only moods **AII** and **EIO** are valid here.

3. In figure 3 both premises would have to be negative, in violation of Rule 4. In all other figures (1, 2, 4) it is possible, as is shown by the validity of **EAE–1**, **EAE–2**, and **AEE–4**.

4. None. Regardless of figure, **II_** would violate Rule 2 and **OO_** would violate Rule 4. If one premise is **I** and one is **O** then by Rule 5 the conclusion would be negative and would distribute its predicate. By Rule 3 the major term would have to be distributed in the major premise, but **I** and **O** (together) distribute only one term, and it is not possible also to distribute the middle term in at least one premise. Rule 2 would then be violated.

5. Plainly this is **possible in the first figure**, where **AII–1**, which is valid, has only one term distributed, and that term only once. It also is **possible in the third figure**, where **AII–3**

(as well as **IAI–3**) are valid and also have only one term distributed, and distributed only once. It also is **possible in the fourth figure**, where **IAI–4**, which is valid, has only one term distributed, and distributed only once. But where the middle term is the predicate term of both premises, **in the second figure, it is not possible**. Consider: to avoid breaking Rule 2, which requires that the middle term be distributed in at least one premise, one of the premises in this figure must be negative. But then, by Rule 5, the conclusion would have to be negative and would distribute its predicate. Thus, if only one term can be distributed, in the second figure that would have to be in the conclusion; but if the distributed term can be distributed only once, that would break Rule 3, because if it is distributed in the conclusion it must be distributed in the premises.

6. None. By Rule 2 one of the distributed terms would have to be the middle term. One premise would have to distribute both its terms and would have to be an **E.** Then, by Rule 5, the conclusion would have to be negative and, since it is, to distribute only one of its terms it would have to be an **O**. The other premise is to distribute only the middle term and so could be only an **A** or an **O**, but it could not be an **A** by Rule 6 and it could not be an **O** by Rule 4.

7. None. The negative conclusion could not be an **E**, for if it were by Rule 3 both major and minor terms would have to be distributed in the premises, and since an affirmative proposition distributes at most one of its terms, Rule 2 would be violated. Nor could the negative conclusion be an **O**, for if it were then by Rule 6 at least one premise would have to be particular; and therefore an **I**. But an **I** proposition distributes neither of its terms, so both premises would distribute only one term between them, thus violating either Rule 2 or 3.

8. None. If the particular premise were an **O** then by Rule 5 the conclusion would be negative and hence an **E**, whence by Rule 3 both major and minor terms would have to be distributed in the premises. Since the middle term must be distributed there also, by Rule 2, and the **O** premise distributes only one of its terms, the other premise would have to distribute both its terms and be an **E**, thus violating Rule 4.

   On the other hand, if the particular premise were an **I** it would distribute neither of its terms. But since the universal conclusion requires by Rule 3 that the minor term be distributed in the minor premise, the **I** premise would have to be the major premise, and, being an **I**, would not distribute the middle term. So by Rule 2, the middle term must also be distributed by the minor premise, which would therefore have to be an **E**. Then by Rule 5 the conclusion would be negative and distribute the major term also, in violation of Rule 3.

9. By Rule 2 the middle term is distributed in at least one premise; hence at least one premise is negative, so by Rule 5 the conclusion is negative also and must be an **E**. Hence by Rule 6 both premises must be universal, and by Rule 4 at least one is affirmative. Hence the only two moods are **AEE** and **EAE**, both valid in Figure 2.

10. None. If the middle term were distributed in both premises, then, in the first figure, the minor premise would have to be negative, whence (by Rule 5) the conclusion would have to be negative, so by Rule 3 the major premise would have to be negative, in violation of Rule 4. In the second figure, both premises would have to be negative, in violation of Rule 4. In the third figure both premises would have to be universal, so the minor premise would have to be negative by Rule 3, and by Rule 5 the conclusion would be negative—so by Rule 3 the major premise would also have to be negative, in violation of Rule 4. In the fourth figure the major premise would have to be negative. Therefore (by Rule 5) the conclusion would have to be negative (**E** or **O**) and it would distribute its major term, which means (by Rule 3) that the major premise would also have to distribute its major term and would therefore be universal (an **E** proposition). The minor premise also must be universal, since it distributes the middle term, and by Rule 4 it cannot be negative, so it must be the **A** proposition *All M is S*. Now Rule 6 precludes the possibility of an **O** proposition in the conclusion, and Rule 3 precludes the possibility of an **E**.

11. No.

    If the major term is undistributed in the conclusion, the conclusion must be affirmative, and by Rule 5, both premises must be affirmative. If the major term is distributed in the major premise, that premise, being affirmative, must be the **A** proposition *All P is M*, which does not distribute the middle term. But the middle term must, by Rule 2, be distributed in at least one of the premises, so it must be distributed in the minor premise. This can only happen if the minor premise is the **A** proposition *All M is S*, which does not distribute the minor term. With two **A** premises, the conclusion cannot be *Some S is P*, which would violate Rule 6, and it cannot be *All S is P,* which would violate Rule 3. Hence the *major* term cannot be distributed in a premise but undistributed in the conclusion of a valid syllogism.

    If the minor term is undistributed in the conclusion, the conclusion must be particular. If the conclusion is also affirmative (*Some S is P*) then by Rule 5 both premises must be affirmative also. If the minor term is distributed in the minor premise, that premise—being affirmative—must be the **A** proposition *All S is M*, which does not distribute the middle term. So the middle term must, by Rule 2, be distributed in the major premise, which—being affirmative—must be the **A** proposition *All M is P*, but now the syllogism would violate Rule 6.

    But if the conclusion is negative *(Some S is not P)* it distributes the major term, which by Rule 3 must be distributed in the major premise. If the minor term is distributed in the minor premise, then since by Rule 2 the middle term must be distributed in at least one premise, whichever premise distributes it must distribute both its terms and be an **E** proposition. The other premise cannot be negative (by Rule 4), and it cannot be universal (by Rule 6), so it must be particular affirmative and hence cannot distribute either of its terms. This, of course, contradicts the fact that in this case the major premise must distribute the major term and the *minor* premise must distribute the minor term. Hence the *minor* term cannot be distributed in a premise but undistributed in the conclusion of a valid syllogism.

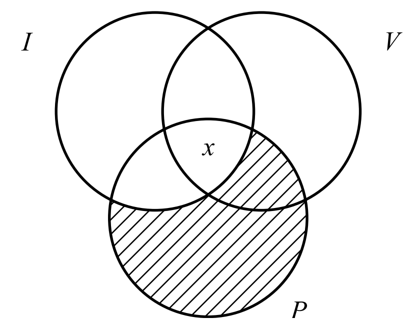
Chapter 7

Section 7.2
Exercises on pages 248–249

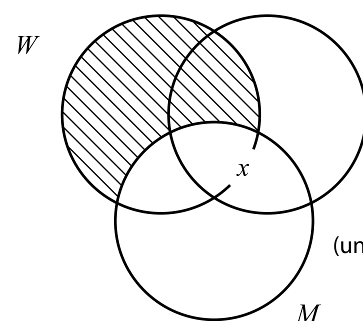
1. Some preachers are persons of unfailing vigor. No preachers are nonintellectuals. Therefore some intellectuals are persons of unfailing vigor.

   Solution

   This may be translated into: Some preachers are persons of unfailing vigor. (Some *P* is *V*.) All preachers are intellectuals. (By obversion: All *P* is *I*.) Therefore some intellectuals are persons of unfailing vigor. (Some *I* is *V*.) Shown on a Venn diagram, this syllogism (in *Disamis*) is seen to be valid:

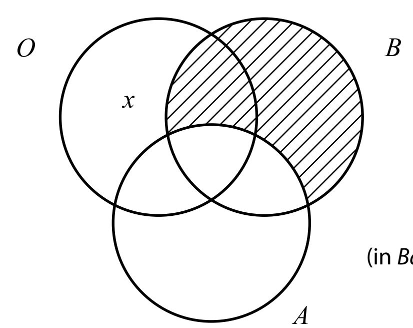


2. Some *M* is *R*.
   All *W* is *M*.
   ∴ Some *W* is *R*.



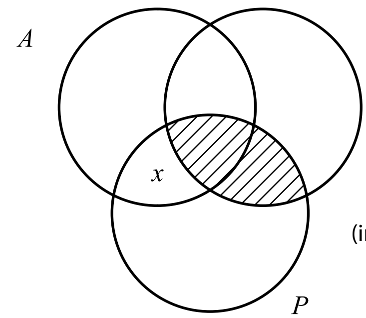
Invalid
(undistributed
middle)

3. All *B* is *A*.
   Some *O* is not *A*.
   ∴ Some *O* is not *B*.



Valid
(in *Baroko*)

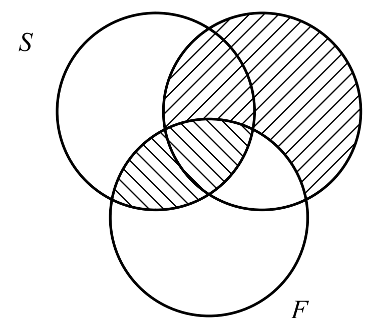
4. No *D* is *P*.
Some *P* is *A*.
∴ Some *A* is not *D*.



Valid
(in *Fresison*)

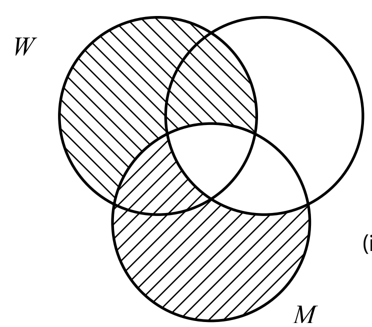
5. Where
E = Explosives
F = Flammable things (note that "flammable" and "inflammable" are synonyms!)
S = Safe things.

This syllogism translates into standard form thus:
All *E* is *F*.
No *F* is *S*.
Therefore no *S* is *E*.



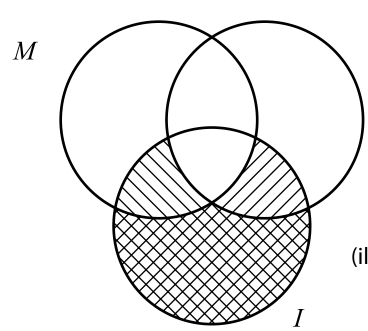
Shown in a Venn diagram, this syllogism (in *Camenes*) is seen to be valid.

6. All *M* is *C*.
All *W* is *M*.
∴ All *W* is *C*.



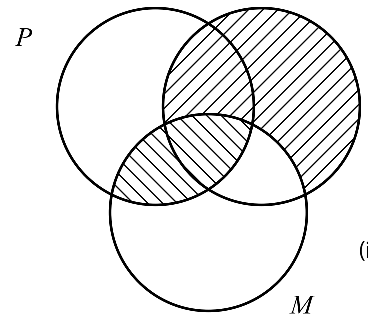
Valid
(in *Barbara*)

7. All *I* is *C*.
All *I* is *M*.
∴ All *M* is *C*.



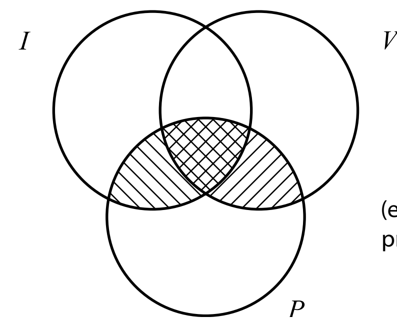
Invalid
(illicit minor)

8. All *H* is *M*.
   No *M* is *P*.
   ∴ No *P* is *H*.



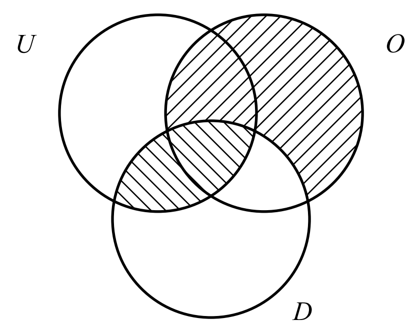
Valid
(in *Camenes*)

9. No *V* is *P*.
   No *P* is *I*.
   ∴ All *I* is *V*.



Invalid
(exclusive
premises)

10. Where
    O = Objects over six feet long
    D = Difficult things to store
    U = Useful things.
    This syllogism translates into standard form thus:
    All *O* is *D*.
    No *D* is *U*.
    Therefore no *U* is *O*.
    Shown in a Venn diagram, this syllogism
    (in *Camenes*) is shown to be valid.



Section 7.3
Exercises on pages 257–258

1. All roses are fragrant things.
2. No orchids are fragrant things.
3. Some persons are beings who have lived to regret their misspent youths.

4. Some persons worth meeting are not persons worth having as friends.

5. All Junkos are the best things that money can buy.

6. All Buds are real beers.

7. No safe things are exciting things.

8. All winners of the Congressional Medal of Honor are brave people.

9. Some persons are nonappreciators of good counselors.
(Or perhaps: Some good counselors are not persons who are appreciated.)

10. No people who face the sun are people who see their own shadows.

11. All persons who hear her sing are persons who become inspired.

12. All persons who take the sword are persons who shall perish by the sword.

13. All persons who can use the front door are members.

14. All persons like Sara Lee.

15. No candidates of the Old Guard are persons supported by the Young Turks.
(Or: No Young Turks are supporters of candidates of the Old Guard.)

16. No styles that are tiresome are good.

17. All persons who only stand and wait are persons who also serve.

18. All women who know their own limitations are happy indeed.

19. All things of beauty are things that are joys forever.

20. All people who love well are people who pray well.

21. Some glittering things are not gold things.

22. All persons who think the great unhappy are great persons.

23. All persons who never felt wounds are persons who jest at scars.
(Always a source of warm discussion!)

24. All things that a man sows are things that that man also reaps.

25. All soft answers are things that turn away wrath.

Section 7.4 – A
Exercises on page 260

1. All times when he is reminded of his loss are times when he groans.
2. No times when she goes to work are times when she drives her car.
3. All places where he chooses to walk are places where he walks.
4. All times when he orders an item on the menu are times when he orders the most expensive item on the menu. (Or perhaps better: All items on the menu that he orders are items that are the most expensive items on the menu.)
5. All cases in which she gives her opinion are cases in which she is asked to give her opinion.
6. All places where she may happen to be are places where she tries to sell life insurance.
7. All times when he gets angry are times when his face gets red.
8. All occasions on which he is asked to say a few words are occasions when he talks for hours.
9. All places where reason is left free to combat error of opinion are places where error of opinion may be tolerated.
10. No times when people do not discuss questions freely are times when people are most likely to settle questions rightly.

Section 7.4 – B
Exercises on pages 260–263

1. Since all knowledge comes from sensory impressions and since there's no sensory impression of substance itself, it follows logically that there is no knowledge of substance.

   —Robert M. Pirsig, *Zen and the Art of Motorcycle Maintenance*

   Solution

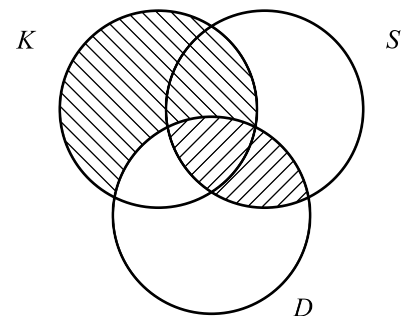
   a. Standard-form translation:

   No things derived from sensory impressions are items of knowledge of substance itself.

   All items of knowledge are things derived from sensory impressions.
   Therefore, no items of knowledge are items of knowledge of substance itself.

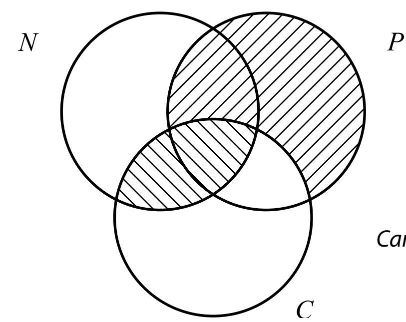
b. Mood and figure: **EAE–1**

c. Valid; *Celarent*



2. All predicables are things that come in contradictory pairs.
No names are things that come in contradictory pairs.
∴ No names are predicables.

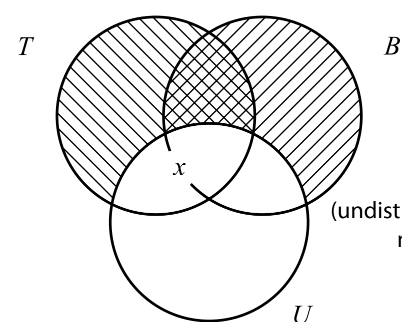
AEE–2



Valid
Camestres

3. All bankrupt companies are companies unable to pay interest on their debts.
Barcelona Traction is a company unable to pay interest on its debts.
∴ Barcelona Traction is a bankrupt company.

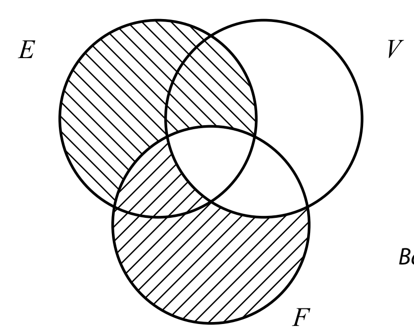
AAA–2
AII–2



Invalid
(undistributed
middle)

4. All fanaticism is vice.
All extremism is fanaticism.
∴ All extremism is vice.

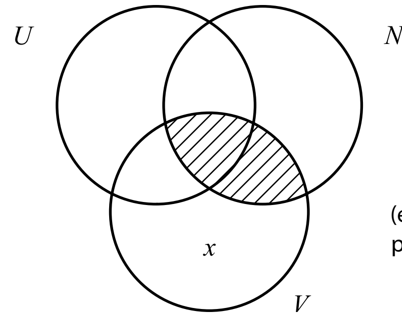
AAA–1



Valid
Barbara

5. No syllogisms having two negative premises are valid syllogisms.
Some valid syllogisms are not unsound arguments.
∴ Some unsound arguments are syllogisms having two negative premises.

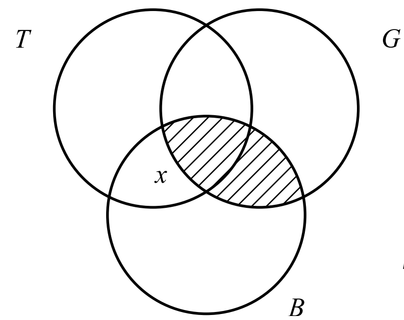
EOI–4



Invalid
(exclusive
premises)

6. No gold is base metal.
Some base metals are things that glitter.
∴ Some things that glitter are not gold.

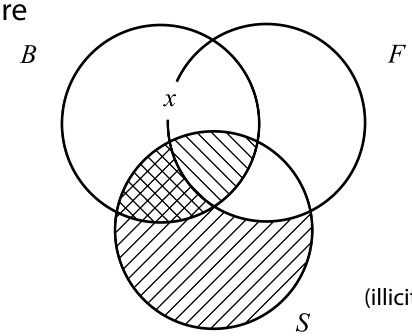
EIO–4



Valid
Fresison

7. All places where there is smoke are places where there is fire.
The basement is not a place where there is smoke.
∴ The basement is not a place where there is a fire.

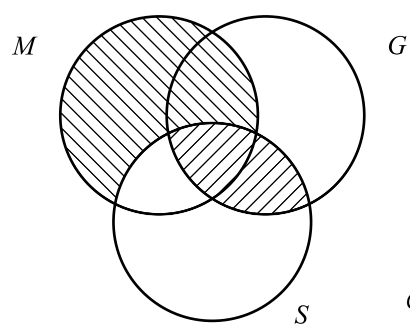
AEE–1
AOO–1



Invalid
(illicit major)

8. No sorrow is a thing that is in God.
All mercy is sorrow.
∴ No mercy is a thing that is in God.

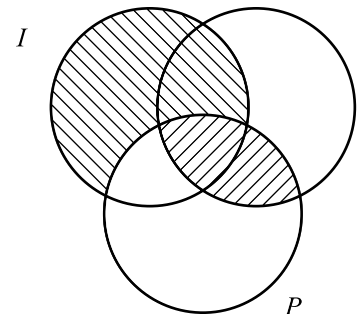
EAE–1



Valid
Celarent

9. No painful sensation is a thing that can exist in an unperceiving corporeal substance.
All intense heat is painful sensation.
∴ No intense heat is a thing that can exist in an unperceiving corporeal substance.

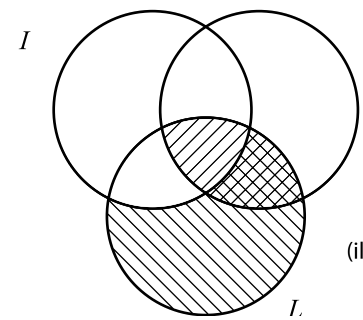
EAE–1



Valid
Celarent

10. No persons who are truly objective are persons likely to be mistaken.
All persons likely to be mistaken are persons who ignore the facts.
∴ No persons who ignore the facts are persons who are truly objective.

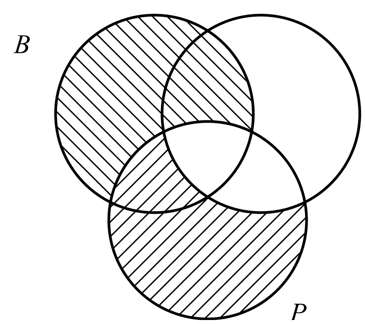
EAE–4



Invalid
(illicit minor)

11. All people are thinkers.
All bridge players are people.
∴ All bridge players are thinkers.

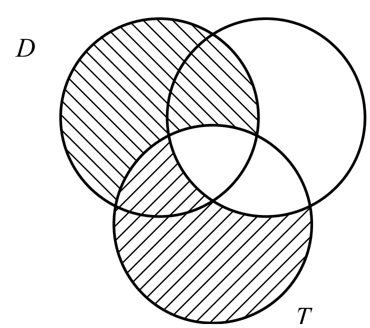
AAA–1



Valid
Barbara

12. All times when I am in trouble are times when I pray.
All days are times when I am in trouble.
∴ All days are times when I pray.

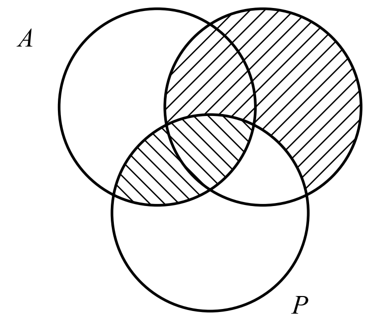
AAA–1



Valid
Barbara

13. All brain-processes are things in physical space.
No after-images are things in physical space.
∴ No after-images are brain-processes.

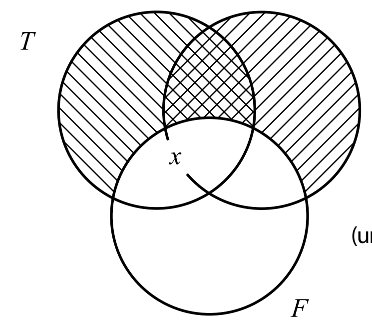
AEE–2



Valid
Camestres

14. All times following rain are times when fish do not bite.
This time is a time when fish do not bite.
∴ This time is a time following rain.

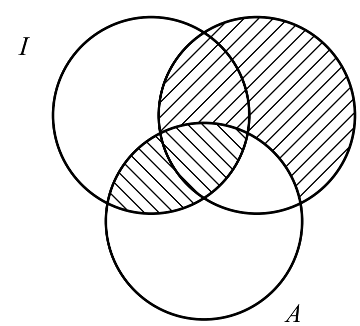
AAA–2
AII–2



Invalid
(undistributed
middle)

15. All things interesting to engineers are approximations.
No approximations are irrationals.
∴ No irrationals are things interesting to engineers.

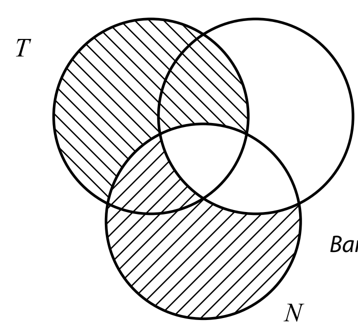
AEE–4



Valid
Camenes

16. All fights against neighbors are evils.
All fights against Thebans are fights against neighbors.
∴ All fights against Thebans are evils.

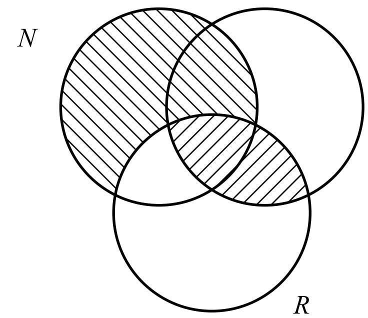
AAA–1



Valid
Barbara

17. No things due to chance are things that reappear constantly or frequently.
All products of Nature are things that reappear constantly or frequently.
∴ No products of Nature are things due to chance.

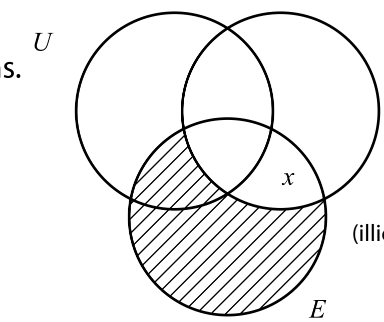
EAE–2



Valid
Cesare

18. All excessive drinkers are debtors.
Some excessive drinkers are not unemployed persons.
∴ Some unemployed persons are not debtors.

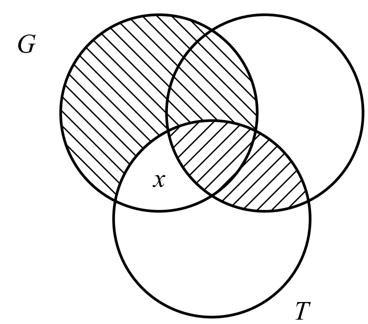
AOO–3



Invalid
(illicit major)

19. No title contests are dull games.
The game tomorrow is a title contest.
∴ The game tomorrow will not be a dull game.

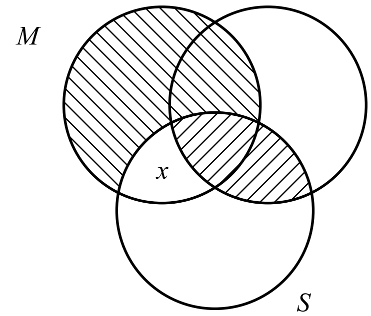
EAE–1
EIO–1



Valid
Celarent

20. No times when Bill goes to work are times when Bill wears a sweater.
This morning was a time when Bill wore a sweater.
∴ This morning was not a time when Bill went to work.

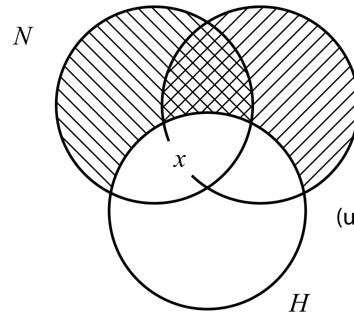
EAE–2
EIO–2



Valid
Cesare
Festino

21. All times that Cynthia compliments Henry are times that Henry is cheerful.
Now is a time that Henry is cheerful.
∴ Now is a time that Cynthia compliments Henry.

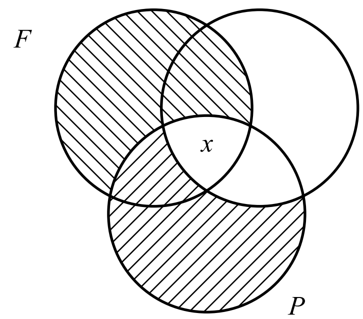
AAA–2
AII–2



Invalid
(undistributed
middle)

22. All places where pickets are present are places where there is a strike.
The factory is a place where pickets are present.
∴ The factory is a place where there is a strike.

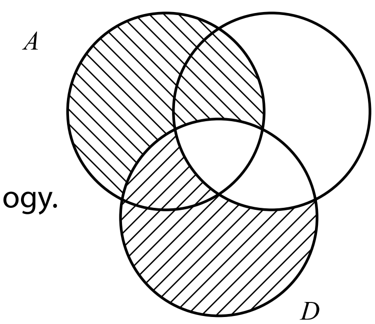
AAA–1
AII–1



Valid
Barbara
Darii

23. All cases of disease are things that can be profitably investigated by the methods of epidemiology.
All cases of drug abuse are cases of disease.
∴ All cases of drug abuse are things that can be profitably investigated by the methods of epidemiology.

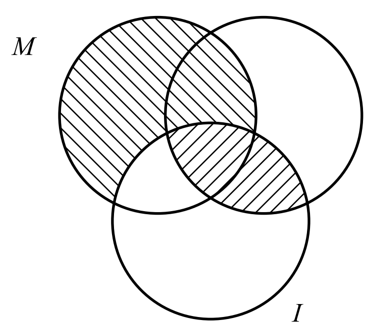
AAA–1



Valid
Barbara

24. No things derived from reason are things that have an influence on the actions and affections.
All morals are things that have an influence on the actions and affections.
∴ No morals are things derived from reason.

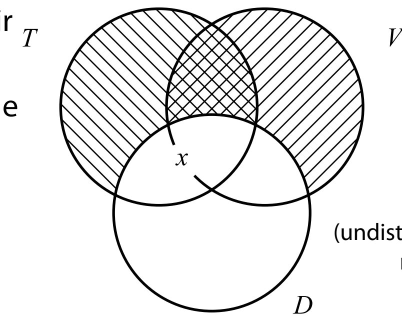
EAE–2



Valid
Cesare

25. All valid syllogisms are syllogisms that distribute their middle terms in at least one premise.
This syllogism is a syllogism that distributes its middle term in at least one premise.
∴ This syllogism is a valid syllogism.

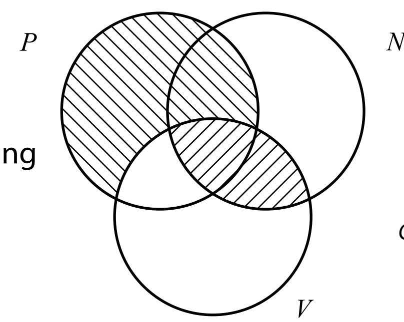
AAA–2
AII–2



Invalid
(undistributed
middle)

26. No valid syllogisms are syllogisms having two negative premises.
All syllogisms on this page are valid syllogisms.
∴ No syllogisms are on this page are syllogisms having two negative premises.

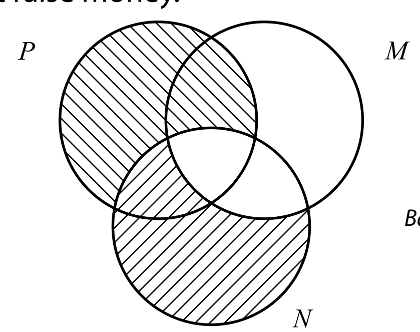
EAE–1



Valid
Celarent

27. All events that result in good poll numbers are events that raise money.
All events that result in good press are events that result in good poll numbers.
∴ All events that result in good press are events that raise money.

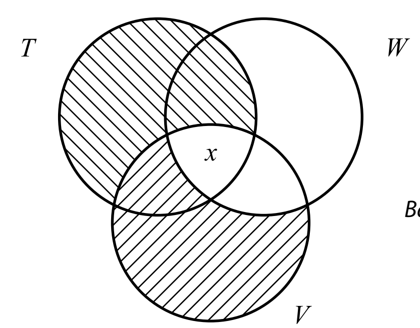
AAA–1



Valid
Barbara

28. All places with vegetation are places where water is present.
This place is a place with vegetation.
∴ This place is a place where water is present.

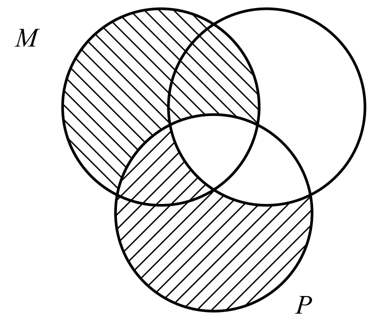
AAA–1
AII–1



Valid
Barbara
Darii

29. All persons present are employed persons.
All members are persons present.
∴ All members are employed persons.

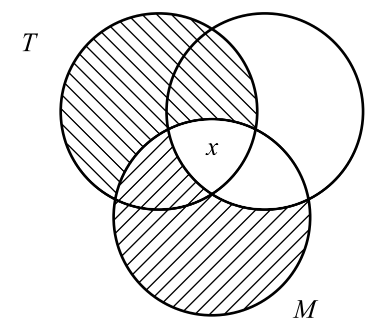
AAA–1



Valid
Barbara

30. All situations in which much money is involved are situations in which competition is stiff.
This situation is a situation in which much money is involved.
∴ This situation is a situation in which competition is stiff.

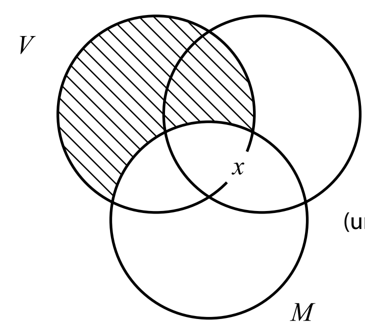
AAA–1
AII–1



Valid
Barbara
Darii

31. Some men are handsome creatures.
All vile creatures are men.
∴ Some vile creatures are handsome creatures.

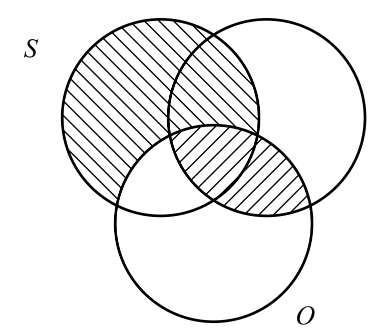
IAI–1



Invalid
(undistributed
middle)

32. No simple objects are things that can be separated from themselves.
All souls are simple objects.
∴ No souls are things that can be separated from themselves.

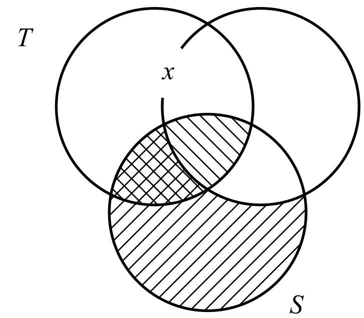
EAE–1



Valid
Celarent

33. All times when he is sick are times when he complains.
This time is not a time when he is sick.
∴ This time is not a time when he complains.

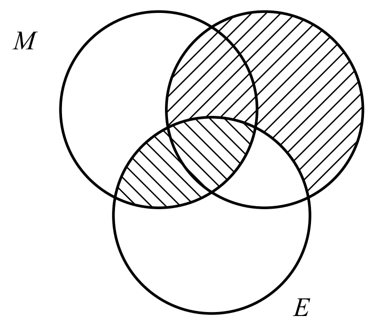
AEE–1
AOO–1



Invalid (illicit major)

34. All significant propositions are expressions of either tautologies or empirical hypotheses.
No metaphysical assertions are expressions of either tautologies or empirical hypotheses.
∴ No metaphysical assertions are significant propositions.

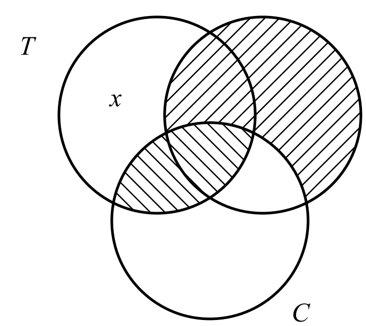
AEE–2



Valid
Camestres

35. All invalid syllogisms are syllogisms that commit an illicit process.
This syllogism is not a syllogism that commits an illicit process.
∴ This syllogism is not an invalid syllogism.

AEE–2
AOO–2



Valid
Camestres
Baroko

Section 7.5
Exercises on pages 266–269

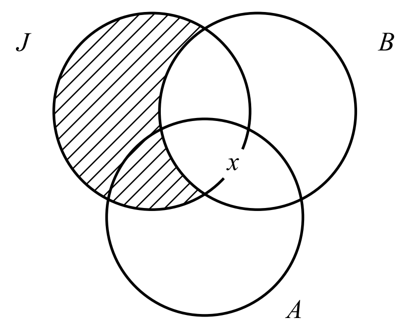
1. Transgenic animals are manmade and as such are patentable.
—Alan E. Smith, cited in *Genetic Engineering (San Diego, Greenhaven Press, 1990)*

Solution

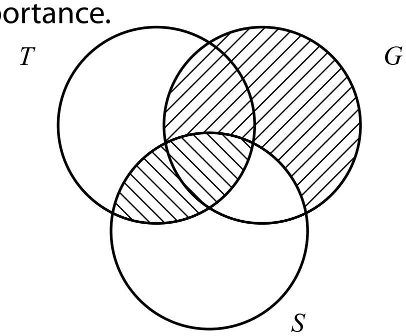
a. The premise understood but not stated here is that whatever is manmade is patentable.

b. Standard-form translation:
   All manmade things are patentable things.
   All transgenic animals are manmade things.
   Therefore, all transgenic animals are patentable things.
c. The enthymeme is of the first order, since the premise taken as understood was the major premise of the completed argument.
d. This is a valid syllogism of the from **AAA–1**, *Barbara*.

2. a. Unstated conclusion: Abe knows the job.
   b. Standard-form translation:
      All persons who know the job are persons who know the buck.
      Abe is a person who knows the buck.
      ∴ Abe is a person who knows the job.
   c. Third-order enthymeme.
   d. Invalid (undistributed middle).



3. a. Unstated premise: All things of great worth and importance are likely to be stolen.
   b. Standard-form translation:
      All things of great worth and importance are things that are likely to be stolen.
      No textbooks are things that are likely to be stolen.
      ∴ No textbooks are things of great worth and importance.
   c. First-order enthymeme.
   d. Valid (in *Camestres*).

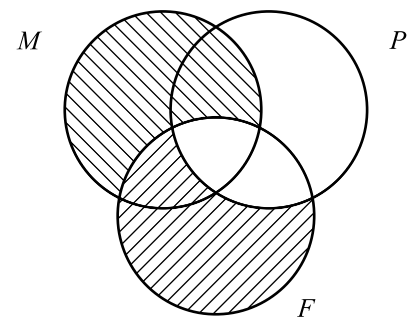


4. a. Unstated premise: All flesh is passive, the plaything of its hormones and of the species, the restless prey of its desires.
   b. Standard-form translation:
      All flesh is passive, the plaything of its hormones and of the species, the restless prey of its desires.
      Man is flesh.

∴ Man is passive, the plaything of his hormones and of the species, the restless prey of his desires.

c. First-order enthymeme.

d. Valid (in *Barbara*).



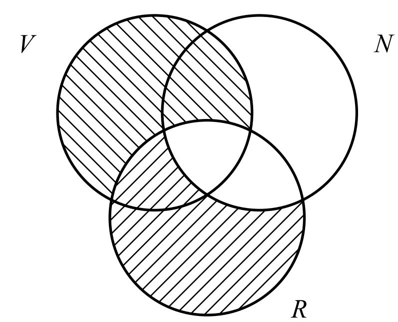
5. a. Unstated conclusion: Those persons who are vicious competitors you do not hate.

b. Standard-form translation:

All persons whom you respect are persons whom you do not hate.
All persons who are vicious competitors are persons whom you respect.
∴ All persons who are vicious competitors are persons whom you do not hate.

c. Third-order enthymeme.

d. Valid (in *Barbara*).



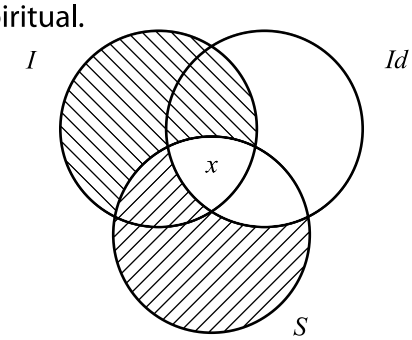
6. a. Unstated premise: All persons who believe that all that exists is spiritual are idealists.

b. Standard-form translation:

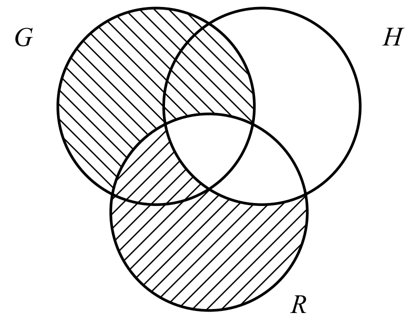
All persons who believe that all that exists is spiritual are idealists.
I am a person who believes that all that exists is spiritual.
∴ I am an idealist.

c. First-order enthymeme.

d. Valid (in *Barbara* or *Darii*).

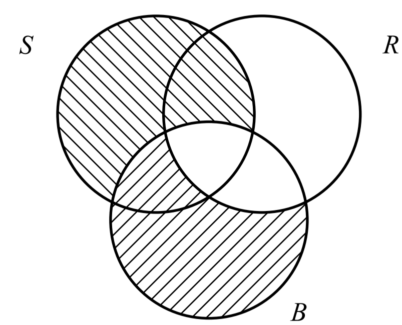


7. a. Unstated premise: The gods are beings that reason.
   b. Standard-form translation:
      All beings that reason are beings that have a human figure.
      All gods are beings that reason.
      ∴ All gods are beings that have a human figure.
   c. Second-order enthymeme.
   d. Valid (in *Barbara*).

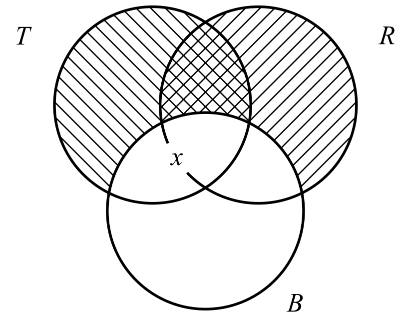


NOTE: This argument of the "anthropomorphite" (the argument ascribed by Hume to Epicurus) is an enthymeme whose generally accepted minor premise—that the gods reason—is here plausibly taken for granted. But of course the leap (from the claim that no man had ever seen reason but in a human figure) to the major premise (that all reasoning beings do have a human figure) is seriously problematic. With that supposition made, the enthymeme is valid.

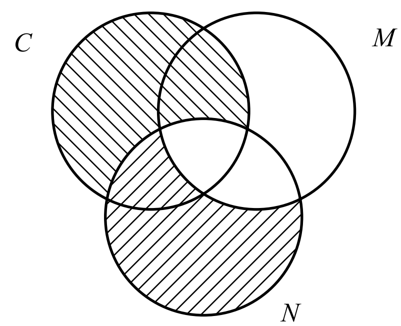
8. a. Unstated premise: Countries for which history often turns out badly tend to remember history especially well.
   b. Standard-form translation:
      All countries for which history often turns out badly are countries that tend to remember history especially well.
      All small countries are countries for which history often turns out badly.
      ∴ All small countries are countries that tend to remember history especially well.
   c. First-order enthymeme.
   d. Valid (in *Barbara*).



9. a. Unstated premise: Fish do not bite after it rains.
   b. Standard-form translation:
      All times following rain are times when fish do not bite.
      This time is a time when fish do not bite.
      ∴ This time is a time following rain.
   c. First-order enthymeme.
   d. Invalid (undistributed middle).

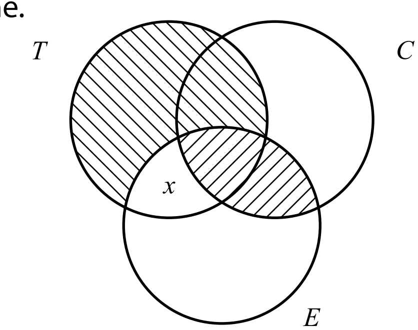


10. a. Unstated premise: All lies, misstatements, and omissions that are not the result of ignorance are the result of malevolence.
   b. Standard-form translation:
      All lies, misstatements, and omissions that are not the result of ignorance are lies, misstatements, and omissions that are the result of malevolence.
      All lies, misstatements, and omissions in Carter's book are lies, misstatements, and omissions that are not the result of ignorance.
      ∴ All lies, misstatements, and omissions in Carter's book are lies, misstatements, and omissions that are the result of malevolence.
   c. First-order enthymeme.
   d. Valid (in *Barbara*).



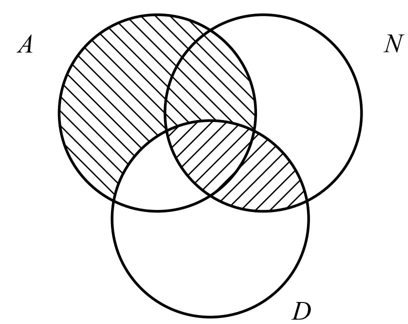
NOTE: The author here intends to present a valid disjunctive syllogism in the form of an enthymeme. The assumed disjunctive premise is disputable, of course.

11. a. Unstated premise: This argument is an enthymeme.
   b. Standard-form translation:
      No enthymemes are complete arguments.
      This argument is an enthymeme.
      ∴ This argument is not a complete argument.
   c. Second-order enthymeme.
   d. Valid (in *Celarent* or *Ferio*).

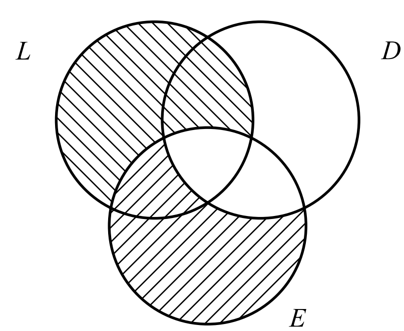


12. a. Unstated premise: Only those have free-speech rights who have need of them.
b. Standard-form syllogism:
All persons who need free-speech rights are persons who have free-speech rights.
No persons who are members of the white majority are persons who need free-speech rights.
∴ All persons who have free-speech rights are members of victimized minorities.
c. First-order enthymeme.
d. Invalid (four terms). In the conclusion is buried the tacit assumption that all those who are not members of the white majority are members of victimized minorities (that "victimized minorities" and "the white majority" are complementary classes, which is dubious).

13. a. Unstated conclusion: Nothing in Nature should be able to make you abandon the theory of the Creation.
b. Standard-form translation:
No demonstrative proofs that there was no Creation are things that exist in Nature.
All things that should be able to make you abandon the theory of the Creation are demonstrative proofs that there was no Creation.
∴ No things that should be able to make you abandon the theory of the Creation are things that exist in Nature.
c. Third-order enthymeme.
d. Valid (in *Celarent*).



14. a. Unstated premise: Weapons that make it easier for a nuclear war to begin are probably the most dangerous.
b. Standard-form translation:
All weapons that make it easier for a nuclear war to begin are weapons that are probably the most dangerous.
The least destructive nuclear weapons are weapons that make it easier for a nuclear war to begin.
∴ The least destructive nuclear weapons are weapons that are probably the most dangerous.
c. First-order enthymeme.
d. Valid (in *Barbara*).



15. a. Unstated premise: Species that tend to increase at a greater rate than their means of subsistence are occasionally subject to a severe struggle for existence.

b. Standard-form translation:

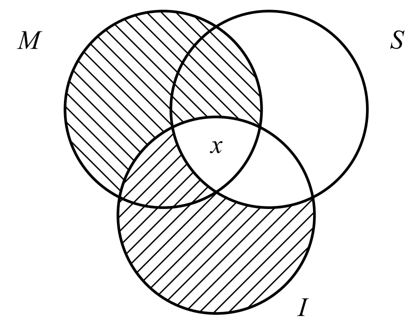
All species that tend to increase at a greater rate than their means of subsistence are species that are occasionally subject to a severe struggle for existence.

Man is a species that tends to increase at a greater rate than his means of subsistence.

∴ Man is a species that is occasionally subject to a severe struggle for existence.

c. First-order enthymeme.

d. Valid (in *Barbara* or *Darii*).



16. a. Unstated conclusion.

b. Standard-form syllogism:

No internal combustion engines are pollution-free devices.

No internal combustion engines are completely efficient machines.

[**EE_–3**: Both premises are negative, so no syllogistic conclusion follows validly from them.]

c. Third-order enthymeme.

d. Invalid regardless of context.

17. a. Unstated conclusion: A nation without a conscience cannot live.

b. Standard-form translation:

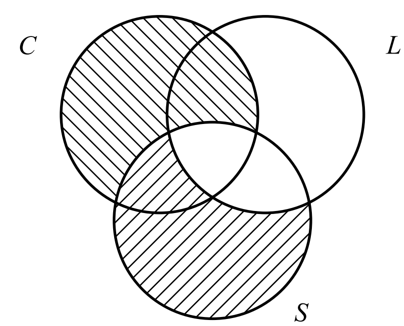
All nations without souls are nations that cannot live.

All nations without consciences are nations without souls.

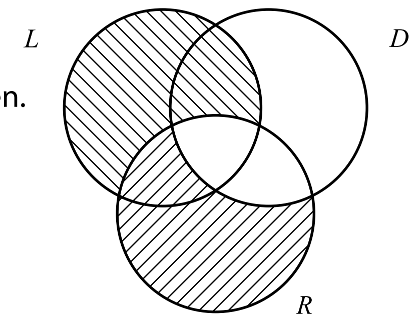
∴ All nations without consciences are nations that cannot live.

c. Third-order enthymeme.

d. Valid (in *Barbara*).

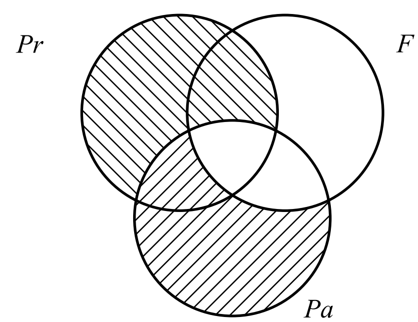


18. a. Unstated premise: Most men dread responsibility.
  b. Standard-form translation:
   All responsibilities are things dreaded by most men.
   All liberties are responsibilities.
   ∴ All liberties are things dreaded by most men.
  c. First-order enthymeme.
  d. Valid (in *Barbara*).

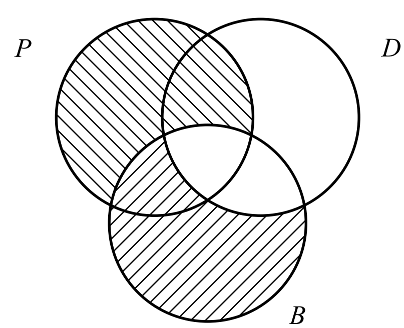


NOTE: If this passage is an argument then it is a valid enthymeme. However, it may not express an argument at all, but rather an explanation of why men dread liberty. (See Section 1.4.)

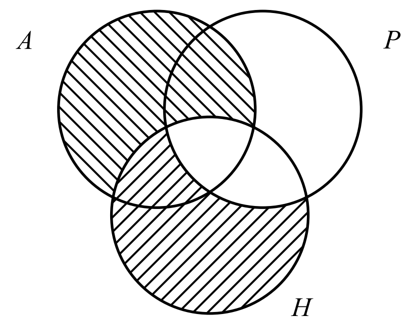
19. a. Unstated conclusion: Those who control the present control the future.
  b. Standard-form translation:
   All persons who control the past are persons who control the future.
   All persons who control the present are persons who control the past.
   ∴ All persons who control the present are persons who control the future.
  c. Third-order enthymeme.
  d. Valid (in *Barbara*).



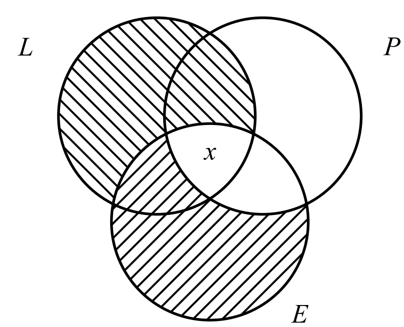
20. a. Unstated premise: All that betters the condition of the vast majority of the people is desirable.
  b. Standard-form translation:
   All things that better the condition of the vast majority of the people are things that are desirable.
   All productivity is a thing that betters the condition of the vast majority of the people.
   ∴ All productivity is a thing that is desirable.
  c. First-order enthymeme.
  d. Valid (in *Barbara*).



21. a. Unstated premise: All that helps to bring buyers and sellers together performs a vital function in almost any society.
 b. Standard-form translation:
 All things that help to bring buyers and sellers together perform a vital function in almost any society.
 All advertisements are things that help to bring buyers and sellers together.
 ∴ All advertisements are things that perform a vital function in almost any society.
 c. First-order enthymeme.
 d. Valid (in *Barbara*).



22. a. Unstated premise: All that is empirically founded and experimentally applied is a matter of profound human importance.
 b. Standard-form translation:
 All things that are empirically founded and experimentally applied are matters of profound human importance.
 Logic is a thing that is empirically founded and experimentally applied.
 ∴ Logic is a matter of profound human importance.
 c. First-order enthymeme.
 d. Valid (in *Barbara* or *Darii*).



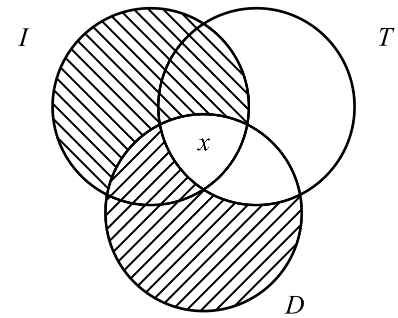
23. a. Unstated premise: Anything that demonstrates inexorably how human character, with its itch to be admired, combines with the malice of heaven to produce wars which no one in his right mind would want and which turn out to be utterly disastrous for everybody, is a tragedy.
 b. Standard-form translation:
 All things that demonstrate inexorably how human character, with its itch to be admired, combines with the malice of heaven to produce wars which no one in his right mind would want and which turn out to be utterly disastrous for everybody, are tragedies.

Iphigeneia at Aulis demonstrates inexorably how human character, with its itch to be admired, combines with the malice of heaven to produce wars which no one in his right mind would want and which turn out to be utterly disastrous for everybody.

∴ *Iphigeneia at Aulis* is a tragedy.

c. First-order enthymeme.

d. Valid (in *Barbara* or *Darii*).



24. a. Unstated conclusion: Suicide is forbidden by the law.

b. Standard-form translation:

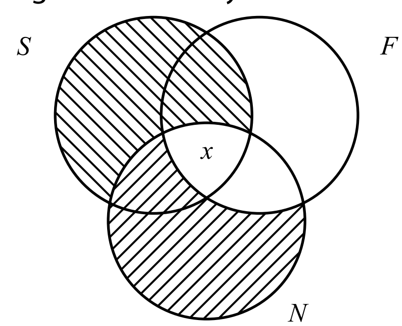
All things not expressly permitted by the law are things forbidden by the law.

Suicide is a thing not expressly permitted by the law.

∴ Suicide is a thing forbidden by the law.

c. Third-order enthymeme.

d. Valid (in *Barbara* or *Darii*).



25. a. Unstated premise: The man who says that all things come to pass by necessity cannot criticize those who, by his own admission, do what they do by necessity.

b. Standard-form translation:

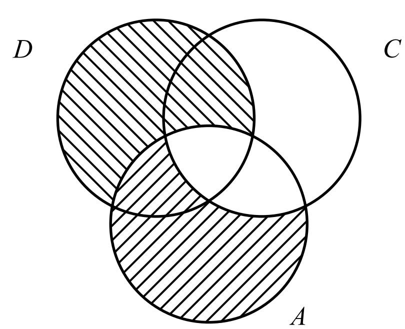
All people who are admitted to do what they do by necessity by the man who says that all things come to pass by necessity are people who cannot be criticized by the man who says that all things come to pass by necessity.

All people who deny that all things come to pass by necessity are people who are admitted to do what they do by necessity by the man who says that all things come to pass by necessity.

∴ All people who deny that all things come to pass by necessity are people who cannot be criticized by the man who says that all things come to pass by necessity.

c. First-order enthymeme.

d. Valid (in *Barbara*).

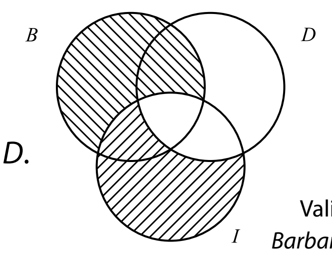


Section 7.6 – A
Execises on pages 270–272

1. (1) Babies are illogical.
   (2) Nobody is despised who can manage a crocodile.
   (3) Illogical persons are despised.
   Therefore babies cannot manage crocodiles.

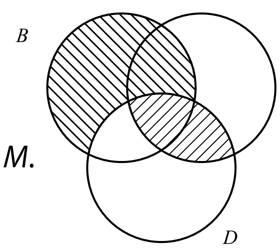
   Solution
   Standard-form translation:
   (1′) All babies are illogical persons.
   (3′) All illogical persons are despised persons.
   (2′) No persons who can manage crocodiles are despised persons.
   Therefore, no babies are persons who can manage crocodiles.
   This sorites consists of two syllogisms, as follows:

All *I* is *D*.
All *B* is *I*.
Therefore all *B* is *D*.



Valid
Barbara

No *M* is *D*.
All *B* is *D*.
Therefore no *B* is *M*.

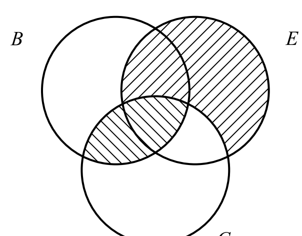


Valid
Cesare

The sorites is valid.

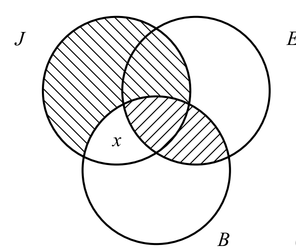
2. (1′) All experienced persons are competent persons.
   (3′) No competent persons are persons who are always blundering.
   (2′) Jenkins is a person who is always blundering.
   ∴ Jenkins is not an experienced person.

All *E* is *C*.
No *C* is *B*.
∴ No *B* is *E*.



Valid
Camenes

No *B* is *E*.
J is *B*.
∴ *J* is not *E*.

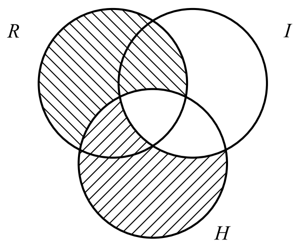


Valid
Celarent
or *Ferio*

The sorites is valid.

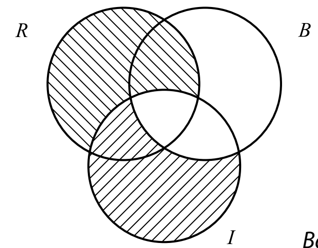
3. (3′) All romances in this library are books in this library that are healthy in tone.
(1′) All books in this library that are healthy in tone are books in this library that I recommend for reading.
(4′) All books in this library that I recommend for reading are bound books in this library.
(2′) All bound books in this library are well-written books.
∴ All romances in this library are well-written books.

All *H* is *I*.
All *R* is *H*.
∴ All *R* is *I*.



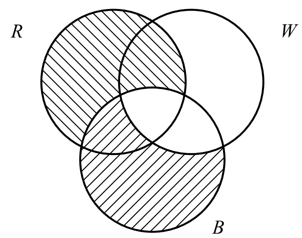
Valid
Barbara

All *I* is *B*.
All *R* is *I*.
∴ All *R* is *B*.



Valid
Barbara

All *B* is *W*.
All *R* is *B*.
∴ All *R* is *W*.

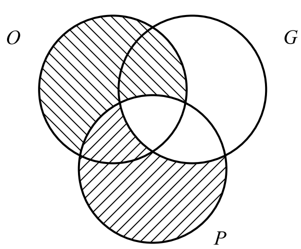


Valid
Barbara

The sorites is valid.

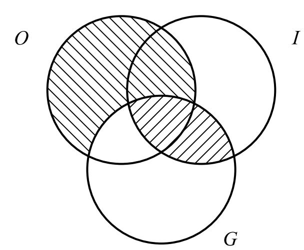
4. (1′) All Oxford dons are profound scholars.
(4′) All profound scholars are great lovers of music.
(2) No insensitive souls are great lovers of music.
(3′) No insensitive souls are Don Juans.
∴ All Oxford dons are Don Juans.

All *P* is *G*.
All *O* is *P*.
∴ All *O* is *G*.



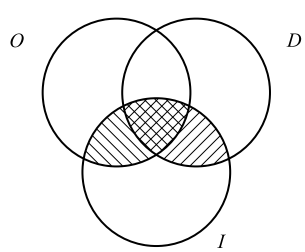
Valid
Barbara

No *I* is *G*.
All *O* is *G*.
∴ No *O* is *I*.



Valid
Cesare

No *I* is *D*.
No *O* is *I*.
∴ All *O* is *D*.

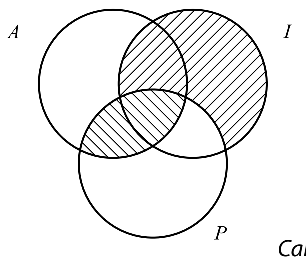


Invalid

The sorites is invalid.

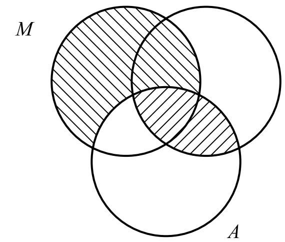
5. (1′) All interesting poems are poems that are popular among people of real taste.
(4′) No affected poems are poems that are popular among people of real taste.
(2′) All modern poems are affected poems.
(5′) All poems on the subject of soap bubbles are modern poems.
(3′) All poems of yours are poems on the subject of soap bubbles.
∴ No poems of yours are interesting poems.

All *I* is *P*.
No *A* is *P*.
∴ All *A* is *I*.



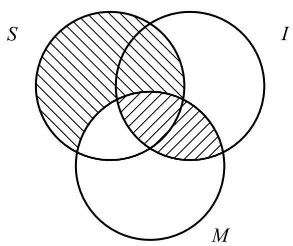
Valid
Camestres

No *A* is *I*.
All *M* is *A*.
∴ No *M* is *I*.



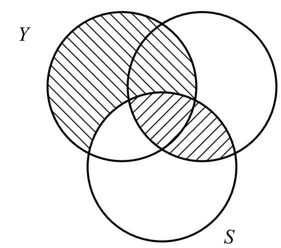
Valid
Celarent

No *M* is *I*.
All *S* is *M*.
∴ No *S* is *I*.



Valid
Celarent

No *S* is *I*.
All *Y* is *S*.
∴ No *Y* is *I*.

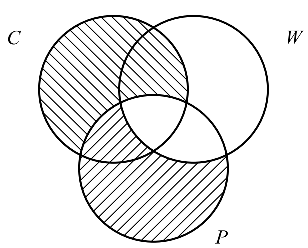


Valid
Celarent

The sorites is valid.

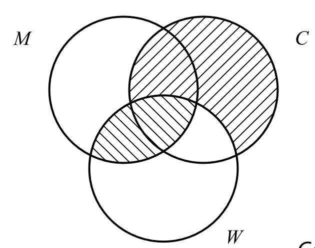
6. (3′) All contributors to the new magazine are poets.
(1′) All poets are writers.
(4′) No military officers are writers.
(2′) All astronauts are military officers.
∴ No astronauts are contributors to the new magazine.

All *P* is *W*.
All *C* is *P*.
∴ All *C* is *W*.



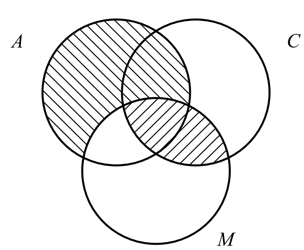
Valid
Barbara

All *C* is *W*.
No *M* is *W*.
∴ No *M* is *C*.



Valid
Camestres

No *M* is *C*.
All *A* is *M*.
∴ No *A* is *C*.



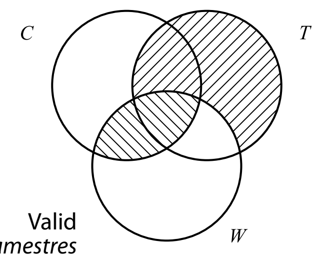
Valid
Celarent

The sorites is valid.

Section 7.6 – B
Exercise on page 272

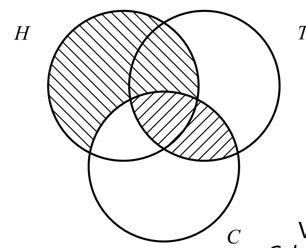
1. (1′) All those who read *The Times* are those who are well educated.
   (3′) No creatures who cannot read are those who are well educated.
   (2′) All hedgehogs are creatures who cannot read.
   ∴ No hedgehogs are those who read *The Times*.

All *T* is *W*.
No *C* is *W*.
∴ No *C* is *T*.



Valid
Camestres

No *C* is *T*.
All *H* is *C*.
∴ No *H* is *T*.

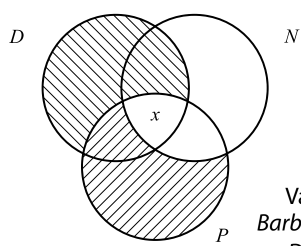


Valid
Celarent

The sorites is valid.

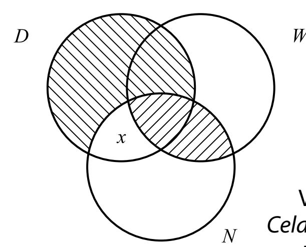
2. (2) This dish is a pudding.
   (1′) All puddings are nice things.
   (3′) No nice things are wholesome things.
   ∴ This dish is not a wholesome thing.

All *P* is *N*.
D is *P*.
∴ *D* is *N*.



Valid
Barbara
or *Darii*

No *N* is *W*.
D is *N*.
∴ *D* is not *W*.

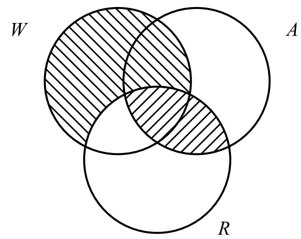


Valid
Celarent
or *Ferio*

The sorites is valid.

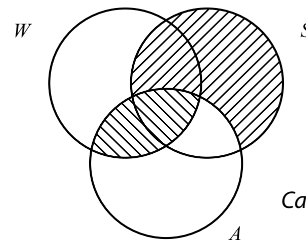
3. (3′) All wedding cakes are very rich articles of food.
   (1′) No articles of food allowed me by my doctor are very rich articles of food.
   (4′) All articles of food that are suitable for supper are articles of food allowed me by my doctor.
   (2′) All articles of food that agree with me are articles of food that are suitable for supper.
   ∴ No wedding cakes are articles of food that agree with me.

No *A* is *R*.
All *W* is *R*.
∴ No *W* is *A*.



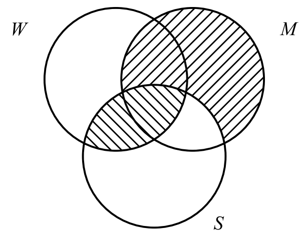
Valid
Cesare

All *S* is *A*.
No *W* is *A*.
∴ No *W* is *S*.



Valid
Camestres

All *M* is *S*.
No *W* is *S*.
∴ No *W* is *M*.

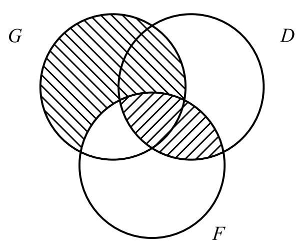


Valid
Camestres

The sorites is valid.

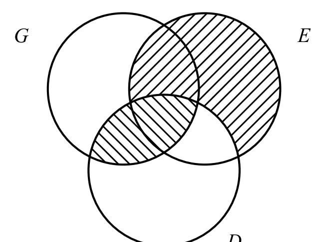
4. (3′) All gluttons who are children of mine are fat persons.
(1′) No daughters of mine are fat persons.
(4′) All children of mine who take exercise are daughters of mine.
(2′) All children of mine who are healthy are children of mine who take exercise.
∴ No children of mine who are healthy are gluttons.

No *D* is *F*.
All *G* is *F*.
∴ No *G* is *D*.



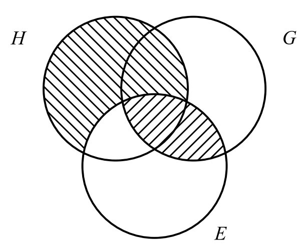
Valid
Cesare

All *E* is *D*.
No *G* is *D*.
∴ No *G* is *E*.



Valid
Camestres

No *G* is *E*.
All *H* is *E*.
∴ No *H* is *G*.



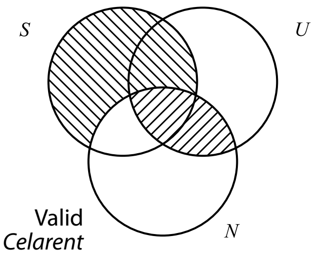
Valid
Cesare

The sorites is valid.

5. (2′) These sorites are examples not arranged in regular order, like the examples I am used to.
(4′) No examples not arranged in regular order, like the examples I am used to, are examples I can understand.

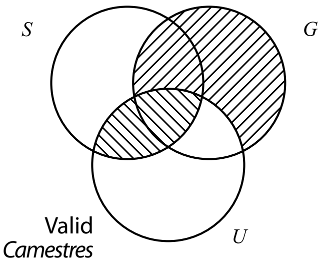
(1′) All examples I do not grumble at are examples I can understand.
(5′) All examples that do not give me a headache are examples I do not grumble at.
(3′) All easy examples are examples that do not give me a headache.
∴ These sorites are not easy examples.

No *N* is *U*.
All *S* is *N*.
∴ No *S* is *U*.



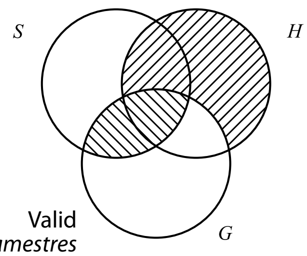
Valid
Celarent

All *G* is *U*.
No *S* is *U*.
∴ No *S* is *G*.



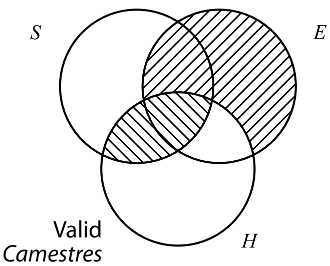
Valid
Camestres

All *H* is *G*.
No *S* is *G*.
∴ No *S* is *H*.



Valid
Camestres

All *E* is *H*.
No *S* is *H*.
∴ No *S* is *E*.



Valid
Camestres

The sorites is valid.

Section 7.7
Exercises on pages 276–278

1. Pure hypothetical syllogism. Valid.
2. Disjunctive syllogism. Valid.
3. Mixed hypothetical syllogism. Invalid (fallacy of Denying the Antecedent).
4. Pure hypothetical syllogism. Invalid.
5. Mixed hypothetical syllogism. ***Modus Ponens***. Valid.
6. Mixed hypothetical syllogism. Invalid (fallacy of Affirming the Consequent).
7. Disjunctive syllogism (but only in the general sense of containing a disjunction). Invalid (the second premise affirms one of the disjuncts instead of denying it).
8. Pure hypothetical syllogism. Valid.
9. Mixed hypothetical syllogism. ***Modus Tollens***. Valid.
10. Disjunctive syllogism. Valid.
11. Disjunctive syllogism. Valid.
12. Mixed hypothetical syllogism. ***Modus Tollens***. Valid.
13. Disjunctive syllogism. Valid.

14. Mixed hypothetical syllogism. ***Modus Tollens***. Valid.
15. Mixed hypothetical syllogism. ***Modus Tollens***. Valid.
16. Mixed hypothetical syllogism. ***Modus Tollens***. Valid.
17. Mixed hypothetical syllogism. ***Modus Tollens***. Valid.
18. Pure hypothetical syllogism. Valid.
19. Mixed hypothetical syllogism. ***Modus Tollens***. Valid.
20. Mixed hypothetical syllogism. ***Modus Tollens***. Valid. (NOTE: Due to the position of the "if," at first glance the antecedent of the first premise is "everyone follows total pacifism," but the thrust of the premise is that total pacifism might be a good principle, *but only* if everyone were to follow it. Therefore the antecedent is in fact "total pacifism is a good principle.")

Section 7.8
Exercises on pages 282–285

1. It is impossible to go between the horns. It is possible to grasp it by either horn, arguing either (a) that liberties do not properly include the right to publish false and harmful doctrines or (b) that we run no risk of losing our own liberties if we vigorously oppose false and harmful doctrines with true and helpful ones. And it could plausibly be rebutted (but not refuted) by the use of its ingredients to construct a counterdilemma: "we must either be guiltless of suppressing the liberties of others or else run no risk of losing our own liberties."

2. Perhaps possible to go between the horns in that we may partially understand something—at least enough for it to be intelligible without it being already understood. Focusing on the different possible meanings of "understand" also suggests plausible ways to grasp the dilemma by either horn. Rebuttal not plausible here, for the conclusion *whatever you say either enlarges my understanding or else is intelligible to me* is not particularly attractive.

3. The key to refuting this dilemma lies in exposing the ambiguity of the key phrase "going beyond," which could mean "going logically beyond to what is not implied" or "going psychologically beyond to what is not suggested." When this is done, it permits grasping it by one horn or the other, depending upon which sense of "going beyond" is intended. A plausible but nonrefuting rebuttal can be constructed here.

4. Very easy to go between the horns here (since the preceding dilemma is so easily refuted). Plausible to grasp by the second horn, since a deductive argument that establishes a familiar conclusion may well be of some value ("bringing nothing new to light" is again an ambiguous phrase). No rebuttal can be made of the original dilemma's ingredients, but other rebutting counterdilemmas can be thought of easily.

5. Very easy to go between the horns here. Plausible to grasp by either horn. A nonrefuting rebuttal can be made here, but it is not very plausible.

6. Very easy to go between the horns here. Grasping by the horns may or may not be plausible—a deliberate point, perhaps. No rebuttal using the original dilemma's ingredients, but other rebutting counterdilemmas can be thought of easily.

7. Perhaps possible to go between the horns in that going to war and stopping soon after U.N. action and threat of intervention may be a third and distinct possibility. Very plausible to grasp by either horn (the second if we note that "unsuccessful in its purpose of preventing war" in the premise has become simply "unsuccessful" in the conclusion since the U.N. may be necessary for purposes other than keeping peace and may be successful in realizing those other purposes. The usual plausible, but nonrefuting, rebuttal can be constructed out of the original dilemma's ingredients.

8. It is very easy to go between the horns here, because people lie on a continuum of virtue stretching from saints to sinners. It can plausibly be grasped by the second horn, arguing that even very bad people may be deterred from wrongdoing by strictly enforced laws. A plausible but nonrefuting rebuttal can be constructed here out of the ingredients of the given dilemma.

9. Perhaps possible to go between the horns if there is or can be a mode of living somewhere between extravagant and modest. Plausible to grasp by either horn: one who lives extravagantly has no money left to contribute; one who lives modestly does so because he has no money for either extravagances or contributions. The usual plausible but nonrefuting rebuttal can be constructed out of the original dilemma's ingredients.

10. Impossible to go between the horns. It is plausible to grasp it by either horn, arguing either (a) that when desiring to preserve we may be motivated simply by inertia and seek to rest in the status quo, even while admitting that a change would not be worse and might even be better—but just "not worth the trouble of changing" or (b) that when desiring to change we may be motivated simply by boredom with the status quo, and seek a change even while admitting that a change might not be better and might even be worse—but "let's have a little variety." These are psychological rather than political or moral considerations, but the original dilemma appears to be itself psychological. The usual rebutting counterdilemma could be used here: when desiring to preserve, we do not wish to bring about something better; when desiring to change, we do not wish to prevent a change to the worse. It is a question, however, how plausible this is.

11. Easy to go between the horns here: a thing moves *from* the place where it is to a place where it is not. It is plausible to grasp by the first horn by observing that a rotating object moves in the place where it is *while* remaining therein. One might grasp the second horn by rejecting (with Whitehead) simple location as a fallacy, and maintaining that everything is everywhere, in the sense of influencing what happens there. It is hard to see what kind of rebuttal might be available here.

12. Plausible to go between the horns here and either request or entreat the young men not to flock to him, or (somehow) arrange with other elders to keep them away. One might grasp the first horn and argue that young men do not have enough influence on their elders to cause them to drive someone out of the city "on request." It is not so easy to grasp the second horn because that is what happened in Athens itself. The usual plausible but nonrefuting rebuttal can be constructed out of the original dilemma's ingredients.

13. Plausible to go between the horns here and say the Socrates' dying was the boundary between the time he was alive and the time he was dead. Perhaps one might grasp the first horn and insist that when the act of dying was finished the living was finished too. It is hard to construct any plausible rebuttal here.

14. It is certainly possible to go between the horns here. A physician may "communicate" a good deal of reassurance without either telling the truth that the placebo administered is without pharmaceutical value or lying and claiming that it is pharmaceutically effective. One may plausibly grasp one horn and insist it would be telling the truth to say that placebo has established "medical" value, which it may have in the broad sense of that term. A rebuttal may well be used here: if the physician "tells the truth" he will build trust, and if he doesn't he will cure the patient.

15. There were in theory a number of ways to go between the horns here: Between defiance and obedience to the Court decision there are many degrees of partial compliance that fall short of full obedience but do not constitute outright defiance. Either horn could be grasped, at least in theory: An emergency situation in the international sphere might prevent defiance from being followed by impeachment; and it is logically possible that the evidence produced by obedience to the order might not have been sufficient to persuade the Congress to impeach. A nonrefuting rebuttal is barely possible, but not very persuasive or helpful. (If he defied the order he would not "be impeached on the evidence," and if he obeyed the order he would not be impeached for defiance—but he would still have been impeached for one or the other.)

16. Impossible to go between the horns. But either horn may plausibly be grasped. The claim that having peace requires that the competitive spirit not be encouraged may be contested; that spirit, it could be argued, results in the productivity that alone can yield the contentment that peace requires. Or the claim that progress requires the encouragement of the competitive spirit may be contested; cooperation in place of competition may produce progress of a more lasting and more satisfying kind.

17. Impossible to between the horns. The horns may easily be grasped. Indeed, the Southern Secession and the American Civil War show that political realities are not as easily mapped as Madison believed.

18. This argument, as stated, is not persuasive. The leap to the explicit conclusion is clearly wild, being an *ignoratio elenchi*. But there is a highly enthymematic dilemma present, with only its conjunctive premise stated. The suppressed disjunctive premise may be understood to assert of every thing that a man either knows it or does not know it. And the understood conclusion asserts of every thing that either a man has no need to enquire about it or cannot enquire about it. It is perhaps possible to go between the horns in that we may have partial knowledge of a thing: enough to know the subject but not enough to have no need to enquire further. This suggests plausible ways to grasp the dilemma by either horn.

19. It is easy to go between the horns here. On the continuum of possible salaries, there is surely a range (though it might be narrow) of salaries that are neither too high nor too low. And either horn may be grasped, though with different degrees of plausibility. If "too high" a salary is asked for, employers may see that the job or the applicant is worth more than they first thought. And if "too low" a salary is asked for, the applicant may also express a willingness to work at that low salary with a conviction that the employer is likely soon to recognize that a higher salary is deserved. The employer may also see hiring the applicant as a bargain.

20. This is a rather informal version of Pascal's argument, which has been much discussed for more than three hundred years. If it is interpreted as having the disjunctive premise that either God exists or God does not exist, then it is obviously impossible to go between the horns. But each of the horns can be grasped to refute the given argument. It might be argued that if you live a life of conspicuous virtue even though you are not a believer, you will be condemned to spend eternity in the flames of Hell. (Of course, it might be argued that you could not be virtuous without being a believer, but this is another argument.) Or it might be argued that if you live as believer you will suffer the loss of all those earthly pleasures that you might otherwise have enjoyed, and that that is a very grave penalty indeed.

    Of this argument William James wrote in his essay, "The Will to Believe":

    > You probably feel that when religious faith expresses itself thus, in the language of the gamin-table, it is put to its last trumps. Surely Pascal's own personal belief in masses and holy water had far other springs: and this celebrated page of his is but an argument for others, a last desperate snatch at a weapon against the hardness of the unbelieving heart. We feel that a faith in masses and holy water adopted willfully after such a mechanical calculation would lack the inner soul of faith's reality; and if we were ourselves in the place of the Deity, we should probably take particular pleasure in cutting off believers of this pattern from their infinite reward.

Chapter 8

Section 8.2–A
Exercises on pages 297–299

1. T
2. T
3. F
4. F
5. T
6. T
7. F
8. T
9. F
10. T
11. F
12. T
13. F
14. F
15. F
16. T
17. T
18. F
19. T
20. T
21. F
22. T
23. F
24. T
25. F

Section 8.2–B
Exercises on page 299

1. T
2. F
3. T
4. T
5. F
6. T
7. T
8. F
9. T
10. T
11. T
12. F
13. F
14. F
15. T
16. T
17. F
18. F
19. T
20. F
21. F
22. T
23. F
24. T
25. F

Section 8.2–C

Exercises on pages 299–300

1. $I \bullet \sim L$
2. $I \vee L$
3. $I \vee L$
4. $\sim(I \bullet L)$
5. $\sim I \bullet \sim L$
6. $(I \vee L) \bullet \sim(I \bullet L)$
7. $S \bullet (I \vee J)$
8. $(S \bullet I) \vee J$
9. $\sim E \bullet J$
10. $\sim(E \vee J)$
11. $\sim E \vee J$
12. $\sim(E \bullet J)$
13. $J \vee S$
14. $E \vee L$
15. $\sim I \vee L$
16. $(I \bullet L) \vee (\sim I \bullet \sim L)$
17. $L \bullet E$
18. $\sim(\sim I \bullet \sim L)$
19. $(E \bullet J) \vee (\sim I \bullet \sim L)$
20. $(I \bullet E) \vee \sim(J \bullet S)$
21. $(E \bullet S) \vee (J \vee L)$
22. $S \bullet [J \vee (L \bullet I)]$
23. $(E \vee J) \bullet (\sim L \bullet \sim I)$
24. $E \bullet (S \bullet L)$
25. $(L \bullet E) \bullet (S \bullet J)$

Section 8.3–A

Exercises on page 308

1. T
2. F
3. F
4. T
5. F
6. F
7. T
8. T
9. F
10. T
11. F
12. F
13. T
14. T
15. F
16. T
17. F
18. F
19. T
20. F
21. T
22. F
23. F
24. F
25. T

Section 8.3–B
Exercises on pages 308–310

1. $A \supset (B \supset C)$
2. $A \supset (B \vee C)$
3. $A \supset (B \bullet C)$
4. $(A \supset B) \bullet C$
5. $(A \bullet B) \supset C$
6. $(A \vee B) \supset C$
7. $A \vee (B \supset C)$
8. $\sim A \supset (\sim B \vee \sim C)$
9. $\sim A \supset (\sim B \bullet \sim C)$
10. $\sim[A \supset (B \bullet C)]$
11. $(\sim A \supset \sim B) \bullet C$
12. $A \supset B$
13. $B \supset A$
14. $C \supset (A \bullet B)$
15. $B \supset (A \vee C)$
16. $(B \vee C) \supset A$
17. $B \vee C$
18. $A \supset (B \vee C)$
19. $\sim B \vee A$
20. $B \vee C$
21. $A \supset B$
22. $C \supset A$
23. $(A \bullet B) \supset (C \bullet D)$
24. $(A \bullet B) \supset (C \vee D)$
25. $(\sim C \bullet \sim D) \supset (\sim B \vee A)$

Section 8.4
Exercises on pages 312–313

a. 3 is the specific form of a.
b. 6 is the specific form of b.
c. 4 is the specific form of c.
d. 9 is the specific form of d.
e. 10 is the specific form of e.
f. 16 is the specific form of f.
g. 8 is the specific form of g.
h. 11 is the specific form of h.
i. 12 is the specific form of i.
j. 23 is the specific form of j. Also j is a substitution instance of 6.
k. 4 has k as a substitution instance.
l. 5 has l as a substitution instance.
m. m is a substitution instance of 3 and 4.
n. 8 has n as a substitution instance, and 21 is the specific form of n.
o. 3 has o as a substitution instance, and 24 is the specific form of o.

Section 8.7–A
Exercises on page 322 (referring to 8.4)

1.

p	q	$p \supset q$	$\sim q$	$\sim p$	$\sim q \supset \sim p$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T
Valid					

2.

p	q	$\sim p$	$\sim q$	$p \supset q$	$\sim p \supset \sim q$
T	T	F	F	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T
Invalid—shown by row 3					

3.

p	q	$p \bullet q$
T	T	T
T	F	F
F	T	F
F	F	F
Valid		

4.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F
Valid		

5.

p	q	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

Invalid—shown by row 2

6.

p	q	$p \bullet q$	$p \supset q$	$p \supset (p \bullet q)$
T	T	T	T	T
T	F	F	F	F
F	T	F	T	T
F	F	F	T	T

Valid

7.

p	q	$p \bullet q$	$p \vee q$	$p \supset q$	$q \supset p$	$(p \vee q) \supset (p \bullet q)$	$(p \supset q) \bullet (q \supset p)$
T	T	T	T	T	T	T	T
T	F	F	T	F	T	F	F
F	T	F	T	T	F	F	F
F	F	F	F	T	T	T	T

Valid

8.

p	q	$\sim p$	$\sim q$	$p \supset q$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Invalid—shown by row 3

9.

p	q	$\sim p$	$\sim q$	$p \supset q$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Valid

10.

p	q	$p \bullet q$
T	T	T
T	F	F
F	T	F
F	F	F
Valid		

11.

p	q	r	$p \supset q$	$p \supset r$	$q \vee r$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	F
Invalid—shown by row 8					

12.

p	q	r	$p \supset q$	$q \supset r$	$r \supset p$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	F
F	T	F	T	F	T
F	F	T	T	T	F
F	F	F	T	T	T
Invalid—shown by row 5 or 7					

13.

p	q	r	$q \supset r$	$p \supset (q \supset r)$	$p \supset q$	$p \supset r$
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	T	T	F	T
T	F	F	T	T	F	F
F	T	T	T	T	T	T
F	T	F	F	T	T	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

Valid

14.

p	q	r	$q \bullet r$	$p \supset (q \bullet r)$	$q \vee r$	$\sim p$	$(q \vee r) \supset \sim p$
T	T	T	T	T	T	F	F
T	T	F	F	F	T	F	F
T	F	T	F	F	T	F	F
T	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	F	T	T

Valid

15.

p	q	r	$q \supset r$	$p \supset (q \supset r)$	$p \supset r$	$q \supset (p \supset r)$	$p \vee q$	$(p \vee q) \supset r$
T	T	T	T	T	T	T	T	T
T	T	F	F	F	F	F	T	F
T	F	T	T	T	T	T	T	T
T	F	F	T	T	F	T	T	F
F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	F
F	F	T	T	T	T	T	F	T
F	F	F	T	T	T	T	F	T

Invalid (shown by fourth and sixth rows)

16.

p	q	r	s	$p \supset q$	$r \supset s$	$(p \supset q) \bullet (r \supset s)$	$p \vee r$	$q \vee s$
T	T	T	T	T	T	T	T	T
T	T	T	F	T	F	F	T	T
T	T	F	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	T	F	T	F	T	T
T	F	T	F	F	F	F	T	F
T	F	F	T	F	T	F	T	T
T	F	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T	T
F	T	T	F	T	F	F	T	T
F	T	F	T	T	T	T	F	T
F	T	F	F	T	T	T	F	T
F	F	T	T	T	T	T	T	T
F	F	T	F	T	F	F	T	F
F	F	F	T	T	T	T	F	T
F	F	F	F	T	T	T	F	F
Valid								

17.

p	q	r	s	$p \supset q$	$r \supset s$	$(p \supset q) \bullet (r \supset s)$	$\sim q$	$\sim s$	$\sim q \vee \sim s$	$\sim p$	$\sim p \vee \sim s$
T	T	T	T	T	T	T	F	F	F	F	F
T	T	T	F	T	F	F	F	T	T	F	T
T	T	F	T	T	T	T	F	F	F	F	F
T	T	F	F	T	T	T	F	T	T	F	T
T	F	T	T	F	T	F	T	F	T	F	F
T	F	T	F	F	F	F	T	T	T	F	T
T	F	F	T	F	T	F	T	F	T	F	F
T	F	F	F	F	T	F	T	T	T	F	T
F	T	T	T	T	T	T	F	F	F	T	T
F	T	T	F	T	F	F	F	T	T	T	T
F	T	F	T	T	T	T	F	F	F	T	T
F	T	F	F	T	T	T	F	T	T	T	T
F	F	T	T	T	T	T	T	F	T	T	T
F	F	T	F	T	F	F	T	T	T	T	T
F	F	F	T	T	T	T	T	F	T	T	T
F	F	F	F	T	T	T	T	T	T	T	T
Valid											

18.

p	q	r	s	$q \supset r$	$p \supset (q \supset r)$	$r \supset s$	$q \supset (r \supset s)$	$p \supset s$
T	T	T	T	T	T	T	T	T
T	T	T	F	T	T	F	F	F
T	T	F	T	F	F	T	T	T
T	T	F	F	F	F	T	T	F
T	F	T	T	T	T	T	T	T
T	F	T	F	T	T	F	T	F
T	F	F	T	T	T	T	T	T
T	F	F	F	T	T	T	T	F
F	T	T	T	T	T	T	T	T
F	T	T	F	T	T	F	F	T
F	T	F	T	F	T	T	T	T
F	T	F	F	F	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T

Invalid—shown by row 6 or 8

19.

p	q	r	s	$q \supset r$	$p \supset (q \supset r)$	$(q \supset r) \supset s$	$p \supset s$
T	T	T	T	T	T	T	T
T	T	T	F	T	T	F	F
T	T	F	T	F	F	T	T
T	T	F	F	F	F	T	F
T	F	T	T	T	T	T	T
T	F	T	F	T	T	F	F
T	F	F	T	T	T	T	T
T	F	F	F	T	T	F	F
F	T	T	T	T	T	T	T
F	T	T	F	T	T	F	T
F	T	F	T	F	T	T	T
F	T	F	F	F	T	T	T
F	F	T	T	T	T	T	T
F	F	T	F	T	T	F	T
F	F	F	T	T	T	T	T
F	F	F	F	T	T	F	T

Valid

20.

p	q	r	s	$p \bullet q$	$p \supset q$	$(p \bullet q) \supset r$	$r \supset s$	$p \supset (r \supset s)$	$(p \supset q) \bullet [(p \bullet q) \supset r]$	$p \supset s$
T	T	T	T	T	T	T	T	T	T	T
T	T	T	F	T	T	T	F	F	T	F
T	T	F	T	T	T	F	T	T	F	T
T	T	F	F	T	T	F	T	T	F	F
T	F	T	T	F	F	T	T	T	F	T
T	F	T	F	F	F	T	F	F	F	F
T	F	F	T	F	F	T	T	T	F	T
T	F	F	F	F	F	T	T	T	F	F
F	T	T	T	F	T	T	T	T	T	T
F	T	T	F	F	T	T	F	T	T	T
F	T	F	T	F	T	T	T	T	T	T
F	T	F	F	F	T	T	T	T	T	T
F	F	T	T	F	T	T	T	T	T	T
F	F	T	F	F	T	T	F	T	T	T
F	F	F	T	F	T	T	T	T	T	T
F	F	F	F	F	T	T	T	T	T	T
Valid										

21.

p	q	$p \vee q$	$p \bullet q$	$(p \vee q) \supset (p \bullet q)$	$\sim(p \vee q)$	$\sim(p \bullet q)$
T	T	T	T	T	F	F
T	F	T	F	F	F	T
F	T	T	F	F	F	T
F	F	F	F	T	T	T
Valid						

22.

p	q	$p \vee q$	$p \bullet q$	$(p \vee q) \supset (p \bullet q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T
Valid				

23.

p	q	r	s	$p \bullet q$	$r \bullet s$	$(p \bullet q) \supset (r \bullet s)$	$(p \bullet q) \bullet (r \bullet s)$	$(p \bullet q) \supset [(p \bullet q) \bullet (r \bullet s)]$
T	T	T	T	T	T	T	T	T
T	T	T	F	T	F	F	F	F
T	T	F	T	T	F	F	F	F
T	T	F	F	T	F	F	F	F
T	F	T	T	F	T	T	F	T
T	F	T	F	F	F	T	F	T
T	F	F	T	F	F	T	F	T
T	F	F	F	F	F	T	F	T
F	T	T	T	F	T	T	F	T
F	T	T	F	F	F	T	F	T
F	T	F	T	F	F	T	F	T
F	T	F	F	F	F	T	F	T
F	F	T	T	F	T	T	F	T
F	F	T	F	F	F	T	F	T
F	F	F	T	F	F	T	F	T
F	F	F	F	F	F	T	F	T
Valid								

24.

p	q	r	s	$p \supset q$	$r \supset s$	$(p \supset q) \bullet (r \supset s)$
T	T	T	T	T	T	T
T	T	T	F	T	F	F
T	T	F	T	T	T	T
T	T	F	F	T	T	T
T	F	T	T	F	T	F
T	F	T	F	F	F	F
T	F	F	T	F	T	F
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	T	F	T	F	F
F	T	F	T	T	T	T
F	T	F	F	T	T	T
F	F	T	T	T	T	T
F	F	T	F	T	F	F
F	F	F	T	T	T	T
F	F	F	F	T	T	T
Valid						

Section 8.7 – B
Exercises on page 322

1. $(A \vee B) \supset (A \bullet B)$ has the specific form $(p \vee q) \supset (p \bullet q)$
   $A \vee B$ $\qquad$ $p \vee q$
   $\therefore A \bullet B$ $\qquad$ $\therefore p \bullet q$

p	q	$p \vee q$	$p \bullet q$	$(p \vee q) \supset (p \bullet q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T
Valid				

2. $(C \vee D) \supset (C \bullet D)$ has the specific form $(p \vee q) \supset (p \bullet q)$
   $C \bullet D$ $\qquad$ $p \bullet q$
   $\therefore C \vee D$ $\qquad$ $\therefore p \vee q$

p	q	$p \bullet q$	$p \vee q$	$(p \vee q) \supset (p \bullet q)$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	F
F	F	F	F	T
Valid				

3. $E \supset F$ has the specific form $p \supset q$
   $F \supset E$ $\qquad$ $q \supset p$
   $\therefore E \vee F$ $\qquad$ $\therefore p \vee q$

p	q	$p \supset q$	$q \supset p$	$p \vee q$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	F
Invalid, line 4				

4. $(G \vee H) \supset (G \bullet H)$ has the $(p \vee q) \supset (p \bullet q)$
$\sim(G \bullet H)$ specific form $\sim(p \bullet q)$
$\therefore \sim(G \vee H)$ $\therefore \sim(p \vee q)$

p	q	$p \bullet q$	$p \vee q$	$\sim(p \bullet q)$	$\sim(p \vee q)$	$(p \vee q) \supset (p \bullet q)$
T	T	T	T	F	F	T
T	F	F	T	T	F	F
F	T	F	T	T	F	F
F	F	F	F	T	T	T
Valid						

5. $(I \vee J) \supset (I \bullet J)$ has the $(p \vee q) \supset (p \bullet q)$
$\sim(I \vee J)$ specific form $\sim(p \vee q)$
$\therefore \sim(I \bullet J)$ $\therefore \sim(p \bullet q)$

p	q	$p \vee q$	$p \bullet q$	$(p \vee q) \supset (p \bullet q)$	$\sim(p \vee q)$	$\sim(p \bullet q)$
T	T	T	T	T	F	F
T	F	T	F	F	F	T
F	T	T	F	F	F	T
F	F	F	F	T	T	T
Valid (Note: Fallacy of denying the antecedent is not committed here!)						

6. $K \vee L$ has the $p \vee q$
K specific form p
$\therefore \sim L$ $\therefore \sim q$

p	q	$p \vee q$	$\sim q$
T	T	T	F
T	F	T	T
F	T	T	F
F	F	F	T
Invalid, line 1			

7. $M \vee (N \bullet {\sim}N)$ has the $p \vee (q \bullet {\sim}q)$
M specific form p
$\therefore {\sim}(N \bullet {\sim}N)$ $\therefore {\sim}(q \bullet {\sim}q)$

p	q	${\sim}q$	$q \bullet {\sim}q$	$p \vee (q \bullet {\sim}q)$	${\sim}(q \bullet {\sim}q)$
T	T	F	F	T	T
T	F	T	F	T	T
F	T	F	F	F	T
F	F	T	F	F	T
Valid					

8. $(O \vee P) \supset Q$ has the $(p \vee q) \supset$ r
$Q \supset (O \bullet P)$ specific form $r \supset (p \bullet q)$
$\therefore (O \vee P) \supset (O \bullet P)$ $\therefore (p \vee q) \supset (p \bullet q)$

p	q	r	$p \vee q$	$(p \vee q) \supset r$	$p \bullet q$	$r \supset (p \bullet q)$	$(p \vee q) \supset (p \bullet q)$
T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T
T	F	T	T	T	F	F	F
T	F	F	T	F	F	T	F
F	T	T	T	T	F	F	F
F	T	F	T	F	F	T	F
F	F	T	F	T	F	F	T
F	F	F	F	T	F	T	T
Valid							

9. $(R \vee S) \supset T$ has the $(p \vee q) \supset r$
$T \supset (R \bullet S)$ specific form $r \supset (p \bullet q)$
$\therefore (R \bullet S) \supset (R \vee S)$ $\therefore (p \bullet q) \supset (p \vee q)$

p	q	r	$p \vee q$	$(p \vee q) \supset r$	$p \bullet q$	$r \supset (p \bullet q)$	$(p \bullet q) \supset (p \vee q)$
T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T
T	F	T	T	T	F	F	T
T	F	F	T	F	F	T	T
F	T	T	T	T	F	F	T
F	T	F	T	F	F	T	T
F	F	T	F	T	F	F	T
F	F	F	F	T	F	T	T
Valid							

10. $U \supset (V \vee W)$ has the $p \supset (q \vee r)$
$(V \bullet W) \supset {\sim}U$ specific form $(q \bullet r) \supset {\sim}p$
$\therefore {\sim}U$ $\therefore {\sim}p$

p	q	r	$q \vee r$	$p \supset (q \vee r)$	$q \bullet r$	${\sim}p$	$(q \bullet r) \supset {\sim}p$
T	T	T	T	T	T	F	F
T	T	F	T	T	F	F	T
T	F	T	T	T	F	F	T
T	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	F	T	F	T	T
Invalid (shown by second and third rows)							

Section 8.7 – C
Exercises on pages 322–323

1. $A \supset (B \bullet C)$ has the specific form $p \supset (q \bullet r)$
$\sim B$ $\sim q$
$\therefore \sim A$ $\therefore \sim p$

p	q	r	$q \bullet r$	$p \supset (q \bullet r)$	$\sim q$	$\sim p$
T	T	T	T	T	F	F
T	T	F	F	F	F	F
T	F	T	F	F	T	F
T	F	F	F	F	T	F
F	T	T	T	T	F	T
F	T	F	F	T	F	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T
Valid						

2. $D \supset (E \supset F)$ has the specific form $p \supset (q \supset r)$
E q
$\therefore D \supset F$ $\therefore p \supset r$

p	q	r	$q \supset r$	$p \supset (q \supset r)$	$p \supset r$
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	T	T	T
F	T	F	F	T	T
F	F	T	T	T	T
F	F	F	T	T	T
Valid					

3. $G \supset H$ has the $p \supset q$
$G \supset I$ specific form $p \supset r$
$\therefore H \supset I$ $\therefore q \supset r$

p	q	r	$p \supset q$	$p \supset r$	$q \supset r$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	T	T
F	F	F	T	T	T

Invalid, line 6

4. $J \supset (K \vee L)$ has the $p \supset (q \vee r)$
$\sim K$ specific form $\sim q$
$\therefore J \supset L$ $\therefore p \supset r$

p	q	r	$q \vee r$	$p \supset (q \vee r)$	$\sim q$	$p \supset r$
T	T	T	T	T	F	T
T	T	F	T	T	F	F
T	F	T	T	T	T	T
T	F	F	F	F	T	F
F	T	T	T	T	F	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T
F	F	F	F	T	T	T

Valid

5. $M \supset (N \supset O)$ has the specific form $p \supset (q \supset r)$
N q
$\therefore O \supset M$ $\therefore r \supset p$

p	q	r	$q \supset r$	$p \supset (q \supset r)$	$r \supset p$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	F
F	T	F	F	T	T
F	F	T	T	T	F
F	F	F	T	T	T

Invalid (shown by fifth row)

6. $E \supset D$ has the specific form $p \supset q$
$D \supset P$ $q \supset r$
$P \supset {\sim}E$ $r \supset {\sim}p$

$\therefore {\sim}E$ $\therefore {\sim}p$

p	q	r	$p \supset q$	$q \supset r$	${\sim}p$	$r \supset {\sim}p$
T	T	T	T	T	F	F
T	T	F	T	F	F	T
T	F	T	F	T	F	F
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

Valid

7. $T \supset L$ / $\sim T \supset I$ / $\therefore L \vee I$ has the specific form $p \supset q$ / $\sim p \supset r$ / $\therefore q \vee r$

p	q	r	$p \supset q$	$\sim p$	$\sim p \supset r$	$q \vee r$
T	T	T	T	F	T	T
T	T	F	T	F	T	T
T	F	T	F	F	T	T
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T
F	F	F	T	T	F	F

Valid

8. $R \supset (A \vee D)$ / $\sim A$ / $\therefore \sim D \supset \sim R$ has the specific form $p \supset (q \vee r)$ / $\sim q$ / $\therefore \sim r \supset \sim p$

p	q	r	$q \vee r$	$p \supset (q \vee r)$	$\sim q$	$\sim r$	$\sim p$	$\sim r \supset \sim p$
T	T	T	T	T	F	F	F	T
T	T	F	T	T	F	T	F	F
T	F	T	T	T	T	F	F	T
T	F	F	F	F	T	T	F	F
F	T	T	T	T	F	F	T	T
F	T	F	T	T	F	T	T	T
F	F	T	T	T	T	F	T	T
F	F	F	F	T	T	T	T	T

Valid

9. $G \supset (I \vee D)$ has the specific form $p \supset (q \vee r)$
$(I \bullet D) \supset B$ $\quad$ $(q \bullet r) \supset s$
$\therefore G \supset B$ $\quad$ $\therefore p \supset s$

p	q	r	s	$q \vee r$	$p \supset (q \vee r)$	$q \bullet r$	$(q \bullet r) \supset s$	$p \supset s$
T	T	T	T	T	T	T	T	T
T	T	T	F	T	T	T	F	F
T	T	F	T	T	T	F	T	T
T	T	F	F	T	T	F	T	F
T	F	T	T	T	T	F	T	T
T	F	T	F	T	T	F	T	F
T	F	F	T	F	F	F	T	T
T	F	F	F	F	F	F	T	F
F	T	T	T	T	T	T	T	T
F	T	T	F	T	T	T	F	T
F	T	F	T	T	T	F	T	T
F	T	F	F	T	T	F	T	T
F	F	T	T	T	T	F	T	T
F	F	T	F	T	T	F	T	T
F	F	F	T	F	T	F	T	T
F	F	F	F	F	T	F	T	T

Invalid, lines 4 and 6

10. $G \supset (I \bullet D)$ has the specific form $p \supset (q \bullet r)$
$(I \vee D) \supset B$ $(q \vee r) \supset s$
$\therefore G \supset B$ $\therefore p \supset s$

p	q	r	s	$q \bullet r$	$p \supset (q \bullet r)$	$q \vee r$	$(q \vee r) \supset s$	$p \supset s$
T	T	T	T	T	T	T	T	T
T	T	T	F	T	T	T	F	F
T	T	F	T	F	F	T	T	T
T	T	F	F	F	F	T	F	F
T	F	T	T	F	F	T	T	T
T	F	T	F	F	F	T	F	F
T	F	F	T	F	F	F	T	T
T	F	F	F	F	F	F	T	F
F	T	T	T	T	T	T	T	T
F	T	T	F	T	T	T	F	T
F	T	F	T	F	T	T	T	T
F	T	F	F	F	T	T	F	T
F	F	T	T	F	T	T	T	T
F	F	T	F	F	T	T	F	T
F	F	F	T	F	T	F	T	T
F	F	F	F	F	T	F	T	T

Valid

Section 8.8 – A
Exercises on page 328

1. *c* is the specific form of 1.
2. *a* has 2 as a substitution instance, and *d* is the specific form of 2.
3. *b* has 3 as a substitution instance.
4. *b* has 4 as a substitution instance.
5. *c* has 5 as a substitution instance, and *i* is the specific form of 5.
6. *b* has 6 as a substitution instance.
7. *c* has 7 as a substitution instance, and *f* has 7 as a substitution instance.
8. *b* has 8 as a substitution instance, and *j* has 8 as a substitution instance.
9. *b* has 9 as a substitution instance, and *g* has 9 as a substitution instance, and *h* is the specific form of 9.
10. *e* has 10 as a substitution instance.

Section 8.8 – B
Exercises on page 329

1. Contingent—final column T T T F
2. Tautologous—final column T T T T
3. Self-contradictory—final column F F F F
4. Tautologous—final column T T T T
5. Contingent—final column F F T T
6. Self-contradictory—final column F F F F
7. Tautologous—final column T T T T T T T T
8. Self-contradictory—final column F F F F F F F F
9. Tautologous—final column T T T T T T T T T T T T T T T T
10. Contingent—final column T T T T T T T T T T F F T T F T

Section 8.8 – C
Exercises on page 329

1.

p	q	$p \supset q$	$\sim q$	$\sim p$	$\sim q \supset \sim p$	$(p \supset q) \equiv (\sim q \supset \sim p)$
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Tautology

2.

p	q	$\sim p$	$\sim q$	$p \supset q$	$\sim p \supset \sim q$	$(p \supset q) \equiv (\sim p \supset \sim q)$
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

Not a tautology

3.

p	q	r	$p \supset q$	$(p \supset q) \supset r$	$q \supset p$	$(q \supset p) \supset r$	$[(p \supset q) \supset r] \equiv [(q \supset p) \supset r]$
T	T	T	T	T	T	T	T
T	T	F	T	F	T	F	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	F	F
F	T	T	T	T	F	T	T
F	T	F	T	F	F	T	F
F	F	T	T	T	T	T	T
F	F	F	T	F	T	F	T

Not a tautology

4.

p	q	r	$q \supset r$	$p \supset (q \supset r)$	$p \supset r$	$q \supset (p \supset r)$	$[p \supset (q \supset r)] \equiv [q \supset (p \supset r)]$
T	T	T	T	T	T	T	T
T	T	F	F	F	F	F	T
T	F	T	T	T	T	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Tautology

5.

p	q	$p \vee q$	$p \bullet (p \vee q)$	$p \equiv [p \bullet (p \vee q)]$
T	T	T	T	T
T	F	T	T	T
F	T	T	F	T
F	F	F	F	T

Tautology

6.

p	q	$p \bullet q$	$p \vee (p \bullet q)$	$p \equiv [p \vee (p \bullet q)]$
T	T	T	T	T
T	F	F	T	T
F	T	F	F	T
F	F	F	F	T

Tautology

7.

p	q	$p \supset q$	$p \bullet (p \supset q)$	$p \equiv [p \bullet (p \supset q)]$
T	T	T	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

Not a tautology

8.

p	q	$q \supset p$	$p \bullet (q \supset p)$	$p \equiv [p \bullet (q \supset p)]$
T	T	T	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

Tautology

9.

p	q	$p \supset q$	$p \vee (p \supset q)$	$p \equiv [p \vee (p \supset q)]$
T	T	T	T	T
T	F	F	T	T
F	T	T	T	F
F	F	T	T	F

Not a tautology

10.

p	q	$p \supset q$	$p \vee q$	$(p \vee q) \equiv q$	$(p \supset q) \equiv [(p \vee q) \equiv q]$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	T	T

Tautology

11.

p	q	$\sim q$	$q \bullet \sim q$	$p \vee (q \bullet \sim q)$	$p \equiv [p \vee (q \bullet \sim q)]$
T	T	F	F	T	T
T	F	T	F	T	T
F	T	F	F	F	T
F	F	T	F	F	T

Tautology

12.

p	q	$\sim q$	$q \bullet \sim q$	$p \bullet (q \bullet \sim q)$	$p \equiv [p \bullet (q \bullet \sim q)]$
T	T	F	F	F	F
T	F	T	F	F	F
F	T	F	F	F	T
F	F	T	F	F	T

Not a tautology

13.

p	q	$\sim q$	$q \vee \sim q$	$p \bullet (q \vee \sim q)$	$p \equiv [p \bullet (q \vee \sim q)]$
T	T	F	T	T	T
T	F	T	T	T	T
F	T	F	T	F	T
F	F	T	T	F	T

Tautology

14.

p	q	$\sim q$	$q \vee \sim q$	$p \vee (q \vee \sim q)$	$p \equiv [p \vee (q \vee \sim q)]$
T	T	F	T	T	T
T	F	T	T	T	T
F	T	F	T	T	F
F	F	T	T	T	F

Not a tautology

15.

p	q	r	$q \vee r$	$p \bullet (q \vee r)$	$p \bullet q$	$p \bullet r$	$(p \bullet q) \vee (p \bullet r)$	$[p \bullet (q \vee r)] \equiv [(p \bullet q) \vee (p \bullet r)]$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T	T
T	F	T	T	T	F	T	T	T
T	F	F	F	F	F	F	F	T
F	T	T	T	F	F	F	F	T
F	T	F	T	F	F	F	F	T
F	F	T	T	F	F	F	F	T
F	F	F	F	F	F	F	F	T

Tautology

16.

p	q	r	$q \vee r$	$p \bullet (q \vee r)$	$p \vee q$	$p \vee r$	$(p \vee q) \bullet (p \vee r)$	$[p \bullet (q \vee r)] \equiv [(p \vee q) \bullet (p \vee r)]$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T	T
T	F	T	T	T	T	T	T	T
T	F	F	F	F	T	T	T	F
F	T	T	T	F	T	T	T	F
F	T	F	T	F	T	F	F	T
F	F	T	T	F	F	T	F	T
F	F	F	F	F	F	F	F	T

Not a tautology

17.

p	q	r	$q \bullet r$	$p \vee (q \bullet r)$	$p \bullet q$	$p \bullet r$	$(p \bullet q) \vee (p \bullet r)$	$[p \vee (q \bullet r)] \equiv [(p \bullet q) \vee (p \bullet r)]$
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	F	T	T
T	F	T	F	T	F	T	T	T
T	F	F	F	T	F	F	F	F
F	T	T	T	T	F	F	F	F
F	T	F	F	F	F	F	F	T
F	F	T	F	F	F	F	F	T
F	F	F	F	F	F	F	F	T

Not a tautology

18.

p	q	r	$q \bullet r$	$p \vee (q \bullet r)$	$p \vee q$	$p \vee r$	$(p \vee q) \bullet (p \vee r)$	$[p \vee (q \bullet r)] \equiv [(p \vee q) \bullet (p \vee r)]$
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F	T
F	F	T	F	F	F	T	F	T
F	F	F	F	F	F	F	F	T

Tautology

19.

p	q	r	$p \bullet q$	$(p \bullet q) \supset r$	$q \supset r$	$p \supset (q \supset r)$	$[(p \bullet q) \supset r] \equiv [p \supset (q \supset r)]$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	F	T	T	T	T
F	T	F	F	T	F	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

Tautology

20.

p	q	$p \supset q$	$q \supset p$	$(p \supset q) \bullet (q \supset p)$	$p \bullet q$	$\sim p$	$\sim q$	$\sim p \bullet \sim q$	$(p \bullet q) \vee (\sim p \bullet \sim q)$	$[(p \supset q) \bullet (q \supset p)] \equiv [(p \bullet q) \vee (\sim p \bullet \sim q)]$
T	T	T	T	T	T	F	F	F	T	T
T	F	F	T	F	F	F	T	F	F	T
F	T	T	F	F	F	T	F	F	F	T
F	F	T	T	T	F	T	T	T	T	T

Tautology

Chapter 9

Section 9.2
Exercises on pages 344–345

1. Absorption (Abs.)
2. Simplification (Simp.)
3. Addition (Add.)
4. Simplification (Simp.)
5. Constructive Dilemma (C.D.)
6. *Modus Tollens* (M.T.)
7. Disjunctive Syllogism (D.S.)
8. *Modus Ponens* (M.P.)
9. Hypothetical Syllogism (H.S.)
10. Hypothetical Syllogism (H.S.)
11. *Modus Ponens* (M.P.)
12. Disjunctive Syllogism (D.S.)
13. *Modus Tollens* (M.T.)
14. *Modus Ponens* (M.P.)
15. Conjunction (Conj.)
16. Conjunction (Conj.)
17. Absorption (Abs.)
18. Addition (Add.)
19. Constructive Dilemma (C.D.)
20. Hypothetical Syllogism (H.S.)

Section 9.3
Exercises on pages 347–348

1. 3. 1, Simp.
   4. 3, Add.
   5. 2, 4, M.P.
   6. 3, 5, Conj.

2. 4. 1, Simp.
   5. 2, 4, C.D.
   6. 5, 3, D.S.

3. 5. 1, 2, H.S.
   6. 5, 3, Conj.
   7. 6, 4, C.D.

4. 4. 1, Abs.
   5. 4, 2, H.S.
   6. 5, Abs.
   7. 6, 3, M.T.

5. 5. 2, 4, M.P.
   6. 1, 5, Conj.
   7. 3, 4, D.S.
   8. 6, 7, C.D.

6. 5. 1, Abs.
   6. 5, 3, H.S.
   7. 2, 6, M.P.
   8. 7, 4, D.S.

7. 4. 3, Simp.
   5. 4, Add.
   6. 1, 5, M.P.
   7. 6, Add.
   8. 2, 7, M.P.
   9. 8, 4, M.P.

8.
5. 4, Add.
6. 3, 5, M.P.
7. 1, 6, M.T.
8. 2, 7, M.P.
9. 8, 6, M.T.

10.
6. 4, 5, Conj.
7. 3, 6, M.P.
8. 7,1, H.S.
9. 2, 8, Conj.
10. 9, 4, C.D.

9.
5. 1, Abs.
6. 5, 4, M.T.
7. 2, 6, D.S.
8. 7, Simp.
9. 3, 8, M.T.
10. 9, Add.

Section 9.4
Exercises on pages 349–350

1.

1.	A	
2.	B	
	∴ (A ∨ C) • B	
3.	(A ∨ C)	1, Add.
4.	(A ∨ C) • B	3, 2, Conj.

2.

1.	D ⊃ E	
2.	D • F	
	∴ E	
2.	D	2, Simp.
3.	E	1, 3, M.P.

3.

1.	G	
2.	H	
	∴ (G • H) ∨ I	
3.	G • H	1, 2, Conj.
4.	(G • H) ∨ I	3, Add.

4.

1.	J ⊃ K	
2.	J	
	∴ K ∨ L	
3.	K	1, 2, M.P.
4.	K ∨ L	3, Add.

5.	1.	$M \vee N$	
	2.	$\sim M \bullet \sim O$	
		$\therefore\ N$	
	3.	$\sim M$	2, Simp.
	4.	N	1, 3, D.S.
6.	1.	$P \bullet Q$	
	2.	R	
		$\therefore\ P \bullet R$	
	3.	P	1, Simp.
	4.	$P \bullet R$	3, 2, Conj.
7.	1.	$S \supset T$	
	2.	$\sim T \bullet \sim U$	
		$\therefore\ \sim S$	
	3.	$\sim T$	2, Simp.
	4.	$\sim S$	1, 3, M.T.
8.	1.	$V \vee W$	
	2.	$\sim V$	
		$\therefore\ W \vee X$	
	3.	W	1, 2, D.S.
	4.	$W \vee X$	3, Add.
9.	1.	$Y \supset Z$	
	2.	Y	
		$\therefore\ Y \bullet Z$	
	3.	Z	1, 2, M.P.
	4.	$Y \bullet Z$	2, 3, Conj.
10.	1.	$A \supset B$	
	2.	$(A \bullet B) \supset C$	
		$\therefore\ A \supset C$	
	3.	$A \supset (A \bullet B)$	1, Abs.
	4.	$A \supset C$	3, 2, H.S.

11.	1.	$D \supset E$	
	2.	$(E \supset F) \bullet (F \supset D)$	
		$\therefore D \supset F$	
	3.	$E \supset F$	2, Simp.
	4.	$D \supset F$	1, 3, H.S.
12.	1.	$(G \supset H) \bullet (I \supset J)$	
	2.	G	
		$\therefore H \vee J$	
	3.	$G \vee I$	2, Add.
	4.	$H \vee J$	1, 3, C.D.
13.	1.	$\sim (K \bullet L)$	
	2.	$K \supset L$	
		$\therefore \sim K$	
	3.	$K \supset (K \bullet L)$	2, Abs.
	4.	$\sim K$	3, 1, M.T.
14.	1.	$(M \supset N) \bullet (M \supset O)$	
	2.	$N \supset O$	
		$\therefore M \supset O$	
	3.	$M \supset N$	1, Simp.
	4.	$M \supset O$	3, 2, H.S.
15.	1.	$(P \supset Q) \bullet (R \supset S)$	
	2.	$(P \vee R) \bullet (Q \vee R)$	
		$\therefore Q \vee S$	
	3.	$P \vee R$	2, Simp.
	4.	$Q \vee S$	1, 3, C.D.
16.	1.	$(T \supset U) \bullet (T \supset V)$	
	2.	T	
		$\therefore U \vee V$	
	3.	$T \vee T$	2, Add.
	4.	$U \vee V$	1, 3, C.D.

17.	1.	$(W \vee X) \supset Y$	
	2.	W	
		$\therefore Y$	
	3.	$W \vee X$	2, Add.
	4.	Y	1, 3, M.P.

18.	1.	$(Z \bullet A) \supset (B \bullet C)$	
	2.	$Z \supset A$	
		$\therefore Z \supset (B \bullet C)$	
	3.	$Z \supset (Z \bullet A)$	2, Abs.
	4.	$Z \supset (B \bullet C)$	3, 1, H.S.

19.	1.	$D \supset E$	
	2.	$[D \supset (D \bullet E)] \supset (F \supset \sim G)$	
		$\therefore F \supset \sim G$	
	3.	$D \supset (D \bullet E)$	1, Abs.
	4.	$F \supset \sim G$	2, 3, M.P.

20.	1.	$(\sim H \vee I) \vee J$	
	2.	$\sim (\sim H \vee I)$	
		$\therefore J \vee \sim H$	
	3.	J	1, 2, D.S.
	4.	$J \vee \sim H$	3, Add.

21.	1.	$(K \supset L) \supset M$	
	2.	$\sim M \bullet \sim (L \supset K)$	
		$\therefore \sim (K \supset L)$	
	3.	$\sim M$	2, Simp.
	4.	$\sim (K \supset L)$	1, 3, M.T.

22.	1.	$(N \supset O) \supset (P \supset Q)$	
	2.	$[P \supset (N \supset O)] \bullet [N \supset (P \supset Q)]$	
		$\therefore P \supset (P \supset Q)$	
	3.	$P \supset (N \supset O)$	2, Simp.
	4.	$P \supset (P \supset Q)$	3, 1, H.S.

23.	1.	$R \supset S$	
	2.	$S \supset (S \bullet R)$	
		$\therefore [R \supset (R \bullet S)] \bullet [S \supset (S \bullet R)]$	
	3.	$R \supset (R \bullet S)$	1, Abs.
	4.	$[R \supset (R \bullet S)] \bullet [S \supset (S \bullet R)]$	3, 2, Conj.
24.	1.	$[T \supset (U \vee V)] \bullet [U \supset (T \vee V)]$	
	2.	$(T \vee U) \bullet (U \vee V)$	
		$\therefore (U \vee V) \vee (T \vee V)$	
	3.	$T \vee U$	2, Simp.
	4.	$(U \vee V) \vee (T \vee V)$	1, 3, C.D.
25.	1.	$(W \bullet X) \supset (Y \bullet Z)$	
	2.	$\sim [(W \bullet X) \bullet (Y \bullet Z)]$	
		$\therefore \sim (W \bullet X)$	
	3.	$(W \bullet X) \supset [(W \bullet X) \bullet (Y \bullet Z)]$	1, Abs.
	4.	$\sim (W \bullet X)$	3, 2, M.T.
26.	1.	$A \supset B$	
	2.	$A \vee C$	
		$C \supset D$	
	3.	$\therefore B \vee D$	
	4.	$(A \supset B) \bullet (C \supset D)$	1, 3, Conj.
	5.	$B \vee D$	4, 2, C.D.
27.	1.	$(E \bullet F) \vee (G \supset H)$	
	2.	$I \supset G$	
	3.	$\sim (E \bullet F)$	
		$\therefore I \supset H$	
	4.	$G \supset H$	1, 3, D.S.
	5.	$I \supset H$	2, 4, H.S.
28.	1.	$J \vee \sim K$	
	2.	$K \vee (L \supset J)$	
	3.	$\sim J$	
		$\therefore L \supset J$	
	4.	$\sim K$	1, 3, D.S.
	5.	$L \supset J$	2, 4, D.S.

29.	1.	$(M \supset N) \bullet (O \supset P)$	
	2.	$N \supset P$	
	3.	$(N \supset P) \supset (M \vee O)$	
		$\therefore N \vee P$	
	4.	$M \vee O$	3, 2, M.P.
	5.	$N \vee P$	1, 4, C.D.

30.	1.	$Q \supset (R \vee S)$	
	2.	$(T \bullet U) \supset R$	
	3.	$(R \vee S) \supset (T \bullet U)$	
		$\therefore Q \supset R$	
	4.	$Q \supset (T \bullet U)$	1, 3, H.S.
	5.	$Q \supset R$	4, 2, H.S.

Section 9.5 – A
Exercises on pages 351–352

1.	1.	$A \vee (B \supset A)$	
	2.	$\sim A \bullet C$	
		$\therefore \sim B$	
	3.	$\sim A$	2, Simp.
	4.	$B \supset A$	1, 3, D.S.
	5.	$\sim B$	4, 3, M.T.

2.	1.	$(D \vee E) \supset (F \bullet G)$	
	2.	D	
		$\therefore F$	
	3.	$D \vee E$	2, Add.
	4.	$F \bullet G$	1, 3, M.P.
	5.	F	4, Simp.

3.	1.	$(H \supset I) \bullet (H \supset J)$	
	2.	$H \bullet (I \vee J)$	
		$\therefore I \vee J$	
	3.	H	2, Simp.
	4.	$H \vee H$	3, Add.
	5.	$I \vee J$	1, 4, C.D.

4.	1.	(K • L) ⊃ M	
	2.	K ⊃ L	
		∴ K ⊃ [(K • L) • M]	
	3.	K ⊃ (K • L)	2, Abs.
	4.	(K • L) ⊃ [(K • L) M]	1, Abs.
	5.	K ⊃ [(K • L) • M]	3, 4, H.S.

5.	1.	N ⊃ [(N • O) ⊃ P]	
	2.	N • O	
		∴ P	
	3.	N	2, Simp.
	4.	(N • O) ⊃ P	1, 3, M.P.
	5.	P	4, 2, M.P.

6.	1.	Q ⊃ R	
	2.	R ⊃ S	
	3.	~ S	
		∴ ~ Q • ~ R	
	4.	~ R	2, 3, M.T.
	5.	~ Q	1, 4, M.T.
	6.	~ Q • ~ R	5, 4, Conj.

7.	1.	T ⊃ U	
	2.	V v ~ U	
	3.	~ V • ~ W	
		∴ ~ T	
	4.	~ V	3, Simp.
	5.	~ U	2, 4, D.S.
	6.	~ T	1, 5, M.T.

8.	1.	~ X ⊃ Y	
	2.	Z ⊃ X	
	3.	~ X	
		∴ Y • ~ Z	
	4.	Y	1, 3, M.P.
	5.	~ Z	2, 3, M.T.
	6.	Y • ~ Z	4, 5, Conj.

9.	1.	$(A \vee B) \supset \sim C$	
	2.	$C \vee D$	
	3.	A	
		$\therefore D$	
	4.	$A \vee B$	3, Add.
	5.	$\sim C$	1, 4, M.P.
	6.	D	2, 5, D.S.

10.	1.	$E \vee \sim F$	
	2.	$F \vee (E \vee G)$	
	3.	$\sim E$	
		$\therefore G$	
	4.	$\sim F$	1, 3, D.S.
	5.	$E \vee G$	2, 4, D.S.
	6.	G	5, 3, D.S.

11.	1.	$(H \supset I) \bullet (J \supset K)$	
	2.	$K \vee H$	
	3.	$\sim K$	
		$\therefore I$	
	4.	$H \supset I$	1, Simp.
	5.	H	2, 3, D.S.
	6.	I	4, 5, M.P.

12.	1.	$L \vee (M \supset N)$	
	2.	$\sim L \supset (N \supset O)$	
	3.	$\sim L$	
		$\therefore M \supset O$	
	4.	$M \supset N$	1, 3, D.S.
	5.	$N \supset O$	2, 3, M.P.
	6.	$M \supset O$	4, 5, H.S.

13.	1.	$(P \supset Q) \bullet (Q \supset P)$	
	2.	$R \supset S$	
	3.	$P \vee R$	
		$\therefore Q \vee S$	
	4.	$P \supset Q$	1, Simp.
	5.	$(P \supset Q) \bullet (R \supset S)$	4, 2, Conj.
	6.	$Q \vee S$	5, 3, C.D.

14.	1.	$(T \supset U) \bullet (V \supset W)$	
	2.	$(U \supset X) \bullet (W \supset Y)$	
	3.	T	
		$\therefore X \vee Y$	
	4.	$T \vee V$	3, Add.
	5.	$U \vee W$	1, 4, C.D.
	6.	$X \vee Y$	2, 5, C.D.

15.	1.	$(Z \bullet A) \supset B$	
	2.	$B \supset A$	
	3.	$(B \bullet A) \supset (A \bullet B)$	
		$\therefore (Z \bullet A) \supset (A \bullet B)$	
	4.	$B \supset (B \bullet A)$	2, Abs.
	5.	$B \supset (A \bullet B)$	4, 3, H.S.
	6.	$(Z \bullet A) \supset (A \bullet B)$	1, 5, H.S.

Section 9.5 – B
Exercises on pages 353–354

1.	1.	$A \supset B$	
	2.	$A \vee (C \bullet D)$	
	3.	$\sim B \bullet \sim E$	
		$\therefore C$	
	4.	$\sim B$	3, Simp.
	5.	$\sim A$	1, 4, M.T.
	6.	$C \bullet D$	2, 5, D.S.
	7.	C	6, Simp.

2.	1.	$(F \supset G) \bullet (H \supset I)$	
	2.	$J \supset K$	
	3.	$(F \vee J) \bullet (H \vee L)$	
		$\therefore G \vee K$	
	4.	$F \supset G$	1, Simp.
	5.	$(F \supset G) \bullet (J \supset K)$	4, 2, Conj.
	6.	$F \vee J$	3, Simp.
	7.	$G \vee K$	5, 6, C.D.

3.	1.	$(\sim M \bullet \sim N) \supset (O \supset N)$	
	2.	$N \supset M$	
	3.	$\sim M$	
		$\therefore \sim O$	
	4.	$\sim N$	2, 3, M.T.
	5.	$\sim M \bullet \sim N$	3, 4, Conj.
	6.	$O \supset N$	1, 5, M.P.
	7.	$\sim O$	6, 4, M.T.
4.	1.	$(K \vee L) \supset (M \vee N)$	
	2.	$(M \vee N) \supset (O \bullet P)$	
	3.	K	
		$\therefore O$	
	4.	$K \vee L$	3, Add.
	5.	$M \vee N$	1, 4, M.P.
	6.	$O \bullet P$	2, 5, M.P.
	7.	O	6, Simp.
5.	1.	$(Q \supset R) \bullet (S \supset T)$	
	2.	$(U \supset V) \bullet (W \supset X)$	
	3.	$Q \vee U$	
		$\therefore R \vee V$	
	4.	$Q \supset R$	1, Simp.
	5.	$U \supset V$	2, Simp.
	6.	$(Q \supset R) \bullet (U \supset V)$	4, 5, Conj.
	7.	$R \vee V$	6, 3, C.D.
6.	1.	$W \supset X$	
	2.	$(W \bullet X) \supset Y$	
	3.	$(W \bullet Y) \supset Z$	
		$\therefore W \supset Z$	
	4.	$W \supset (W \bullet X)$	1, Abs.
	5.	$W \supset Y$	4, 2, H.S.
	6.	$W \supset (W \bullet Y)$	5, Abs.
	7.	$W \supset Z$	6, 3, H.S.

7.	1.	$A \supset B$	
	2.	$C \supset D$	
	3.	$A \vee C$	
		$\therefore (A \bullet B) \vee (C \bullet D)$	
	4.	$A \supset (A \bullet B)$	1, Abs.
	5.	$C \supset (C \bullet D)$	2, Abs.
	6.	$[A \supset (A \bullet B)] \bullet [C \supset (C \bullet D)]$	4, 5, Conj.
	7.	$(A \bullet B) \vee (C \bullet D)$	6, 3, C.D.

8.	1.	$(E \vee F) \supset (G \bullet H)$	
	2.	$(G \vee H) \supset I$	
	3.	E	
		$\therefore I$	
	4.	$E \vee F$	3, Add.
	5.	$G \bullet H$	1, 4, M.P.
	6.	G	5, Simp.
	7.	$G \vee H$	6, Add.
	8.	I	2, 7, M.P.

9.	1.	$J \supset K$	
	2.	$K \vee L$	
	3.	$(L \bullet \sim J) \supset (M \bullet \sim J)$	
	4.	$\sim K$	
		$\therefore M$	
	5.	L	2, 4, D.S.
	6.	$\sim J$	1, 4, M.T.
	7.	$L \bullet \sim J$	5, 6, Conj.
	8.	$M \bullet \sim J$	3, 7, M.P.
	9.	M	8, Simp.

10.	1.	$(N \vee O) \supset P$	
	2.	$(P \vee Q) \supset R$	
	3.	$Q \vee N$	
	4.	$\sim Q$	
		$\therefore R$	
	5.	N	3, 4, D.S.
	6.	$N \vee O$	5, Add.
	7.	P	1, 6, M.P.
	8.	$P \vee Q$	7, Add.
	9.	R	2, 8, M.P.

Section 9.5 – C
Exercises on pages 354–356

1.

1.	$(G \vee H) \supset (J \bullet K)$	
2.	G	
	$\therefore J$	
3.	$G \vee H$	2, Add.
4.	$J \bullet K$	1, 3, M.P.
5.	J	4, Simp.

2.

1.	$(A \supset S) \bullet (B \supset F)$	
2.	$A \vee B$	
3.	$(S \supset B) \bullet (F \supset W)$	
	$\therefore B \vee W$	
4.	$S \vee F$	1, 2, C.D.
5.	$B \vee W$	3, 4, C.D.

3.

1.	$(R \supset P) \bullet (P \supset \sim L)$	
2.	$T \supset L$	
3.	$R \vee T$	
	$\therefore P \vee L$	
4.	$R \supset P$	1, Simp.
5.	$(R \supset P) \bullet (T \supset L)$	4, 2, Conj.
6.	$P \vee L$	5, 3, C.D.

4.

1.	$(N \supset O) \bullet (P \supset Q)$	
2.	$(R \supset S) \bullet (S \supset T)$	
3.	$N \vee R$	
	$\therefore O \vee S$	
4.	$N \supset O$	1, Simp.
5.	$R \supset S$	2, Simp.
6.	$(N \supset O) \bullet (R \supset S)$	4, 5, Conj.
7.	$O \vee S$	6, 3, C.D.

5.	1.	$C \supset R$	
	2.	$(C \bullet R) \supset B$	
	3.	$(C \supset B) \supset \sim S$	
	4.	$S \vee M$	
		$\therefore M$	
	5.	$C \supset (C \bullet R)$	1, Abs.
	6.	$C \supset B$	5, 2, H.S.
	7.	$\sim S$	3, 6, M.P.
	8.	M	4, 7, D.S.

6.	1.	$(J \supset R) \bullet (\sim J \supset E)$	
	2.	$R \supset I$	
	3.	$[(J \supset R) \bullet (R \supset I)] \supset [(J \bullet I) \vee (\sim J \bullet \sim I)]$	
	4.	$(J \bullet I) \supset T$	
	5.	$(\sim J \bullet \sim I) \supset D$	
		$\therefore T \vee D$	
	6	$J \supset R$	1, Simp.
	7.	$(J \supset R) \bullet (R \supset I)$	6, 2, Conj.
	8.	$(J \bullet I) \vee (\sim J \bullet \sim I)$	3, 7, M.P.
	9.	$[(J \bullet I) \supset T] \bullet [(\sim J \bullet \sim I) \supset D]$	4, 5, Conj.
	10.	$T \vee D$	9, 8, C.D.

7.	1.	$A \supset B$	
	2.	$(A \bullet B) \supset (C \vee D)$	
	3.	$(C \vee D) \supset \sim E$	
	4.	$(A \supset \sim E) \supset F$	
		$\therefore F$	
	5.	$A \supset (A \bullet B)$	1, Abs.
	6.	$A \supset (C \vee D)$	5, 2, H.S.
	7.	$A \supset \sim E$	6, 3, H.S.
	8.	F	4, 7, M.P.

8.	1.	$J \supset E$	
	2.	$E \supset T$	
	3.	$\sim T$	
	4.	$R \supset H$	
	5.	$D \supset \sim H$	
	6.	D	
	7.	$\sim J \supset (R \vee S)$	
		$\therefore S$	

	8.	$J \supset T$	1, 2, H.S.
	9.	$\sim J$	8, 3, M.T.
	10.	$R \vee S$	7, 9, M.P.
	11.	$\sim H$	5, 6, M.P.
	12.	$\sim R$	4, 11, M.T.
	13.	S	10, 12, D.S.

9.	1.	$S \supset W$	
	2.	$W \supset \sim L$	
	3.	S	
	4.	$D \supset \sim I$	
	5.	D	
	6.	$L \vee (I \vee C)$	
	7.	$C \supset B$	
		$\therefore B$	
	8.	W	1, 3, M.P.
	9.	$\sim L$	2, 8, M.P.
	10.	$I \vee C$	6, 9, D.S.
	11.	$\sim I$	4, 5, M.P.
	12.	C	10, 11, D.S.
	13.	B	7, 12, M.P.

10.	1.	$O \supset \sim M$	
	2.	O	
	3.	$B \supset \sim N$	
	4.	B	
	5.	$(\sim M \bullet \sim N) \supset F$	
	6.	$(B \bullet F) \supset G$	
		$\therefore G$	
	7.	$\sim M$	1, 2, M.P.
	8.	$\sim N$	3, 4, M.P.
	9.	$\sim M \bullet \sim N$	7, 8, Conj.
	10.	F	5, 9, M.P.
	11.	$B \bullet F$	4, 10, Conj.
	12.	G	6, 11, M.P.

Section 9.6
Exercises on pages 363–364

1. Transposition (Trans.)
2. Material Implication (Impl.)
3. Exportation (Exp.)
4. Tautology (Taut.)
5. Material Equivalence (Equiv.)
6. De Morgan's Theorem (De M.)
7. Exportation (Exp.)
8. Distribution (Dist.)
9. Double Negation (D.N.)
10. Association (Assoc.)
11. Material Implication (Impl.)
12. Transposition (Trans.)
13. De Morgan's Theorem (De M.)
14. Association (Assoc.)
15. Distribution (Dist.)
16. Tautology (Taut.)
17. Material Equivalence (Equiv.)
18. Double Negation (D.N.)
19. Distribution (Dist.)
20. De Morgan's Theorem (De M.)

Section 9.8 – A
Exercises on pages 370–371

1. 3. 2, Trans.
   4. 3, D.N.
   5. 1, 4, H.S.

2. 3. 1, Com.
   4. 3, Exp.
   5. 4, 2, H.S.

3. 3. 2, Add.
   4. 3, Com.
   5. 1, 4, M.P.
   6. 5, Assoc.
   7. 6, Simp.

4. 3. 2, Add.
   4. 3, De M.
   5. 1, 4, M.T.
   6. 5, De M.
   7. 6, Simp.

5. 3. 2, Dist.
   4. 3, Com.
   5. 4, Simp.
   6. 5, Taut.
   7. 1, Assoc.
   8. 7, 6, D.S.
   9. 8, Impl.

6. 4. 2, Exp.
   5. 3, De M.
   6. 4, 5, Conj.
   7. 1, Dist.
   8. 6, 7, C.D.
   9. 8, Equiv.

7. 5. 3, Equiv.
   6. 5, 4, D.S.
   7. 6, De M.
   8. 1, 2, H.S.
   9. 8, Exp.
   10. 9, Taut.
   11. 10, 7, M.T.

8. 5. 1, 2, H.S.
6. 5, 3, Conj.
7. 6, Equiv.
8. 7, Equiv.
9. 4, Impl.
10. 9, De M.
11. 8, 10, D.S.

9. 4. 3, Equiv.
5. 4, Simp.
6. 5, Abs.
7. 6, 1, H.S.
8. 2, Dist.
9. 8, Simp.
10. 9, D.N.
11. 10, Impl.
12. 7, 11, H.S.

10. 3. 2, Trans.
4. 3, Exp.
5. 1, D.N.
6. 5, Com.
7. 6, Dist.
8. 7, Com.
9. 4, 8, C.D.
10. 9, Com.
11. 10, D.N.
12. 11, De M.

Section 9.8 – B
Exercises on pages 372–374

1.	1.	$A \supset \sim A$	
		$\therefore \sim A$	
	2.	$\sim A \vee \sim A$	1, Impl.
	3.	$\sim A$	2, Taut.
2.	1.	$B \bullet (C \bullet D)$	
		$\therefore C \bullet (D \bullet B)$	
	2.	$(C \bullet D) \bullet B$	1, Com.
	3.	$C \bullet (D \bullet B)$	2, Assoc.
3.	1.	E	
		$\therefore (E \vee F) \bullet (E \vee G)$	
	2.	$E \vee (F \bullet G)$	1, Add.
	3.	$(E \vee F) \bullet (E \vee G)$	2, Dist.
4.	1.	$H \vee (I \bullet J)$	
		$\therefore H \vee I$	
	2.	$(H \vee I) \bullet (H \vee J)$	1, Dist.
	3.	$H \vee I$	2, Simp.

5.	1.	$\sim K \vee (L \supset M)$	
		$\therefore (K \bullet L) \supset M$	
	2.	$K \supset (L \supset M)$	1, Impl.
	3.	$(K \bullet L) \supset M$	2, Exp.
6.	1.	$(N \bullet O) \supset P$	
		$\therefore (N \bullet O) \supset [N \bullet (O \bullet P)]$	
	2.	$(N \bullet O) \supset [(N \bullet O) \bullet P]$	1, Abs.
	3.	$(N \bullet O) \supset [N \bullet (O \bullet P)]$	2, Assoc.
7.	1.	$Q \bullet [R \supset (S \supset T)]$	
	2.	$Q \supset (Q \bullet R)$	
		$\therefore Q \supset (S \supset T)$	
	3.	$(Q \bullet R) \supset (S \supset T)$	1, Exp.
	4.	$Q \supset (S \supset T)$	2, 3, H.S.
8.	1.	$U \supset \sim V$	
	2.	V	
		$\therefore \sim U$	
	3.	$\sim\sim V$	2, D.N.
	4.	$\sim U$	1, 3, M.T.
9.	1.	$W \supset X$	
	2.	$\sim Y \supset \sim X$	
		$\therefore W \supset Y$	
	3.	$X \supset Y$	2, Trans.
	4.	$W \supset Y$	1, 3, H.S.
10.	1.	$Z \supset A$	
	2.	$\sim A \vee B$	
		$\therefore Z \supset B$	
	3.	$A \supset B$	2, Impl.
	4.	$Z \supset B$	1, 3, H.S.
11.	1.	$C \supset \sim D$	
	2.	$\sim E \supset D$	
		$\therefore C \supset \sim\sim E$	
	3.	$\sim D \supset \sim\sim E$	2, Trans.
	4.	$C \supset \sim\sim E$	1, 3, H.S.

12.	1.	$F \equiv G$	
	2.	$\sim (F \bullet G)$	
		$\therefore \sim F \bullet \sim G$	
	3.	$(F \bullet G) \vee (\sim F \bullet \sim G)$	1, Equiv.
	4.	$\sim F \bullet \sim G$	3, 2, D.S.
13.	1.	$H \supset (I \bullet J)$	
	2.	$I \supset (J \supset K)$	
		$\therefore H \supset K$	
	3.	$(I \bullet J) \supset K$	2, Exp.
	4.	$H \supset K$	1, 3, H.S.
14.	1.	$(L \supset M) \bullet (N \supset M)$	
	2.	$L \vee N$	
		$\therefore M$	
	3.	$M \vee M$	1, 2, C.D.
	4.	M	3, Taut.
15.	1.	$(O \vee P) \supset (Q \vee R)$	
	2.	$P \vee O$	
		$\therefore Q \vee R$	
	3.	$O \vee P$	2, Com.
	4.	$Q \vee R$	1, 3, M.P.
16.	1.	$(S \bullet T) \vee (U \bullet V)$	
	2.	$\sim S \vee \sim T$	
		$\therefore U \bullet V$	
	3.	$\sim (S \bullet T)$	2, De M.
	4.	$U \bullet V$	1, 3, D.S.
17.	1.	$(W \bullet X) \supset Y$	
	2.	$(X \supset Y) \supset Z$	
		$\therefore W \supset Z$	
	3.	$W \supset (X \supset Y)$	1, Exp.
	4.	$W \supset Z$	3, 2, H.S.

18.	1.	(A ∨ B) ⊃ (C ∨ D)	
	2.	~ C • ~ D	
		∴ ~ (A ∨ B)	
	3.	~ (C ∨ D)	2, De M.
	4.	~ (A ∨ B)	1, 3, M.T.
19.	1.	(E • F) ⊃ (G • H)	
	2.	F • E	
		∴ G • H	
	3.	E • F	2, Com.
	4.	G • H	1, 3, M.P.
20.	1.	I ⊃ [J ∨ (K ∨ L)]	
	2.	~ [(J ∨ K) ∨ L]	
		∴ ~ I	
	3.	~ [J ∨ (K ∨ L)]	2, Assoc.
	4.	~ I	1, 3, M.T.
21.	1.	(M ⊃ N) • (~ O ∨ P)	
	2.	M ∨ O	
		∴ N ∨ P	
	3.	(M ⊃ N) • (O ⊃ P)	1, Impl.
	4.	N ∨ P	3, 2, C.D.
22.	1.	(~ Q ⊃ ~ R) • (~ S ⊃ ~ T)	
	2.	~ ~ (~ Q ∨ ~ S)	
		∴ ~ R ∨ ~ T	
	3.	~ Q ∨ ~ S	2, D.N.
	4.	~ R ∨ ~ T	1, 3, C.D.
23.	1.	~ [(U ⊃ V) • (V ⊃ U)]	
	2.	(W ≡ X) ⊃ (U ≡ V)	
		∴ ~ (W ≡ X)	
	3.	~ (U ≡ V)	1, Equiv.
	4.	~ (W ≡ X)	2, 3, M.T.
24.	1.	(Y ⊃ Z) • (Z ⊃ Y)	
		∴ (Y • Z) ∨ (~ Y • ~ Z)	
	2.	Y ≡ Z	1, Equiv.
	3.	(Y • Z) ∨ (~ Y • ~ Z)	2, Equiv.

25.	1.	$A \vee B$	
	2.	$C \vee D$	
		$\therefore [(A \vee B) \bullet C] \vee [(A \vee B) \bullet D]$	
	3.	$(A \vee B) \bullet (C \vee D)$	1, 2, Conj.
	4.	$[(A \vee B) \bullet C] \vee [(A \vee B) \bullet D]$	3, Dist.

26.	1.	$[(E \vee F) \bullet (G \vee H)] \supset (F \bullet I)$	
	2.	$(G \vee H) \bullet (E \vee F)$	
		$\therefore F \bullet I$	
	3.	$(E \vee F) \bullet (G \vee H)$	2, Com.
	4.	$F \bullet I$	1, 3, M.P.

27.	1.	$(J \bullet K) \supset [(L \bullet M) \vee (N \bullet O)]$	
	2.	$\sim (L \bullet M) \bullet \sim (N \bullet O)$	
		$\therefore \sim (J \bullet K)$	
	3.	$\sim [(L \bullet M) \vee (N \bullet O)]$	2, De M.
	4.	$\sim (J \bullet K)$	1, 3, M.T.

28.	1.	$(P \supset Q) \supset [(R \vee S) \bullet (T \equiv U)]$	
	2.	$(R \vee S) \supset [(T \equiv U) \supset Q]$	
		$\therefore (P \supset Q) \supset Q$	
	3.	$[(R \vee S) \bullet (T \equiv U)] \supset Q$	2, Exp.
	4.	$(P \supset Q) \supset Q$	1, 3, H.S.

29.	1.	$[V \bullet (W \vee X)] \supset (Y \supset Z)$	
	2.	$\sim (Y \supset Z) \vee (\sim W \equiv A)$	
		$\therefore [V \bullet (W \vee X)] \supset (\sim W \equiv A)$	
	3.	$(Y \supset Z) \supset (\sim W \equiv A)$	2, Impl.
	4.	$[V \bullet (W \vee X)] \supset (\sim W \equiv A)$	1, 3, H.S.

30.	1.	$\sim [(B \supset \sim C) \bullet (\sim C \supset B)]$	
	2.	$(D \bullet E) \supset (B \equiv \sim C)$	
		$\therefore \sim (D \bullet E)$	
	3.	$\sim (B \equiv \sim C)$	1, Equiv.
	4.	$\sim (D \bullet E)$	2, 3, M.T.

Section 9.8 – C
Exercises on page 375

1.	1.	$\sim A \supset A$	
		$\therefore A$	
	2.	$\sim\sim A \vee A$	1, Impl.
	3.	$A \vee A$	2, D.N.
	4.	A	3, Taut.

2.	1.	$\sim B \vee (C \bullet D)$	
		$\therefore B \supset C$	
	2.	$(\sim B \vee C) \bullet (\sim B \vee D)$	1, Dist.
	3.	$\sim B \vee C$	2, Simp.
	4.	$B \supset C$	3, Impl.

3.	1.	$E \vee (F \bullet G)$	
		$\therefore E \vee G$	
	2.	$E \vee (G \bullet F)$	1, Com.
	3.	$(E \vee G) \bullet (E \vee F)$	2, Dist.
	4.	$E \vee G$	3, Simp.

4.	1.	$H \bullet (I \bullet J)$	
		$\therefore J \bullet (I \bullet H)$	
	2.	$(I \bullet J) \bullet H$	1, Com.
	3.	$(J \bullet I) \bullet H$	2, Com.
	4.	$J \bullet (I \bullet H)$	3, Assoc.

5.	1.	$[(K \vee L) \vee M] \vee N$	
		$\therefore (N \vee K) \vee (L \vee M)$	
	2.	$[K \vee (L \vee M)] \vee N$	1, Assoc.
	3.	$N \vee [K \vee (L \vee M)]$	2, Com.
	4.	$(N \vee K) \vee (L \vee M)$	3, Assoc.

6.	1.	$O \supset P$	
	2.	$P \supset \sim P$	
		$\therefore \sim O$	
	3.	$\sim P \vee \sim P$	2, Impl.
	4.	$\sim P$	3, Taut.
	5.	$\sim O$	1, 4, M.T.

7.	1.	$Q \supset (R \supset S)$	
	2.	$Q \supset R$	
		$\therefore Q \supset S$	
	3.	$(Q \bullet R) \supset S$	1, Exp.
	4.	$Q \supset (Q \bullet R)$	2, Abs.
	5.	$Q \supset S$	4, 3, H.S.
8.	1.	$T \supset U$	
	2.	$\sim (U \vee V)$	
		$\therefore \sim T$	
	3.	$\sim U \bullet \sim V$	2, De M.
	4.	$\sim U$	3, Simp.
	5.	$\sim T$	1, 4, M.T.
9.	1.	$W \bullet (X \vee Y)$	
	2.	$\sim W \vee \sim X$	
		$\therefore W \bullet Y$	
	3.	$(W \bullet X) \vee (W \bullet Y)$	1, Dist.
	4.	$\sim (W \bullet X)$	2, De M.
	5.	$W \bullet Y$	3, 4,D.S.
10.	1.	$(Z \vee A) \vee B$	
	2.	$\sim A$	
		$\therefore Z \vee B$	
	3.	$(A \vee Z) \vee B$	1, Com.
	4.	$A \vee (Z \vee B)$	3, Assoc.
	5.	$Z \vee B$	4, 2, D.S.
11.	1.	$(C \vee D) \supset (E \bullet F)$	
	2.	$D \vee C$	
		$\therefore E$	
	3.	$C \vee D$	2, Com.
	4.	$E \bullet F$	1, 3, M.P.
	5.	E	4, Simp.

12.	1.	$G \supset H$	
	2.	$H \supset G$	
		$\therefore (G \bullet H) \vee (\sim G \bullet \sim H)$	
	3.	$(G \supset H) \bullet (H \supset G)$	1, 2, Conj.
	4.	$G \equiv H$	3, Equiv.
	5.	$(G \bullet H) \vee (\sim G \bullet \sim H)$	4, Equiv.

13.	1.	$(I \supset J) \bullet (K \supset L)$	
	2.	$I \vee (K \bullet M)$	
		$\therefore J \vee L$	
	3.	$(I \vee K) \bullet (I \vee M)$	2, Dist.
	4.	$I \vee K$	3, Simp.
	5.	$J \vee L$	1, 4, C.D.

14.	1.	$(N \bullet O) \supset P$	
	2.	$(\sim P \supset \sim O) \supset Q$	
		$\therefore N \supset Q$	
	3.	$N \supset (O \supset P)$	1, Exp.
	4.	$N \supset (\sim P \supset \sim O)$	3, Trans.
	5.	$N \supset Q$	4, 2, H.S.

15.	1.	$[R \supset (S \supset T)] \bullet [(R \bullet T) \supset U]$	
	2.	$R \bullet (S \vee T)$	
		$\therefore T \vee U$	
	3.	$(R \bullet S) \vee (R \bullet T)$	2, Dist.
	4.	$[(R \bullet S) \supset T] \bullet [(R \bullet T) \supset U]$	1, Exp.
	5.	$T \vee U$	4, 3, C.D.

Section 9.8 – D
Exercises on page 377

1.	1.	$\sim A$	
		$\therefore A \supset B$	
	2.	$\sim A \vee B$	1, Add.
	3.	$A \supset B$	2, Impl.

2.	1.	C	
		$\therefore D \supset C$	
	2.	$C \vee \sim D$	1, Add.
	3.	$\sim D \vee C$	2, Com.
	4.	$D \supset C$	3, Impl.
3.	1.	$E \supset (F \supset G)$	
		$\therefore F \supset (E \supset G)$	
	2.	$(E \bullet F) \supset G$	1, Exp.
	3.	$(F \bullet E) \supset G$	2, Com.
	4.	$F \supset (E \supset G)$	3, Exp.
4.	1.	$H \supset (I \bullet J)$	
		$\therefore H \supset I$	
	2.	$\sim H \vee (I \bullet J)$	1, Impl.
	3.	$(\sim H \vee I) \bullet (\sim H \vee J)$	2, Dist.
	4.	$\sim H \vee I$	3, Simp.
	5.	$H \supset I$	4, Impl.
5.	1.	$K \supset L$	
		$\therefore K \supset (L \vee M)$	
	2.	$\sim K \vee L$	1, Impl.
	3.	$(\sim K \vee L) \vee M$	2, Add.
	4.	$\sim K \vee (L \vee M)$	3, Assoc.
	5.	$K \supset (L \vee M)$	4, Impl.
6.	1.	$N \supset O$	
		$\therefore (N \bullet P) \supset O$	
	2.	$(N \supset O) \vee \sim P$	1, Add.
	3.	$\sim P \vee (N \supset O)$	2, Com.
	4.	$P \supset (N \supset O)$	3, Impl.
	5.	$(P \bullet N) \supset O$	4, Exp.
	6.	$(N \bullet P) \supset O$	5, Com.
7.	1.	$(Q \vee R) \supset S$	
		$\therefore Q \supset S$	
	2.	$\sim (Q \vee R) \vee S$	1, Impl.

	3.	$S \vee \sim (Q \vee R)$	2, Com.
	4.	$S \vee (\sim Q \bullet \sim R)$	3, De M.
	5.	$(S \vee Q) \bullet (S \vee \sim R)$	4, Dist.
	6.	$S \vee \sim Q$	5, Simp.
	7.	$\sim Q \vee S$	6, Com.
	8.	$Q \supset S$	7, Impl.
8.	1.	$T \supset U$	
	2.	$T \supset V$	
		$\therefore T \supset (U \bullet V)$	
	3.	$(T \supset U) \bullet (T \supset V)$	1, 2, Conj.
	4.	$(\sim T \vee U) \bullet (T \supset V)$	3, Impl.
	5.	$(\sim T \vee U) \bullet (\sim T \vee V)$	4, Impl.
	6.	$\sim T \vee (U \bullet V)$	5, Dist.
	7.	$T \supset (U \bullet V)$	6, Impl.
9.	1.	$W \supset X$	
	2.	$Y \supset X$	
		$\therefore (W \vee Y) \supset X$	
	3.	$(W \supset X) \bullet (Y \supset X)$	1, 2, Conj.
	4.	$(\sim W \vee X) \bullet (Y \supset X)$	3, Impl.
	5.	$(\sim W \vee X) \bullet (\sim Y \vee X)$	4, Impl.
	6.	$(X \vee \sim W) \bullet (\sim Y \vee X)$	5, Com.
	7.	$(X \vee \sim W) \bullet (X \vee \sim Y)$	6, Com.
	8.	$X \vee (\sim W \bullet \sim Y)$	7, Dist.
	9.	$(\sim W \bullet \sim Y) \vee X$	8, Com.
	10.	$\sim (W \vee Y) \vee X$	9, De M.
	11.	$(W \vee Y) \supset X$	10, Impl.
10.	1.	$Z \supset A$	
	2.	$Z \vee A$	
		$\therefore A$	
	3.	$A \vee Z$	2, Com.
	4.	$\sim\sim A \vee Z$	3, D.N.
	5.	$\sim A \supset Z$	4, Impl.
	6.	$\sim A \supset A$	5, 1, H.S.
	7.	$\sim\sim A \vee A$	6, Impl.
	8.	$A \vee A$	7, D.N.
	9.	A	8, Taut.

Section 9.8 – E
Exercises on page 380

1.	1.	$A \supset \sim B$	
	2.	$\sim (C \bullet \sim A)$	
		$\therefore C \supset \sim B$	
	3.	$\sim C \vee \sim\sim A$	2, De M.
	4.	$C \supset \sim\sim A$	3, Impl.
	5.	$C \supset A$	4, D.N.
	6.	$C \supset \sim B$	5, 1, H.S.
2.	1.	$(D \bullet \sim E) \supset F$	
	2.	$\sim (E \vee F)$	
		$\therefore \sim D$	
	3.	$D \supset (\sim E \supset F)$	1, Exp.
	4.	$\sim (\sim\sim E \vee F)$	2, D.N.
	5.	$\sim (\sim E \supset F)$	4, Impl.
	6.	$\sim D$	3, 5, M.T.
3.	1.	$(G \supset \sim H) \supset I$	
	2.	$\sim (G \bullet H)$	
		$\therefore I \vee \sim H$	
	3.	$\sim G \vee \sim H$	2, De M.
	4.	$G \supset \sim H$	3, Impl.
	5.	I	1, 4, M.P.
	6.	$I \vee \sim H$	5, Add.
4.	1.	$(J \vee K) \supset \sim L$	
	2.	L	
		$\therefore \sim J$	
	3.	$\sim \sim L$	2, D.N.
	4.	$\sim (J \vee K)$	1, 3, M.T.
	5.	$\sim J \bullet \sim K$	4, De M.
	6.	$\sim J$	5, Simp.
5.	1.	$[(M \bullet N) \bullet O] \supset P$	
	2.	$Q \supset [(O \bullet M) \bullet N]$	
		$\therefore \sim Q \vee P$	
	3.	$[O \bullet \sim (M \bullet N)] \supset P$	1, Com.

	4.	[(O • M) • N] ⊃ P	3, Assoc.
	5.	Q ⊃ P	2, 4, H.S.
	6.	~ Q ∨ P	5, Impl.
6.	1.	R ∨ (S • ~T)	
	2.	(R ∨ S) ⊃ (U ∨ ~T)	
		∴ T ⊃ U	
	3.	(R ∨ S) • (R ∨ ~T)	1, Dist.
	4.	R ∨ S	3, Simp.
	5.	U ∨ ~ T	2, 4, M.P.
	6.	~ T ∨ U	5, Com.
	7.	T ⊃ U	6, Impl.
7.	1.	(~ V ⊃ W) • (X ⊃ W)	
	2.	~ (~ X • V)	
		∴ W	
	3.	~~ X ∨ ~ V	2, De M.
	4.	X ∨ ~ V	3, D.N.
	5.	~ V ∨ X	4, Com.
	6.	W ∨ W	1, 5, C.D.
	7.	W	6, Taut.
8.	1.	[(Y • Z) ⊃ A] • [(Y • B) ⊃ C]	
	2.	(B ∨ Z) • Y	
		∴ A ∨ C	
	3.	Y • (B ∨ Z)	2, Com.
	4.	Y • (Z • B)	3, Com.
	5.	(Y • Z) ∨ (Y • B)	4, Dist.
	6.	A ∨ C	1, 5, C.D.
9.	1.	~ D ⊃ (~ E ⊃ ~ F)	
	2.	~ (F • ~ D) ⊃ ~ G	
		∴ G ⊃ E	
	3.	~ D ⊃ (F ⊃ E)	1, Trans.
	4.	(~ D • F) ⊃ E	3, Exp.
	5.	(F • ~ D) ⊃ E	4, Com.
	6.	G ⊃ (F • ~ D)	2, Trans.
	7.	G ⊃ E	6, 5, H.S.

10.	1.	$[H \vee (I \vee J)] \supset (K \supset J)$	
	2.	$L \supset [I \vee (J \vee H)]$	
		$\therefore (L \bullet K) \supset J$	
	3.	$[(I \vee J) \vee H] \supset (K \supset J)$	1, Com.
	4.	$[I \vee (J \vee H)] \supset (K \supset J)$	3, Assoc.
	5.	$L \supset (K \supset J)$	2, 4, H.S.
	6.	$(L \bullet K) \supset J$	5, Exp.
11.	1.	$M \supset N$	
	2.	$M \supset (N \supset O)$	
		$\therefore M \supset O$	
	3.	$M \supset (M \bullet N)$	1, Abs.
	4.	$(M \bullet N) \supset O$	2, Exp.
	5.	$M \supset O$	3, 4, H.S.
12.	1.	$(P \supset Q) \bullet (P \vee R)$	
	2.	$(R \supset S) \bullet (R \vee P)$	
		$\therefore Q \vee S$	
	3.	$P \supset Q$	1, Simp.
	4.	$R \supset S$	2, Simp.
	5.	$(P \supset Q) \bullet (R \supset S)$	3, 4, Conj.
	6.	$(P \vee R) \bullet (P \supset Q)$	1, Com.
	7.	$P \vee R$	6, Simp.
	8.	$Q \vee S$	5, 7, C.D.
13.	1.	$T \supset (U \bullet V)$	
	2.	$(U \vee V) \supset W$	
		$\therefore T \supset W$	
	3.	$\sim T \vee (U \bullet V)$	1, Impl.
	4.	$(\sim T \vee U) \bullet (\sim T \vee V)$	3, Dist.
	5.	$\sim T \vee U$	4, Simp.
	6.	$(\sim T \vee U) \vee V$	5, Add.
	7.	$\sim T \vee (U \vee V)$	6, Assoc.
	8.	$T \supset (U \vee V)$	7, Impl.
	9.	$T \supset W$	8, 2, H.S.
14.	1.	$(X \vee Y) \supset (X \bullet Y)$	
	2.	$\sim(X \vee Y)$	
		$\therefore \sim (X \bullet Y)$	

	3.	$\sim X \bullet \sim Y$	2, De M.
	4.	$\sim X$	3, Simp.
	5.	$\sim X \vee \sim Y$	4, Add.
	6.	$\sim (X \bullet Y)$	5, De M.
15.	1.	$(Z \supset Z) \supset (A \supset A)$	
	2.	$(A \supset A) \supset (Z \supset Z)$	
		$\therefore A \supset A$	
	3.	$[(Z \supset Z) \supset (A \supset A)] \vee \sim A$	1, Add.
	4.	$\sim A \vee [(Z \supset Z) \supset (A \supset A)]$	3, Com.
	5.	$A \supset [(Z \supset Z) \supset (A \supset A)]$	4, Impl.
	6.	$A \supset \{A \bullet [(Z \supset Z) \supset (A \supset A)]\}$	5, Abs.
	7.	$\sim A \vee \{A \bullet [(Z \supset Z) \supset (A \supset A)]\}$	6, Impl.
	8.	$(\sim A \vee A) \bullet \{\sim A \vee [(Z \supset Z) \supset (A \supset A)]\}$	7, Dist.
	9.	$\sim A \vee A$	8, Simp.
	10.	$A \supset A$	9, Impl.
16.	1.	$\sim B \vee [(C \supset D) \bullet (E \supset D)]$	
	2.	$B \bullet (C \vee E)$	
		$\therefore D$	
	3.	B	2, Simp.
	4.	$\sim \sim B$	3, D.N.
	5.	$(C \supset D) \bullet (E \supset D)$	1, 4, D.S.
	6.	$(C \vee E) \bullet B$	2, Com.
	7.	$C \vee E$	6, Simp.
	8.	$D \vee D$	5, 7, C.D.
	9.	D	8, Taut.
17.	1.	$\sim F \vee \sim [\sim (G \bullet H) \bullet (G \vee H)]$	
	2.	$(G \supset H) \supset [(H \supset G) \supset I]$	
		$\therefore F \supset (F \bullet I)$	
	3.	$F \supset \sim [\sim (G \bullet H) \bullet (G \vee H)]$	1, Impl.
	4.	$F \supset [\sim \sim (G \bullet H) \vee \sim (G \vee H)]$	3, De M.
	5.	$F \supset [(G \bullet H) \vee \sim (G \vee H)]$	4, D.N.
	6.	$F \supset [(G \bullet H) \vee (\sim G \bullet \sim H)]$	5, De M.
	7.	$F \supset (G \equiv H)$	6, Equiv.
	8.	$[(G \supset H) \bullet (H \supset G)] \supset I$	2, Exp.
	9.	$(G \equiv H) \supset I$	8, Equiv.
	10.	$F \supset I$	7, 9, H.S.
	11.	$F \supset (F \bullet I)$	10, Abs.

18.	1.	$J \vee (\sim J \bullet K)$	
	2.	$J \supset L$	
		$\therefore (L \bullet J) \equiv J$	
	3.	$J \supset (J \bullet L)$	2, Abs.
	4.	$J \supset (L \bullet J)$	3, Com.
	5.	$(J \vee \sim J) \bullet (J \vee K)$	1, Dist.
	6.	$J \vee \sim J$	5, Simp.
	7.	$\sim J \vee J$	6, Com.
	8.	$(\sim J \vee J) \vee \sim L$	7, Add.
	9.	$\sim L \vee (\sim J \vee J)$	8, Com.
	10.	$(\sim L \vee \sim J) \vee J$	9, Assoc.
	11.	$\sim (L \bullet J) \vee J$	10, De M.
	12.	$(L \bullet J) \supset J$	11, Impl.
	13.	$[(L \bullet J) \supset J] \bullet [J \supset (L \bullet J)]$	12, 4, Conj.
	14.	$(L \bullet J) \equiv J$	13, Equiv.

19.	1.	$(M \supset N) \bullet (O \supset P)$	
	2.	$\sim N \vee \sim P$	
	3.	$\sim (M \bullet O) \supset Q$	
		$\therefore Q$	
	4.	$(\sim N \supset \sim M) \bullet (O \supset P)$	1, Trans.
	5.	$(\sim N \supset \sim M) \bullet (\sim P \supset \sim O)$	4, Trans.
	6.	$\sim M \vee \sim O$	5, 2, C.D.
	7.	$\sim (M \bullet O)$	6, De M.
	8.	Q	3, 7, M.P.

20.	1.	$(R \vee S) \supset (T \bullet U)$	
	2.	$\sim R \supset (V \supset \sim V)$	
	3.	$\sim T$	
		$\therefore \sim V$	
	4.	$\sim T \vee \sim U$	3, Add.
	5.	$\sim (T \bullet U)$	4, De M.
	6.	$\sim (R \vee S)$	1, 5, M.T.
	7.	$\sim R \bullet \sim S$	6, De M.
	8.	$\sim R$	7, Simp.
	9.	$V \supset \sim V$	2, 8, M.P.
	10.	$\sim V \vee \sim V$	9, Impl.
	11.	$\sim V$	10, Taut.

Section 9.8 – F
Exercises on pages 380–383

1.
1.	~ N ∨ A	
2.	N	
	∴ A	
3.	N ⊃ A	1, Impl.
4.	A	3, 2, M.P.

2.
1.	C ∨ V	
2.	~ V	
	∴ C	
3.	V ∨ C	1, Com.
4.	C	3, 2, D.S.

3.
1.	(~ A ⊃ D) • (A ⊃ I)	
2.	A ∨ ~A	
	∴ D ∨ I	
3.	~ A ∨ A	2, Com.
4.	D ∨ I	1, 3, C.D.

4.
1.	~ (F ∨ ~ A)	
	∴ A	
2.	~ F • ~~ A	1, De M.
3.	~~ A • ~ F	2, Com.
4.	~~ A	3, Simp.
5.	A	4, D.N.

5.
1.	R ⊃ A	
	∴ R ⊃ (A ∨ W)	
2.	~ R ∨ A	1, Impl.
3.	(~ R ∨ A) ∨ W	2, Add.
4.	~ R ∨ (A ∨ W)	3, Assoc.
5.	R ⊃ (A ∨ W)	4, Impl.

6.
1.	F ⊃ R	
2.	R ⊃ ~ E	
3.	F	
	∴ ~ E	

	4.	R	1, 3, M.P.
	5.	$\sim E$	2, 4, M.P.
7.	1.	$M \supset \sim R$	
	2.	$R \vee V$	
	3.	M	
		$\therefore V$	
	4.	$\sim R$	1, 3, M.P.
	5.	V	2, 4, D.S.
8.	1.	$U \supset C$	
	2.	$L \vee U$	
	3.	$\sim L$	
		$\therefore C$	
	4.	U	2, 3, D.S.
	5.	C	1, 4, M.P.
9.	1.	$C \supset M$	
	2.	$M \supset P$	
	3.	$P \supset I$	
		$\therefore C \supset I$	
	4.	$C \supset P$	1, 2, H.S.
	5.	$C \supset I$	4, 3, H.S.
10.	1.	$(G \bullet S) \supset D$	
	2.	$(S \supset D) \supset P$	
	3.	G	
		$\therefore P$	
	4.	$G \supset (S \supset D)$	1, Exp.
	5.	$S \supset D$	4, 3, M.P.
	6.	P	2, 5, M.P.
11.	1.	$G \supset F$	
	2.	$F \supset \sim P$	
	3.	P	
		$\therefore \sim G$	
	4.	$G \supset \sim P$	1, 2, H.S.
	5.	$\sim \sim P \supset \sim G$	4, Trans.
	6.	$P \supset \sim G$	5, D.N.
	7.	$\sim G$	6, 3, M.P.

12.	1.	$F \supset W$	
		$\therefore (F \bullet S) \supset W$	
	2.	$(F \supset W) \vee \sim S$	1, Add.
	3.	$\sim S \vee (F \supset W)$	2, Com.
	4.	$S \supset (F \supset W)$	3, Impl.
	5.	$(S \bullet F) \supset W$	4, Exp.
	6.	$(F \bullet S) \supset W$	5, Com.
13.	1.	$(C \supset H) \bullet (A \supset L)$	
	2.	$(H \bullet L) \supset O$	
	3.	$\sim O$	
		$\therefore \sim C \vee \sim A$	
	4.	$\sim (H \bullet L)$	2, 3, M.T.
	5.	$\sim H \vee \sim L$	4, De M.
	6.	$(\sim H \supset \sim C) \bullet (A \supset L)$	1, Trans.
	7.	$(\sim H \supset \sim C) \bullet (\sim L \supset \sim A)$	6, Trans.
	8.	$\sim C \vee \sim A$	7, 5, C.D.
14.	1.	$I \supset (M \supset C)$	
	2.	$\sim C \bullet I$	
		$\therefore \sim M$	
	3.	$\sim C$	2, Simp.
	4.	$I \bullet \sim C$	2, Com.
	5.	I	4, Simp.
	6.	$M \supset C$	1, 5, M.P.
	7.	$\sim M$	6, 3, M.T.
15.	1.	$M \supset \sim C$	
	2.	$\sim C \supset \sim A$	
	3.	$D \vee A$	
		$\therefore \sim M \vee D$	
	4.	$M \supset \sim A$	1, 2, H.S.
	5.	$A \vee D$	3, Com.
	6.	$\sim\sim A \vee D$	5, D.N.
	7.	$\sim A \supset D$	6, Impl.
	8.	$M \supset D$	4, 7, H.S.
	9.	$\sim M \vee D$	8, Impl.
16.	1.	$(T \vee C) \supset (V \bullet P)$	
	2.	$P \supset O$	

	3.	$\sim O$	
		$\therefore \sim T$	
	4.	$\sim P$	2, 3, M.T.
	5.	$\sim P \vee \sim V$	4, Add.
	6.	$\sim V \vee \sim P$	5, Com.
	7.	$\sim (V \bullet P)$	6, De M.
	8.	$\sim (T \vee C)$	1, 7, M.T.
	9.	$\sim T \bullet \sim C$	8, De M.
	10.	$\sim T$	9, Simp.

17.	1.	$(D \supset F) \bullet (P \supset N)$	
	2.	$D \vee P$	
	3.	$(D \supset \sim N) \bullet (P \supset \sim F)$	
		$\therefore F \equiv \sim N$	
	4.	$F \vee N$	1, 2, C.D.
	5.	$\sim N \vee \sim F$	3, 2, C.D.
	6.	$N \vee F$	4, Com.
	7.	$\sim \sim N \vee F$	6, D.N.
	8.	$\sim N \supset F$	7, Impl.
	9.	$\sim F \vee \sim N$	5, Com.
	10.	$F \supset \sim N$	9, Impl.
	11.	$(F \supset \sim N) \bullet (\sim N \supset F)$	10, 8, Conj.
	12.	$F \equiv \sim N$	11, Equiv.

18.	1.	$W \bullet (A \bullet M)$	
	2.	$(A \bullet W) \supset [N \vee (R \vee H)]$	
	3.	$\sim N \bullet (\sim P \bullet \sim H)$	
		$\therefore R$	
	4.	$(W \bullet A) \bullet M$	1, Assoc.
	5.	$W \bullet A$	4, Simp.
	6.	$A \bullet W$	5, Com.
	7.	$N \vee (R \vee H)$	2, 6, M.P.
	8.	$\sim N$	3, Simp.
	9.	$R \vee H$	7, 8, D.S.
	10.	$H \vee R$	9, Com.
	11.	$(\sim N \bullet \sim P) \bullet \sim H$	3, Assoc.
	12.	$\sim H \bullet (\sim N \bullet \sim P)$	11, Com.
	13.	$\sim H$	12, Simp.
	14.	R	10, 13, D.S.

19.	1.	D ∨ (I • S)	
	2.	(D ⊃ L) • (L ⊃ S)	
		∴ S	
	3.	(L ⊃ S) • (D ⊃ L)	2, Com.
	4.	D ⊃ L	2, Simp.
	5.	L ⊃ S	3, Simp.
	6.	D ⊃ S	4, 5, H.S.
	7.	D ∨ (S • I)	1, Com.
	8.	(D ∨ S) • (D ∨ I)	7, Dist.
	9.	D ∨ S	8, Simp.
	10.	S ∨ D	9, Com.
	11.	~ ~ S ∨ D	10, D.N.
	12.	~ S ⊃ D	11, Impl.
	13.	~ S ⊃ S	12, 6, H.S.
	14.	~ ~ S ∨ S	13, Impl.
	15.	S ∨ S	14, D.N.
	16.	S	15, Taut.

20.	1.	P ⊃ ~ M	
	2.	C ⊃ M	
	3.	~ L ∨ C	
	4.	(~ P ⊃ ~ E) • (~ E ⊃ ~ C)	
	5.	P ∨ ~ P	
		∴ ~ L	
	6.	(~ E ⊃ ~ C) • (~ P ⊃ ~ E)	4, Com.
	7.	~ P ⊃ ~ E	4, Simp.
	8.	~ E ⊃ ~ C	6, Simp.
	9.	~ P ⊃ ~ C	7, 8, H.S
	10.	~ M ⊃ ~ C	2, Trans.
	11.	P ⊃ ~ C	1, 10, H.S.
	12.	(P ⊃ ~ C) • (~ P ⊃ ~ C)	11, 9, Conj.
	13.	~ C ∨ ~ C	12, 5, C.D.
	14.	~ C	13, Taut.
	15.	C ∨ ~ L	3, Com.
	16.	~ L	15, 14, D.S.

Section 9.8 – G
Exercises on page 383

1.
1. (H ⊃ P) • (S ⊃ W)
   ∴ (H ∨ S) ⊃ (P ∨ W)
2. H ⊃ P — 1, Simpl.
3. ~ H ∨ P — 2, Impl.
4. (~ H ∨ P) ∨ W — 3, Add.
5. ~ H ∨ (P ∨ W) — 4, Assoc.
6. (P ∨ W) ∨ ~ H — 5, Com.
7. (S ⊃ W) • (H ⊃ P) — 1, Com.
8. S ⊃ W — 7, Simp.
9. ~ S ∨ W — 8, Impl.
10. (~ S ∨ W) ∨ P — 9, Add.
11. ~ S ∨ (W ∨ P) — 10, Assoc.
12. ~ S ∨ (P ∨ W) — 11, Com.
13. (P ∨ W) ∨ ~ S — 12, Com.
14. [(P ∨ W) ∨ ~ H] • [(P ∨ W) ∨ ~ S] — 6, 13, Conj.
15. (P ∨ W) ∨ (~ H • ~S) — 14, Dist.
16. (~ H • ~ S) ∨ (P ∨ W) — 15, Com.
17. ~ (H ∨ S) ∨ (P ∨ W) — 16, De M.
18. (H ∨ S) ⊃ (P ∨ W) — 17, Impl.

2.
1. (H ⊃ P) • (S ⊃ W)
   ∴ (H • S) ⊃ (P • W)
2. H ⊃ P — 1, Simpl.
3. ~ H ∨ P — 2, Impl.
4. (~ H ∨ P) ∨ ~ S — 3, Add.
5. ~ H ∨ (P ∨ ~ S) — 4, Assoc.
6. ~ H ∨ (~ S ∨ P) — 5, Com.
7. (~ H ∨ ~ S) ∨ P — 6, Assoc.
8. (S ⊃ W) • (H ⊃ P) — 1, Com.
9. S ⊃ W — 8, Simp.
10. ~ S ∨ W — 9, Impl.
11. (~ S ∨ W) ∨ ~ H — 10, Add.
12. ~ H ∨ (~ S ∨ W) — 11, Com.
13. (~ H ∨ ~ S) ∨ W — 12, Assoc.
14. [(~ H ∨ ~ S) ∨ P] • [(~ H ∨ ~ S) ∨ W] — 7, 13, Conj.

	15.	(~ H v ~ S) v (P • W)	14, Dist.
	16.	~ (H • S) v (P • W)	15, De M.
	17.	(H • S) ⊃ (P • W)	16, Impl.
3.	1.	F ⊃ ~ A	
	2.	F ⊃ (~ A ⊃ ~ P)	
	3.	~ A ⊃ (~ P ⊃ ~ C)	
		∴ F ⊃ ~ C	
	4.	F ⊃ (F • ~ A)	1, Abs.
	5.	(F • ~ A) ⊃ ~ P	2, Exp.
	6.	F ⊃ ~ P	4, 5, H.S.
	7.	F ⊃ (~ P ⊃ ~ C)	1, 3, H.S.
	8.	F ⊃ (F • ~ P)	6, Abs.
	9.	(F • ~ P) ⊃ ~ C	7, Exp.
	10.	F ⊃ ~ C	8, 9, H.S.
4.	1.	G	
		∴ H v ~ H	
	2.	G v ~ H	1, Add.
	3.	~ H v G	2, Com.
	4.	H ⊃ G	3, Impl.
	5.	H ⊃ (H • G)	4, Abs.
	6.	~ H v (H • G)	5, Impl.
	7.	(~ H v H) • (~ H v G)	6, Dist.
	8.	~ H v H	7, Simp.
	9.	H v ~ H	8, Com.
5.	1.	(H v ~ H) ⊃ G	
		∴ G	
	2.	[(H v ~ H) ⊃ G] v ~ H	1, Add.
	3.	~ H v [(H v ~ H) ⊃ G]	2, Com.
	4.	H ⊃ [(H v ~ H) ⊃ G]	3, Impl.
	5.	H ⊃ {H • [(H v ~ H) ⊃ G]}	4, Abs.
	6.	~ H v {H • [(H v ~ H) ⊃ G]}	5, Impl.
	7.	(~ H v H) • {~ H v [(H v ~ H) ⊃ G]}	6, Dist.
	8.	~ H v H	7, Simp.
	9.	H v ~ H	8, Com.
	10.	G	1, 9, M.P.

Section 9.9
Exercises on page 385

1.

	A	B	C	D
	F	F	F	T

2.

	E	F	G	H
	T	F	F	F
or	F	T	F	F

3.

	I	J	K	L
	T	F	F	F

4.

	M	N	O	P	Q	R
	T	F	T	F	F	F

5.

	S	T	U	V	W	X
	T	F	F	T	T	T
or	T	F	F	T	F	T
or	T	F	F	T	F	F
or	T	F	F	F	T	T
or	T	F	F	F	T	F
or	T	F	F	F	F	T
or	T	F	F	F	F	F
or	F	F	T	T	T	T
or	F	F	T	T	F	T
or	F	F	T	T	F	F
or	F	F	T	F	T	T
or	F	F	T	F	T	F
or	F	F	T	F	F	T
or	F	F	T	F	F	F

6.

	A	B	C
	F	F	F

7.

	D	E	F	G	H	I	J
	T	T	T	F	T	F	F
or	T	T	T	F	F	F	F
or	T	T	F	F	T	F	F
or	T	T	F	F	F	F	F

8.

	K	L	M	N	O	P	Q	R
	T	T	T	T	F	F	F	F

9.

	S	T	U	V	W	X	Y	Z
	T	T	T	F	T	F	F	T
or	T	T	T	F	F	F	F	T

10.

	A	B	C	D	E	F	G	H	I	J
	T	T	F	T	F	T	F	T	F	T
or	F	T	T	T	F	T	F	T	F	T
or	F	T	F	T	F	T	F	T	F	T

Section 9.10 – A
Exercises on page 389

1.

1.	(A ⊃ B) • (C ⊃ D)	
	∴ (A • C) ⊃ (B ∨ D)	
2.	A ⊃ B	1, Simp.
3.	~ A ∨ B	2, Impl.
4.	(~ A ∨ B) ∨ D	3, Add.
5.	~ A ∨ (B ∨ D)	4, Assoc.
6.	[~ A ∨ (B ∨ D)] ∨ ~ C	5, Add.
7.	~ C ∨ [~ A ∨ (B ∨ D)]	6, Com.
8.	(~ C ∨ ~ A) ∨ (B ∨ D)	7, Assoc.
9.	(~ A ∨ ~ C) ∨ (B ∨ D)	8, Com.
10.	~ (A • C) ∨ (B ∨ D)	9, De M.
11.	(A • C) ⊃ (B ∨ D)	10, Impl.

2.

	E	F	G	H
	T	T	F	F
or	F	F	T	T

3.

	I	J	K	L
	T	T	F	F
or	T	F	T	F

4.

1.	M ⊃ (N • O)	
2.	(N ∨ O) ⊃ P	
	∴ M ⊃ P	
3.	~ M ∨ (N • O)	1, Impl.
4.	(~ M ∨ N) • (~ M ∨ O)	3, Dist.
5.	~ M ∨ N	4, Simp.
6.	(~ M ∨ N) ∨ O	5, Add.
7.	~ M ∨ (N ∨ O)	6, Assoc.
8.	M ⊃ (N ∨ O)	7, Impl.
9.	M ⊃ P	8, 2, H.S.

5.

X	Y	Z	A	B	C
T	F	T	F	T	F

6.

1.	$[(D \vee E) \bullet F] \supset G$	
2.	$(F \supset G) \supset (H \supset I)$	
3.	H	
	$\therefore D \supset I$	
4.	$(D \vee E) \supset (F \supset G)$	1, Exp.
5.	$\sim (D \vee E) \vee (F \supset G)$	4, Impl.
6.	$(F \supset G) \vee \sim (D \vee E)$	3, Com.
7.	$(F \supset G) \vee (\sim D \bullet \sim E)$	6, De M.
8.	$[(F \supset G) \vee \sim D] \bullet [(F \supset G) \vee \sim E]$	7, Dist.
9.	$(F \supset G) \vee \sim D$	8, Simp.
10.	$\sim D \vee (F \supset G)$	9, Com.
11.	$D \supset (F \supset G)$	10, Impl.
12.	$D \supset (H \supset I)$	11, 2, H.S.
13.	$(D \bullet H) \supset I$	12, Exp.
14.	$(H \bullet D) \supset I$	13, Com.
15.	$H \supset (D \supset I)$	14, Exp.
16.	$D \supset I$	15, 3, M.P.

7.

1.	$(J \bullet K) \supset (L \supset M)$	
2.	$N \supset \sim M$	
3.	$\sim (K \supset \sim N)$	
4.	$\sim (J \supset \sim L)$	
	$\therefore \sim J$	
5.	$\sim (\sim K \vee \sim N)$	3, Impl.
6.	$\sim \sim K \bullet \sim \sim N$	5, De M.
7.	$K \bullet \sim \sim N$	6, D.N.
8.	$K \bullet N$	7, D.N.
9.	K	8, Simp.
10.	$N \bullet K$	8, Com.
11.	N	10, Simp.
12.	$\sim (\sim J \vee \sim L)$	4, Impl.
13.	$\sim \sim J \bullet \sim \sim L$	12, De M.
14.	$J \bullet \sim \sim L$	13, D.N.
15.	J	14, Simp.
16.	$\sim \sim L \bullet J$	14, Com.
17.	$\sim \sim L$	16, Simp.

18.	$J \bullet K$	15, 9, Conj.
19.	$L \supset M$	1, 18, M.P.
20.	$\sim M$	2, 11, M.P.
21.	$\sim L$	19, 20, M.T.
22.	$\sim L \vee \sim J$	21, Add.
23.	$\sim J$	22, 17, D.S.

8.

O	P	Q	R	S
T	T	F	F	T

9.

1.	$T \supset (U \bullet V)$	
2.	$U \supset (W \bullet X)$	
3.	$(T \supset W) \supset (Y \equiv Z)$	
4.	$(T \supset U) \supset \sim Y$	
5.	$\sim Y \supset (\sim Z \supset X)$	
	$\therefore X$	
6.	$\sim T \vee (U \bullet V)$	1, Impl.
7.	$(\sim T \vee U) \bullet (\sim T \vee V)$	6, Dist.
8.	$\sim T \vee U$	7, Simp.
9.	$T \supset U$	8, Impl.
10.	$\sim Y$	4, 9, M.P.
11.	$\sim Z \supset X$	5, 10, M.P.
12.	$\sim U \vee (W \bullet X)$	2, Impl.
13.	$(\sim U \vee W) \bullet (\sim U \vee X)$	12, Dist.
14.	$\sim U \vee W$	13, Simp.
15.	$U \supset W$	14, Impl.
16.	$T \supset W$	9, 15, H.S.
17.	$Y \equiv Z$	3, 16, M.P.
18.	$(Y \supset Z) \bullet (Z \supset Y)$	17, Equiv.
19.	$(Z \supset Y) \bullet (Y \supset Z)$	18, Com.
20.	$Z \supset Y$	19, Simp.
21.	$\sim Z$	20, 10, M.T.
22.	X	11, 21, M.P.

10.

	A	B	C	D	E	F	G
	F	F	T	T	F	T	T
or	F	F	T	F	F	T	T
or	F	F	F	T	F	T	T
or	F	F	F	F	F	T	T

Section 9.10 – B
Exercises on pages 389–391

1.	1.	$C \supset (M \supset D)$	
	2.	$D \supset V$	
	3.	$(D \supset A) \bullet \sim A$	
		$\therefore M \supset \sim C$	
	4.	$D \supset A$	3, Simp.
	5.	$\sim A \bullet (D \supset A)$	3, Com.
	6.	$\sim A$	5, Simp.
	7.	$\sim D$	4, 6, M.T.
	8.	$(C \bullet M) \supset D$	1, Exp.
	9.	$\sim (C \bullet M)$	8, 7, M.T.
	10.	$\sim C \vee \sim M$	9, De M.
	11.	$\sim M \vee \sim C$	10, Com.
	12.	$M \supset \sim C$	11, Impl.

2.	1.	$(O \bullet T) \supset (S \supset M)$	
	2.	$R \supset \sim M$	
	3.	$T \bullet R$	
	4.	$O \bullet S$	
		$\therefore V$	
	5.	O	4, Simp.
	6.	T	3, Simp.
	7.	$O \bullet T$	5, 6, Conj.
	8.	$S \supset M$	1, 7, M.P.
	9.	$S \bullet O$	4, Com.
	10.	S	9, Simp.
	11.	M	8, 10, M.P.
	12.	$R \bullet T$	3, Com.
	13.	R	12, Simp.
	14.	$\sim M$	2, 13, M.P.
	15.	$M \vee V$	11, Add.
	16.	V	15, 14, D.S.

3.	1.	$[(W \bullet \sim A) \supset I] \bullet [(A \bullet \sim W) \supset M]$	
	2.	$E \supset (\sim W \vee \sim A)$	
	3.	E	
	4.	$G \supset (\sim I \bullet \sim M)$	
		$\therefore \sim G$	

Proved invalid by the following assignment of truth values:

W	A	I	M	E	G
F	F	F	F	T	T

4. 1. (N ∨ F) ⊃ (C • D)
   2. D ⊃ V
   3. V ⊃ I
   4. I ⊃ A
   5. A ⊃ ~ C
      ∴ ~ F
   6. D ⊃ I — 2, 3, H.S.
   7. D ⊃ A — 6, 4, H.S.
   8. D ⊃ ~ C — 7, 5, H.S.
   9. ~ D ∨ ~ C — 8, Impl.
   10. ~ C ∨ ~ D — 9, Com.
   11. ~ (C • D) — 10, De M.
   12. ~ (N ∨ F) — 1, 11, M.T.
   13. ~ N • ~ F — 12, De M.
   14. ~ F • ~ N — 13, Com.
   15. ~ F — 14, Simp.

5. (I • S) ⊃ (G • P)
   [(S • ~ I) ⊃ A] • (A ⊃ P)
   I ⊃ S
   ∴ P

	I	S	G	P	A
proved invalid by	F	F	T	F	F
or	F	F	F	F	F

6. 1. N ⊃ ~ (M • G)
   2. [(P ∨ D) ⊃ ~ W] • [~ W ⊃ ~ (K ∨ S)]
   3. ~ G • (D • K)
      ∴ ~ N
   4. (D • K) • ~ G — 3, Com.
   5. D • K — 4, Simp.
   6. D — 5, Simp.
   7. D ∨ P — 6, Add.
   8. P ∨ D — 7, Com.
   9. (P ∨ D) ⊃ ~ W — 2, Simp.

	10.	$\sim W$	9, 8, M.P.
	11.	$[\sim W \supset \sim (K \vee S)] \bullet [(P \vee D) \supset \sim W]$	2, Com.
	12.	$\sim W \supset \sim (K \vee S)$	11, Simp.
	13.	$\sim (K \vee S)$	12, 10, M.P.
	14.	$\sim K \bullet \sim S$	13, De M.
	15.	$\sim K$	14, Simp.
	16.	$K \bullet D$	5, Com.
	17.	K	16, Simp.
	18.	$K \vee \sim N$	17, Add.
	19.	$\sim N$	18, 15, D.S.

7.	1.	$(P \supset S) \bullet (S \supset Q)$	
	2.	$(Q \supset R) \bullet (R \supset H)$	
	3.	$\sim H$	
	4.	$[(\sim S \bullet \sim H) \supset D] \bullet (D \supset P)$	
		$\therefore Q$	
	5.	$(S \supset Q) \bullet (P \supset S)$	1, Com.
	6.	$(R \supset H) \bullet (Q \supset R)$	2, Com.
	7.	$(D \supset P) \bullet [(\sim S \bullet \sim H) \supset D]$	4, Com.
	8.	$S \supset Q$	5, Simp.
	9.	$Q \supset R$	2, Simp.
	10.	$S \supset R$	8, 9, H.S.
	11.	$R \supset H$	6, Simp.
	12.	$S \supset H$	10, 11, H.S.
	13.	$\sim S$	12, 3, M.T.
	14.	$\sim S \bullet \sim H$	13, 3, Conj.
	15.	$(\sim S \bullet \sim H) \supset D$	4, Simp.
	16.	D	15, 14, M.P.
	17.	$D \supset P$	7, Simp.
	18.	$P \supset S$	1, Simp.
	19.	$D \supset S$	17, 18, H.S.
	20.	$D \supset Q$	19, 8, H.S.
	21.	Q	20, 16, M.P.

8.	1.	$(B \supset W) \bullet (G \supset \sim S)$	
	2.	$(\sim B \bullet \sim G) \supset (C \bullet P)$	
	3.	$\sim W$	
	4.	P	
		$\therefore C \supset \sim G$	

Proved invalid by the following assignment of truth values:

B	W	G	S	C	P
F	F	T	F	T	T

9.

1.	$(F \supset L) \bullet (\sim L \vee F)$	
2.	$(F \bullet L) \supset (D \bullet P)$	
3.	$(\sim F \bullet \sim L) \supset I$	
4.	$(I \supset C) \bullet (C \supset P)$	
	$\therefore P$	
5.	$(C \supset P) \bullet (I \supset C)$	4, Com.
6.	$I \supset C$	4, Simp.
7.	$C \supset P$	5, Simp.
8.	$I \supset P$	6, 7, H.S.
9.	$(\sim F \bullet \sim L) \supset P$	3, 8, H.S.
10.	$(F \supset L) \bullet (L \supset F)$	1, Impl.
11.	$F \equiv L$	10, Equiv.
12.	$(F \bullet L) \vee (\sim F \bullet \sim L)$	11, Equiv.
13.	$[(F \bullet L) \supset (D \bullet P)] \bullet [(\sim F \bullet \sim L) \supset P]$	2, 9, Conj.
14.	$(D \bullet P) \vee P$	13, 12, C. D.
15.	$P \vee (D \bullet P)$	14, Com.
16.	$P \vee (P \bullet D)$	15, Com.
17.	$(P \vee P) \bullet (P \vee D)$	16, Dist.
18.	$P \vee P$	17, Simp.
19.	P	18, Taut.

10. $(H \supset A) \bullet (F \supset C)$
$A \supset (F \bullet E)$
$(O \supset C) \bullet (O \supset M)$
$P \supset (M \supset D)$
$P \bullet (D \supset G)$
$\therefore H \supset G$

Proved invalid by the following assignment of truth values:

H	A	C	F	E	O	M	P	D	G
T	T	T	T	T	F	F	T	F	F

11.	1.	$L \supset H$	
	2.	$L \supset (H \supset F)$	
	3.	$H \supset (F \supset D)$	
		$\therefore L \supset D$	
	4.	$L \supset (L \bullet H)$	1, Abs.
	5.	$(L \bullet H) \supset F$	2, Exp.
	6.	$L \supset F$	4, 5, H.S.
	7.	$L \supset (F \supset D)$	1, 3, H.S.
	8.	$L \supset (L \bullet F)$	6, Abs.
	9.	$(L \bullet F) \supset D$	7, Exp.
	10.	$L \supset D$	8, 9, H.S.

12.	1.	$(L \supset H) \bullet (Q \supset S)$	
		$\therefore (L \bullet Q) \supset (H \bullet S)$	
	2.	$L \supset H$	1, Simp.
	3.	$\sim L \vee H$	2, Impl.
	4.	$(\sim L \vee H) \vee \sim Q$	3, Add.
	5.	$\sim L \vee (H \vee \sim Q)$	4, Assoc.
	6.	$\sim L \vee (\sim Q \vee H)$	5, Com.
	7.	$(\sim L \vee \sim Q) \vee H$	6, Assoc.
	8.	$(Q \supset S) \bullet (L \supset H)$	1, Com.
	9.	$Q \supset S$	8, Simp.
	10.	$\sim Q \vee S$	9, Impl.
	11.	$(\sim Q \vee S) \vee \sim L$	10, Add.
	12.	$\sim L \vee (\sim Q \vee S)$	11, Com.
	13.	$(\sim L \vee \sim Q) \vee S$	12, Assoc.
	14.	$[(\sim L \vee \sim Q) \vee H] \bullet [(\sim L \vee \sim Q) \vee S]$	7, 13, Conj.
	15.	$(\sim L \vee \sim Q) \vee (H \bullet S)$	14, Dist.
	16.	$\sim (L \bullet Q) \vee (H \bullet S)$	15, De M.
	17.	$(L \bullet Q) \supset (H \bullet S)$	16, Impl.

13.	1.	$(L \supset H) \bullet (Q \supset S)$	
		$\therefore (L \vee Q) \supset (H \vee S)$	
	2.	$L \supset H$	1, Simp.
	3.	$\sim L \vee H$	2, Impl.
	4.	$(\sim L \vee H) \vee S$	3, Add.
	5.	$\sim L \vee (H \vee S)$	4, Assoc.
	6.	$(H \vee S) \vee \sim L$	5, Com.
	7.	$(Q \supset S) \bullet (L \supset H)$	1, Com.

	8.	Q ⊃ S	7, Simp.
	9.	~ Q v S	8, Impl.
	10.	(~ Q v S) v H	9, Add.
	11.	~ Q v (S v H)	10, Assoc.
	12.	~ Q v (H v S)	11, Com.
	13.	(H v S) v ~ Q	12, Com.
	14.	[(H v S) v ~ L] • [(H v S) v ~ Q]	6, 13, Conj.
	15.	(H v S) v (~ L • ~ Q)	14, Dist.
	16.	(~ L • ~ Q) v (H v S)	15, Com.
	17.	~ (L v Q) v (H v S)	16, De M.
	18.	(L v Q) ⊃ (H v S)	17, Impl.
14.	1.	J ⊃ (A v S)	
	2.	K ⊃ (S v I)	
	3.	~ S	
		∴ (~ A • ~ I) ⊃ (~ J • ~ K)	
	4.	~ J v (A v S)	1, Impl.
	5.	(~ J v A) v S	4, Assoc.
	6.	S v (~ J v A)	5, Com.
	7.	~ J v A	6, 3, D.S.
	8.	(~ J v A) v I	7, Add.
	9.	~ J v (A v I)	8, Assoc.
	10.	(A v I) v ~ J	9, Com.
	11.	~ K v (S v I)	2, Impl.
	12.	(S v I) v ~ K	11, Com.
	13.	S v (I v ~ K)	12, Assoc.
	14.	I v ~ K	13, 3, D.S.
	15.	(I v ~ K) v A	14, Add.
	16.	A v (I v ~ K)	15, Com.
	17.	(A v I) v ~ K	16, Assoc.
	18.	[(A v I) v ~ J] • [(A v I) v ~ K]	10, 17, Conj.
	19.	(A v I) v (~ J • ~ K)	18, Dist.
	20.	~~ (A v I) v (~ J • ~ K)	19, D.N.
	21.	~ (~ A v ~ I) v (~ J • ~ K)	20, De M.
	22.	(~ A • ~ I) ⊃ (~ J • ~ K)	21, Impl.

15.	1.	$(J \vee A) \supset [(S \vee K) \supset (\sim I \bullet Y)]$	
	2.	$(\sim I \vee \sim M) \supset E$	
		$\therefore J \supset (S \supset E)$	
	3.	$\sim (J \vee A) \vee [(S \vee K) \supset (\sim I \bullet Y)]$	1, Impl.
	4.	$[(S \vee K) \supset (\sim I \bullet Y)] \vee \sim (J \vee A)$	3, Com.
	5.	$[(S \vee K) \supset (\sim I \bullet Y)] \vee (\sim J \bullet \sim A)$	4, De M.
	6.	$\{[(S \vee K) \supset (\sim I \bullet Y)] \vee \sim J\} \bullet \{[(S \vee K) \supset (\sim I \bullet Y)] \vee \sim A\}$	5, Dist.
	7.	$[(S \vee K) \supset (\sim I \bullet Y)] \vee \sim J$	6, Simp.
	8.	$[\sim (S \vee K) \vee (\sim I \bullet Y)] \vee \sim J$	7, Impl.
	9.	$\sim (S \vee K) \vee [(\sim I \bullet Y) \vee \sim J]$	8, Assoc.
	10.	$[(\sim I \bullet Y) \vee \sim J] \vee \sim (S \vee K)$	9, Com.
	11.	$[(\sim I \bullet Y) \vee \sim J] \vee (\sim S \bullet \sim K)$	10, De M.
	12.	$\{[(\sim I \bullet Y) \vee \sim J] \vee \sim S\} \bullet \{[(\sim I \bullet Y) \vee \sim J] \vee \sim K\}$	11, Dist.
	13.	$[(\sim I \bullet Y) \vee \sim J] \vee \sim S$	12, Simp.
	14.	$(\sim I \bullet Y) \vee (\sim J \vee \sim S)$	13, Assoc.
	15.	$(\sim J \vee \sim S) \vee (\sim I \bullet Y)$	14, Com.
	16.	$[(\sim J \vee \sim S) \vee \sim I] \bullet [(\sim J \vee \sim S) \vee Y]$	15, Dist.
	17.	$(\sim J \vee \sim S) \vee \sim I$	16, Simp.
	18.	$[(\sim J \vee \sim S) \vee \sim I] \vee \sim M$	17, Add.
	19.	$(\sim J \vee \sim S) \vee (\sim I \vee \sim M)$	18, Assoc.
	20.	$\sim (J \bullet S) \vee (\sim I \vee \sim M)$	19, De M.
	21.	$(J \bullet S) \supset (\sim I \vee \sim M)$	20, Impl.
	22.	$(J \bullet S) \supset E$	21, 2, H.S.
	23.	$J \supset (S \supset E)$	22, Exp.

Section 9. 10 – C
Exercises on page 392

1.	1.	$(B \supset S) \bullet (D \supset C)$	
	2.	$(S \vee C) \supset I$	
	3.	$\sim I$	
		$\therefore \sim (B \vee D)$	
	4.	$\sim (S \vee C)$	3, 2, M.T.
	5.	$\sim S \bullet \sim C$	4, De M.
	6.	$\sim S$	5, Simp.
	7.	$B \supset S$	1, Simp.
	8.	$\sim B$	7, 6, M.T.
	9.	$(D \supset C) \bullet (B \supset S)$	1, Com.
	10.	$D \supset C$	9, Simp.

11.	~ C • ~ S	5, Com.
12.	~ C	11, Simp.
13.	~ D	10, 12, M.T.
14.	~ B • ~ D	8, 13, Conj.
15.	~ (B ∨ D)	14, De M.

2.
1. I ⊃ H
2. I ⊃ (H ⊃ D)
3. I ⊃ (D ⊃ U)

∴ U ⊃ I

Proved invalid by any of the following assignments of truth values:

	I	H	D	U
	F	T	T	T
or	F	T	F	T
or	F	F	T	T
or	F	F	F	T

3.

1.	(T ⊃ I) • (B ⊃ U)	
2.	K ⊃ (T ∨ B)	
	∴ K ⊃ (I ∨ U)	
3.	(~T ∨ I) • (B ⊃ U)	1, Impl.
4.	(~T ∨ I) • (~B ∨ U)	3, Impl.
5.	~T ∨ I	4, Simp.
6.	(~T ∨ I) ∨ U	5, Add.
7.	~T ∨ (I ∨ U)	6, Assoc.
8.	(~B ∨ U) • (~T ∨ I)	4, Com.
9.	~B ∨ U	8, Simp.
10.	(~B ∨ U) ∨ I	9, Add.
11.	~B ∨ (U ∨ I)	10, Assoc.
12.	~B ∨ (I ∨ U)	11, Com.
13.	[~T ∨ (I ∨ U)] • [~B ∨ (I ∨ U)]	7, 12, Conj.
14.	[(I ∨ U) ∨ ~T] • [~B ∨ (I ∨ U)]	13, Com.
15.	[(I ∨ U) ∨ ~T] • [(I ∨ U) ∨ ~B]	14, Com.
16.	(I ∨ U) ∨ (~T • ~B)	15, Dist.
17.	(~T • ~B) ∨ (I ∨ U)	16, Com.
18.	~(T ∨ B) ∨ (I ∨ U)	17, De M.
19.	(T ∨ B) ⊃ (I ∨ U)	18, Impl.
20.	K ⊃ (I ∨ U)	2, 19, H.S.

4.	1.	W	
		∴ R ∨ ~ R	
	2.	W ∨ ~ R	1, Add.
	3.	~ R ∨ W	2, Com.
	4.	R ⊃ W	3, Impl.
	5.	R ⊃ (R • W)	4, Abs.
	6.	~ R ∨ (R • W)	5, Impl.
	7.	(~ R ∨ R) • (~ R ∨ W)	6, Dist.
	8.	~ R ∨ R	7, Simp.
	9.	R ∨ ~ R	8, Com.

5.	1.	(R ∨ ~ R) ⊃ W	
		∴ W	
	2.	[(R ∨ ~ R) ⊃ W] ∨ ~ R	1, Add.
	3.	~ R ∨ [(R ∨ ~ R) ⊃ W]	2, Com.
	4.	R ⊃ [(R ∨ ~ R) ⊃ W]	3, Impl.
	5.	R ⊃ {R • [(R ∨ ~ R) ⊃ W]}	4, Abs.
	6.	~ R ∨ {R • [(R ∨ ~ R) ⊃ W]}	5, Impl.
	7.	(~ R ∨ R) • {~ R ∨ [(R ∨ ~ R) ⊃ W]}	6, Dist.
	8.	~ R ∨ R	7, Simp.
	9.	R ∨ ~ R	8, Com.
	10.	W	1, 9, M.P.

Section 9.11. – A
Exercises on page 394

1.	1.	A ∨ (B • C)	
	2.	A ⊃ C	
		∴ C	
	!3.	~ C	I.P. (Indirect Proof)
	!4.	~ A	2, 3, M.T.
	!5.	B • C	1, 4, D.S.
	!6.	C • B	5, Com.
	!7.	C	6, Simp.
	!8.	C • ~ C	7, 3, Conj.

2.	1.	$(G \vee H) \supset \sim G$	
		$\therefore \sim G$	
	!2.	G	I.P. (Indirect Proof)
	!3.	$G \vee H$	2, Add.
	!4.	$\sim G$	1, 3, M.P.
	!5.	$G \bullet \sim G$	2, 4, Conj.

3.	1.	$(D \vee E) \supset (F \supset G)$	
	2.	$(\sim G \vee H) \supset (D \bullet F)$	
		$\therefore G$	
	!3.	$\sim G$	I.P. (Indirect Proof)
	!4.	$\sim G \vee H$	3, Add.
	!5.	$D \bullet F$	2, 4, M.P.
	!6.	D	5, Simp.
	!7.	$D \vee E$	6, Add.
	!8.	$F \supset G$	1, 7, M.P.
	!9.	$F \bullet D$	5, Com.
	!10.	F	9, Simp.
	!11.	G	8, 10, M.P.
	!12.	$G \bullet \sim G$	11, 3, Conj.

4.	1.	$(M \vee N) \supset (O \bullet P)$	
	2.	$(O \vee Q) \supset (\sim R \bullet S)$	
	3.	$(R \vee T) \supset (M \bullet U)$	
		$\therefore \sim R$	
	!4.	R	I.P. (Indirect Proof)
	!5.	$R \vee T$	4, Add.
	!6.	$M \bullet U$	3, 5, M.P.
	!7.	M	6, Simp.
	!8.	$M \vee N$	7, Add.
	!9.	$O \vee P$	1, 8, M.P.
	!10.	O	9, Simp.
	!11.	$O \vee Q$	10, Add.
	!12.	$\sim R \bullet S$	2, 11, M.P.
	!13.	$\sim R$	12, Simp.
	!14.	$R \bullet \sim R$	4, 13, Conj.

5.	1.	$D \supset (Z \supset Y)$	
	2.	$Z \supset (Y \supset \sim Z)$	
		$\therefore \sim D \vee \sim Z$	
	!3.	$\sim (\sim D \vee \sim Z)$	I.P. (Indirect Proof)
	!4.	$\sim\sim D \bullet \sim\sim Z$	3, De M.
	!5.	$D \bullet \sim\sim Z$	4, D.N.
	!6.	$D \bullet Z$	5, D.N.
	!7.	D	6, Simp.
	!8.	$Z \supset Y$	1, 7, M.P.
	!9.	$Z \bullet D$	6, Com.
	!10.	Z	9, Simp.
	!11.	$Y \supset \sim Z$	2, 10, M.P.
	!12.	Y	8, 10, M.P.
	!13.	$\sim Z$	11, 12, M.P.
	!14.	$Z \bullet \sim Z$	10, 13, Conj.

6.	1.	$(O \vee P) \supset (D \bullet E)$	
	2.	$(E \vee L) \supset (Q \vee \sim D)$	
	3.	$(Q \vee Z) \supset \sim (O \bullet E)$	
		$\therefore \sim O$	
	!4.	O	I.P. (Indirect Proof)
	!5.	$O \vee P$	4, Add.
	!6.	$D \bullet E$	1, 5, M.P.
	!7.	$E \bullet D$	6, Com.
	!8.	E	7, Simp.
	!9.	$E \vee L$	8, Add.
	!10.	$Q \vee \sim D$	2, 9, M.P.
	!11.	D	6, Simp.
	!12.	$\sim D \vee Q$	10, Com.
	!13.	$\sim\sim D$	11, D.N.
	!14.	Q	12, 13, D.S.
	!15.	$Q \vee Z$	14, Add.
	!16.	$\sim (O \bullet E)$	3, 15, M.P.
	!17.	$\sim O \vee \sim E$	16, De M.
	!18.	$\sim\sim O$	4, D.N.
	!19.	$\sim E$	17, 18, D.S.
	!20.	$E \bullet \sim E$	8, 19, Conj.

7.	1.	(F v G) ⊃ (D • E)	
	2.	(E v H) ⊃ Q	
	3.	F v H	
		∴ Q	
	!4.	~ Q	I.P. (Indirect Proof)
	!5.	~ (E v H)	2, 4, M.T.
	!6.	~ E • ~ H	5, De M.
	!7.	~ E	6, Simp.
	!8.	~ E v ~ D	7, Add.
	!9.	~ D v ~ E	8, Com.
	!10.	~ (D • E)	9, De M.
	!11.	~ (F v G)	1, 10, M.T.
	!12.	~ F • ~ G	11, De M.
	!13.	~ F	12, Simp.
	!14.	H	3, 13, D.S.
	!15.	~ H • ~ E	6, Com.
	!16.	~ H	15, Simp.
	!17.	H • ~ H	14, 16, Conj.

8.	1.	B ⊃ [(O v ~ O) ⊃ (T v U)]	
	2.	U ⊃ ~ (G v ~ G)	
		∴ B ⊃ T	
	!3.	~ (B ⊃ T)	I.P. (Indirect Proof)
	!4.	~ (~B v T)	3, Impl.
	!5.	~~ B • ~ T	4, De M.
	!6.	B • ~ T	5, D.N.
	!7.	B	6, Simp.
	!8.	(O v ~ O) ⊃ (T v U)	1, 7, M.P.
	!9.	B v ~ O	7, Add.
	!10.	~ O v B	9, Com.
	!11.	O ⊃ B	10, Impl.
	!12.	O ⊃ (O • B)	11, Abs.
	!13.	~ O v (O • B)	12, Impl.
	!14.	(~ O v O) • (~ O v B)	13, Dist.
	!15.	~ O v O	14, Simpl.
	!16.	O v ~ O	15, Com.
	!17.	T v U	8, 16, M.P.
	!18.	~ T • B	6, Com.
	!19.	~ T	18, Simp.

!20.	U	17, 19, D.S.
!21.	~ (G v ~ G)	2, 20, M.P.
!22.	~G • ~~G	21, De M.

Section 9. 11 – B
Exercises on page 394

1.

1.	(F ⊃ R) ⊃ I	
2.	(D ⊃ F) ⊃ I	
	∴ I	
!3.	~ I	I.P. (Indirect Proof)
!4.	~ (F ⊃ R)	1, 3, M.T.
!5.	~ (~ F v R)	4, Impl.
!6.	~~ F • ~ R	5, De M.
!7.	~~ F	6, Simp.
!8.	~ (D ⊃ F)	2, 3, M.T.
!9.	~ (~ D v F)	8, Impl.
!10.	~~ D • ~ F	9, De M.
!11.	~ F • ~~ D	10, Com.
!12.	~ F	11, Simp.
!13.	~ F • ~~ F	12, 7, Conj.

2.

1.	(L • G) ⊃ (O • P)	
2.	G ⊃ ~ P	
	∴ ~ L v ~ G	
!3.	~ (~ L v ~ G)	I.P. (Indirect Proof)
!4.	~~ L • ~~ G	3, De M.
!5.	L • ~~ G	4, D.N.
!6.	L • G	5, D.N.
!7.	O • P	1, 6, M.P.
!8.	P • O	7, Com.
!9.	P	8, Simp.
!10.	G • L	6, Com.
!11.	G	10, Simp.
!12.	~ P	2, 11, M.P.
!13.	P • ~ P	9, 13, Conj.

Section 9.11 – C
Exercises on page 394

(a) Direct formal proof

1.	(V ⊃ ~ W) • (X ⊃ Y)	
2.	(~ W ⊃ Z) • (Y ⊃ ~ A)	
3.	(Z ⊃ ~ B) • (~ A ⊃ C)	
4.	V • X	
	∴ ~ B • C	
5.	V	4, Simp.
6.	V ⊃ ~ W	1, Simp.
7.	~ W	6, 5, M.P.
8.	~ W ⊃ Z	2, Simp.
9.	Z	8, 7, M.P.
10.	Z ⊃ ~ B	3, Simp.
11.	~ B	10, 9, M.P.
12.	(X ⊃ Y) • (V ⊃ ~ W)	1, Com.
13.	X ⊃ Y	12, Simp.
14.	X • V	4, Com.
15.	X	14, Simp.
16.	Y	13, 15, M.P.
17.	(Y ⊃ ~ A) • (~ W ⊃ Z)	2, Com.
18.	Y ⊃ ~ A	17, Simp.
19.	~ A	18, 16, M.P.
20.	(~A ⊃ C) • (Z ⊃ ~B)	3, Com.
21.	~ A ⊃ C	20, Simp.
22.	C	21, 19, M.P.
23.	~ B • C	11, 22, Conj.

(b) Indirect proof

1.	(V ⊃ ~ W) • (X ⊃ Y)	
2.	(~ W ⊃ Z) • (Y ⊃ ~ A)	
3.	(Z ⊃ ~ B) • (~ A ⊃ C)	
4.	V • X	
	∴ ~ B • C	
!5.	~ (~ B • C)	I.P. (Indirect Proof)
!6.	~~ B ∨ ~ C	5, De M.
!7.	(~~B ⊃ ~Z) • (~A ⊃ C)	3, Trans.
!8.	(~~ B ⊃ ~ Z) • (~ C ⊃ ~~ A)	7, Trans.

!9.	$\sim Z \vee \sim\sim A$	8, 6, C.D.
!10.	$(\sim Z \supset \sim\sim W) \bullet (Y \supset \sim A)$	2, Trans.
!11.	$(\sim Z \supset \sim\sim W) \bullet (\sim\sim A \supset \sim Y)$	10, Trans.
!12.	$\sim\sim W \vee \sim Y$	11, 9, C.D.
!13.	$(\sim\sim W \supset \sim V) \bullet (X \supset Y)$	1, Trans.
!14.	$(\sim\sim W \supset \sim V) \bullet (\sim Y \supset \sim X)$	13, Trans.
!15.	$\sim V \vee \sim X$	14, 12, C.D.
!16.	$\sim (V \bullet X)$	15, De M.
!17.	$(V \bullet X) \bullet \sim (V \bullet X)$	4, 16, Conj.

Section 9. 12 – A

A. Exercises on page 396, but referring to page 322, Exercise Set B.

In each case we try to assign truth values such that the premises will be made true (**T**), and the conclusion will be made false (**F**). If such assignments result in an unavoidable inconsistency, the argument is proved valid.

1.
   1. $(A \vee B) \supset (A \bullet B)$
   2. $A \vee B$

   $\therefore A \bullet B$

   Valid.
   To make premise 2 **T**, either A or B must be **T**.
   If either A or B is **T**, then premise 1 is true only if A is **T** and B is **T**.
   But to make the conclusion **F**, either A must be **F** or B must be **F**.
   An inconsistent assignment is unavoidable. Absurd.

2.
   1. $(C \vee D) \supset (C \bullet D)$
   2. $C \bullet D$

   $\therefore C \vee D$

   Valid.
   To make premise 2 **T**, both C and D must be **T**.
   But to make the conclusion **F**, either C or D must be **F**.
   An inconsistent assignment is unavoidable. Absurd.

3.
   1. $E \supset F$
   2. $F \supset E$

   $\therefore E \vee F$

E	F
F	F

   Invalid.
   It is possible to assign truth values that make the premises **T** and the conclusion **F**.

4. 1. (G ∨ H) ⊃ (G • H)
   2. ~ (G • H)
   ∴ ~ (G ∨ H)

   Valid.
   To make the conclusion **F**, either G or H must be made **T**.
   Then to make premise 1 **T**, G and H must both be made **T**.
   But to make premise 2 **T**, either G or H must be **F**.
   An inconsistent assignment is unavoidable. Absurd.

5. 1. (I ∨ J) ⊃ (I • J)
   2. ~ (I ∨ J)
   ∴ ~ (I • J)

   Valid.
   To make the conclusion **F**, both I and J must be made **T**.
   But to make premise 2 **T**, both I and J must be **F**.
   An inconsistent assignment is unavoidable. Absurd.

6 1. K ∨ L
  2. K
  ∴ ~ L

K	L
T	T

   Invalid.
   It is possible to assign truth values that make the premises **T** and the conclusion **F**.

7. 1. M ∨ (N • ~ N)
   2. M
   ∴ ~ (N • ~ N)

   Valid.
   To make the conclusion **F**, N must be made both **T** and **F**. Absurd.

8. 1. (O ∨ P) ⊃ Q
   2. Q ⊃ (O • P)
   ∴ (O ∨ P) ⊃ (O • P)

   Valid.
   To make the conclusion **F**, either O or P must be **T**.
   Then, to make premise 1 **T**, Q must be **T**.
   Then, to make premise 2 **T**, both O and P must be **T**.
   But to make the conclusion **F**, either O or P must be **F**.
   An inconsistent assignment is unavoidable. Absurd.

9. 1. $(R \vee S) \supset T$
   2. $T \supset (R \bullet S)$
   $\therefore (R \bullet S) \supset (R \vee S)$

   Valid.
   If the conclusion is to be made **F**, both R and S must be **T**.
   But if the conclusion is to be made **F**, either R or S must also be **F**.
   An inconsistent assignment is unavoidable. Absurd.

10. 1. $U \supset (V \vee W)$
    2. $(V \bullet W) \supset \sim U$
    $\therefore \sim U$

U	V	W
T	T	F

   Invalid.
   It is possible to assign truth values that make the premises **T** and the conclusion **F**.

Section 9. 12 – B

B. Exercises on page 396, but referring to Exercise Set C, pages 322–323.

Here also we try to assign truth values such that the premises will be made true (**T**) and the conclusion will be made false (**F**). If doing that results in an unavoidable inconsistency, the argument is proved valid.

1. 1. $A \supset (B \bullet C)$
   2. $\sim B$
   $\therefore \sim A$

   Valid.
   If the conclusion is made **F**, then A must be made **T**.
   Then if premise 1 is made **T**, B and C must both be **T**.
   But to make premise 2 **T**, B must be **F**.
   An inconsistent assignment is unavoidable. Absurd.

2. 1. $D \supset (E \supset F)$
   2. E
   $\therefore D \supset F$

   Valid.
   If the conclusion is **F**, then D is **T** and F is **F**.
   Then, if premise 1 is **T**, E must be **F**.
   But if premise 2 is **T**, E must be true.
   An inconsistent assignment is unavoidable. Absurd.

3. 1. G ⊃ H
   2. G ⊃ I
   ∴ H ⊃ I

G	H	I
F	T	F

Invalid.
It is possible to assign truth values that make the premises **T** and the conclusion **F**.

4. 1. J ⊃ (K v L)
   2. ~ K
   ∴ J ⊃ L

Valid.
If the conclusion is **F**, then J must be **T**, and L must be **F**.
Then if premise 1 is **T**, either K must be made **T**, or L must be made **T**.
But if premise 2 is **T**, K must be made **F**, and since L must also be **F**, an inconsistent assignment is unavoidable. Absurd.

5. 1. M ⊃ (N ⊃ O)
   2. N
   ∴ O ⊃ M

M	N	O
F	T	T

Invalid.
It is possible to assign truth values that make the premises **T** and the conclusion **F**.

6. 1. E ⊃ D
   2. D ⊃ P
   3. P ⊃ ~ E
   ∴ ~ E

Valid.
If the conclusion is made false, E is made **T**.
If premise 1 is **T**, then D must be **T**.
If premise 2 is **T**, then P must be **T**.
If premise 3 is **T**, then E must be **F**, which is inconsistent. Absurd.

7. 1. T ⊃ L
   2. ~ T ⊃ I
   ∴ L v I

Valid.
If the conclusion is **F**, then both L and I must be made **F**.
If L is made **F**, then for premise 1 to be **T**, T must be **F**.
If I is made **F**, then for premise 2 to be **T**, T must be **T**.
An inconsistent assignment is unavoidable. Absurd.

8. 1. $R \supset (A \vee D)$
   2. $\sim A$
   $\therefore \sim D \supset \sim R$

   Valid.
   If the conclusion is made **F**, D must be **F** and R must be **T**.
   Then for premise 1 to be **T**, either A must be **T** or D must be **T**.
   For premise 2 to be **T**, A must be **F**, and since D is **F** also, an inconsistent assignment is unavoidable. Absurd.

9. 1. $G \supset (I \vee D)$
   2. $(I \bullet D) \supset B$
   $\therefore G \supset B$

G	I	D	B
T	T	F	F

   Invalid.
   It is possible to assign truth values that make the premises **T** and the conclusion **F**.

10. 1. $C \supset (I \bullet D)$
    2. $(I \vee D) \supset B$
    $\therefore C \supset B$

    Valid.
    If the conclusion is made **F**, then C must be made **T**, and B must be made **F**.
    Then if premise 1 is made **T**, both I and D must be made **T**.
    But if premise 2 is made **T**, then both I and D must be made **F**.
    An inconsistent assignment is unavoidable. Absurd.

Chapter 10

Section 10.4 – A
Exercises on pages 411–412

1. $(x)\ (Bx \supset Px)$
2. $(x)\ (Sx \supset {\sim}Mx)$
3. $(\exists x)\ (Rx \bullet Px)$
4. $(x)\ (Nx \supset Cx)$
5. $(\exists x)\ (Dx \bullet {\sim}Rx)$
6. $(x)\ (Sx \supset Px)$
7. $(x)\ (Bx \supset {\sim}Cx)$
8. $(x)\ (Cx \supset Lx)$
9. $(\exists x)\ (Sx \bullet Fx)$
10. $(x)\ (Cx \supset {\sim}Fx)$
11. $(\exists x)\ (Cx \bullet Px)$
12. $(\exists x)\ (Cx \bullet {\sim}Px)$
13. $(\exists x)\ (Gx \bullet {\sim}Ax)$
14. $(x)\ (Dx \supset Bx)$
15. $(x)\ (Vx \supset Cx)$
16. $(x)\ (Ex \supset Ux)$
17. $(\exists x)\ (Ax \bullet {\sim}Hx)$
18. $(x)\ (Ax \supset {\sim}Hx)$
19. $(x)\ (Sx \supset {\sim}Lx)$
20. $(x)\ (Cx \equiv Hx)$

Section 10.4 – B
Exercises on pages 412–413

1. Nothing is attained in war except by calculation.
   (where: $Ax = x$ is attained in war; $Cx = x$ is by calculation.)
   $(x)\ (Ax \supset Cx)$

2. No one doesn't believe in laws of nature.
   (where: $Px = x$ is a person; $Bx = x$ believes in laws of nature.)
   $(x)\ (Px \supset Bx)$

3. He only earns his freedom and existence who daily conquers them anew.
(where: *Ex* = *x* earns his freedom and existence; *Dx* = *x* daily conquers his freedom and existence anew.)
(*x*) (*Ex* ⊃ *Dx*)

4. No man is thoroughly miserable unless he be condemned to live in Ireland.
(where: *Mx* = *x* is a man who is thoroughly miserable; *Cx* = *x* is condemned to live in Ireland.)
(*x*) (~*Mx* ∨ *Cx*)

5. Not everything good is safe, and not everything dangerous is bad.
(where: *Gx* = *x* is good; *Sx* = *x* is safe; *Dx* = *x* is dangerous; *Bx* = *x* is bad.)
[(∃*x*) (*Gx* • ~*Sx*)] • [(∃*x*) (*Dx* • ~*Bx*)]

6. There isn't any business we can't improve.
(where: *Bx* = *x* is a business; *Wx* = *x* is something we can improve.)
(*x*) (*Bx* ⊃ (*Wx*)

7. A problem well stated is a problem half solved.
(where: *Wx* = *x* is problem well stated; *Hx* = *x* is a problem half solved.)
(*x*) (*Wx* ⊃ (*Hx*)

8. There's not a single witch or wizard who went bad who wasn't in Slytherin.
(where: *Bx* = *x* is a witch or wizard who went bad; *Sx* = *x* was in Slytherin.)
(*x*) (*Bx* ⊃ (*Sx*)

9. Everybody doesn't like something, but nobody doesn't like Willie Nelson.
(where: *Px* = *x* is a person; *Sx* = *x* there is something that *x* doesn't like; *Nx* = *x* doesn't like Willie Nelson.)
(*x*) (*Px* ⊃ *Sx*) • (*x*) (~*Px* ∨ ~*Nx*)

10. (*x*) (~*Bx* ⊃ ~*Wx*)

Section 10.4 – C
Exercises on page 413

1. $(\exists x)\ (Ax \bullet {\sim}Bx)$
2. $(\exists x)\ (Cx \bullet Dx)$
3. $(x)\ ({\sim}Ex \vee Fx)$ or $(x)\ (Ex \supset {\sim}Fx)$
4. $(x)\ ({\sim}Gx \vee Hx)$ or $(x)\ (Gx \supset Hx)$
5. $(\exists x)\ (Ix \bullet {\sim}Jx)$
6. $(\exists x)\ (Kx \bullet Lx)$
7. $(x)\ (Mx \vee Nx)$ or $(x)\ ({\sim}Mx \supset Nx)$
8. $(x)\ (Ox \vee {\sim}Px)$ or $(x)\ ({\sim}Ox \supset {\sim}Px)$
9. $(x)\ ({\sim}Qx \vee Rx)$ or $(x)\ (Qx \supset Rx)$
10. $(\exists x)\ (Sx \bullet {\sim}Tx)$
11. $(\exists x)\ ({\sim}Ux \bullet {\sim}Vx)$
12. $(x)\ ({\sim}Wx \vee Xx)$ or $(x)\ (Wx \supset Xx)$

Section 10.5 – A
Exercises on page 423

1.	1.	$(x)\ (Ax \supset {\sim}Bx)$	
	2.	$(\exists x)\ (Cx \bullet Ax)$	
		$\therefore\ (\exists x)\ (Cx \bullet {\sim}Bx)$	
	3.	$Ca \bullet Aa$	2, E.I.
	4.	$Aa \supset {\sim}Ba$	1, U.I.
	5.	$Aa \bullet Ca$	3, Com.
	6.	Aa	5, Simp.
	7.	${\sim}Ba$	4, 6, M.P.
	8.	Ca	3, Simp.
	9.	$Ca \bullet {\sim}Ba$	8, 7, Conj.
	10.	$(\exists x)\ (Cx \bullet {\sim}Bx)$	9, E.G.

2.	1.	$(x)\ (Dx \supset {\sim}Ex)$	
	2.	$(x)\ (Fx \supset Ex)$	
		$\therefore\ (x)\ (Fx \supset {\sim}Dx)$	
	3.	$Fy \supset Ey$	2, U.I
	4.	$Dy \supset {\sim}Ey$	1, U.I.
	5.	${\sim}{\sim}Dy \supset {\sim}Ey$	4, D.N.
	6.	$Ey \supset {\sim}Dy$	5, Trans.
	7.	$Fy \supset {\sim}Dy$	3, 6, H.S.
	8.	$(x)\ (Fx \supset {\sim}Dx)$	7, U.G.

3.	1.	$(x)\ (Gx \supset Hx)$	
	2.	$(x)\ (Ix \supset {\sim}Hx)$	
		$\therefore\ (x)\ (Ix \supset {\sim}Gx)$	
	3.	$Iy \supset {\sim}Hy$	2, U.I.
	4.	$Gy \supset Hy$	1, U.I.
	5.	${\sim}Hy \supset {\sim}Gy$	4, Trans.
	6.	$Iy \supset {\sim}Gy$	3, 5, H.S.
	7.	$(x)\ (Ix \supset {\sim}Gx)$	6, U.G.
4.	1.	$(\exists x)\ (Jx \bullet Kx)$	
	2.	$(x)\ (Jx \supset Lx)$	
		$\therefore\ (\exists x)\ (Lx \bullet Kx)$	
	3.	$Ja \bullet Ka$	1, E.I.
	4.	$Ja \supset La$	2, U.I.
	5.	Ja	3, Simp.
	6.	La	4, 5, M.P.
	7.	$Ka \bullet Ja$	3, Com.
	8.	Ka	7, Simp.
	9.	$La \bullet Ka$	6, 8, Conj.
	10.	$(\exists x)\ (Lx \bullet Kx)$	9, E.G.
5.	1.	$(x)\ (Mx \supset Nx)$	
	2.	$(\exists x)\ (Mx \bullet Ox)$	
		$\therefore\ (\exists x)\ (Ox \bullet Nx)$	
	3.	$Ma \bullet Oa$	2, E.I.
	4.	$Ma \supset Na$	1, U.I.
	5.	Ma	3, Simp.
	6.	Na	4, 5, M.P.
	7.	$Oa \bullet Ma$	3, Com.
	8.	Oa	7, Simp.
	9.	$Oa \bullet Na$	8, 6, Conj.
	10.	$(\exists x)\ (Ox \bullet Nx)$	9, E.G.
6.	1.	$(\exists x)\ (Px \bullet {\sim}Qx)$	
	2.	$(x)\ (Px \supset Rx)$	
		$\therefore\ (\exists x)\ (Rx \bullet {\sim}Qx)$	
	3.	$Pa \bullet {\sim}Qa$	1, E.I.
	4.	$Pa \supset Ra$	2, U.I.
	5.	Pa	3, Simp.
	6.	Ra	4, 5, M.P.

	7.	~*Qa* • *Pa*	3, Com.
	8.	~*Qa*	7, Simp.
	9.	*Ra* • ~*Qa*	6, 8, Conj.
	10.	(∃*x*) (*Rx* • ~*Qx*)	9, E.G.

7.	1.	(*x*) (*Sx* ⊃ ~*Tx*)	
	2.	(∃*x*) (*Sx* • *Ux*)	
		∴ (∃*x*) (*Ux* • ~*Tx*)	
	3.	*Sa* • *Ua*	2, E.I.
	4.	*Sa* ⊃ ~*Ta*	1, U.I.
	5.	*Sa*	3, Simp.
	6.	~*Ta*	4, 5, M.P.
	7.	*Ua* • *Sa*	3, Com.
	8.	*Ua*	7, Simp.
	9.	*Ua* • ~*Ta*	8, 6, Conj.
	10.	(∃*x*) (*Ux* • ~*Tx*)	9, E.G.

8.	1.	(*x*) (*Vx* ⊃ *Wx*)	
	2.	(*x*) (*Wx* ⊃ ~*Xx*)	
		∴ (*x*) (*Xx* ⊃ ~*Vx*)	
	3.	*Vy* ⊃ *Wy*	1, U.I
	4.	*Wy* ⊃ ~*Xy*	2, U.I.
	5.	*Vy* ⊃ ~*Xy*	3, 4, H.S.
	6.	~~*Xy* ⊃ ~*Vy*	5, Trans.
	7.	*Xy* ⊃ ~*Vy*	6, D.N.
	8.	(*x*) (*Xx* ⊃ ~*Vx*)	7, U.G.

9.	1.	(∃*x*) (*Yx* • *Zx*)	
	2.	(*x*) (*Zx* ⊃ *Ax*)	
		∴ (∃*x*) (*Ax* • *Yx*)	
	3.	*Ya* • *Za*	1, E.I.
	4.	*Za* ⊃ *Aa*	2, U.I.
	5.	*Za* • *Ya*	3, Com.
	6.	*Za*	5, Simp.
	7.	*Aa*	4, 6, M.P.
	8.	*Ya*	3, Simp.
	9.	*Aa* • *Ya*	7, 8, Conj.
	10.	(∃*x*) (*Ax* • *Yx*)	9, E.G.

10.	1.	$(x)\ (Bx \supset {\sim}Cx)$	
	2.	$(\exists x)\ (Cx \bullet Dx)$	
		$\therefore\ (\exists x)\ (Dx \bullet {\sim}Bx)$	
	3.	$Ca \bullet Da$	2, E.I.
	4.	$Ba \supset {\sim}Ca$	1, U.I.
	5.	Ca	3, Simp.
	6.	${\sim}{\sim}Ca$	5, D.N.
	7.	${\sim}Ba$	4, 6, M.T.
	8.	$Da \bullet Ca$	3, Com.
	9.	Da	8, Simp.
	10.	$Da \bullet {\sim}Ba$	9, 7, Conj.
	11.	$(\exists x)\ (Dx \bullet {\sim}Bx)$	10, E.G.

11.	1.	$(x)\ (Fx \supset Gx)$	
	2.	$(\exists x)\ (Fx \bullet {\sim}Gx)$	
		$\therefore\ (\exists x)\ (Gx \bullet {\sim}Fx)$	
	3.	$Fa \bullet {\sim}Ga$	2, E.I.
	4.	$Fa \supset Ga$	1, U.I.
	5.	Fa	3, Simp.
	6.	Ga	4, 5, M.P.
	7.	${\sim}Ga \bullet Fa$	3, Com.
	8.	${\sim}Ga$	7, Simp.
	9.	$Ga \vee (\exists x)\ (Gx \bullet {\sim}Fx)$	6, Add.
	10.	$(\exists x)\ (Gx \bullet {\sim}Fx)$	9, 8, D.S.

Section 10.5–B
Exercises on page 424

1.	1.	$(x)\ (Ax \supset {\sim}Bx)$	
	2.	Bc	
		$\therefore\ {\sim}Ac$	
	3.	$Ac \supset {\sim}Bc$	1, U.I.
	4.	${\sim}{\sim}Bc$	2, D.N.
	5.	${\sim}Ac$	3, 4, M.T.

2.	1.	$(x)\ (Dx \supset Ex)$	
	2.	$(\exists x)\ (Fx \bullet {\sim}Ex)$	
		$\therefore\ (\exists x)\ (Fx \bullet {\sim}Dx)$	
	3.	$Fa \bullet {\sim}Ea$	2, E.I.

	4.	$Da \supset Ea$	1, U.I.
	5.	Fa	3, Simp.
	6.	$\sim Ea \bullet Fa$	3, Com.
	7.	$\sim Ea$	6, Simp.
	8.	$\sim Da$	4, 7, M.T.
	9.	$Fa \bullet \sim Da$	5, 8, Conj.
	10.	$(\exists x)\ (Fx \bullet \sim Dx)$	9, E.G.
3.	1.	$(x)\ (Gx \supset \sim Hx)$	
	2.	$(\exists x)\ (Ix \bullet Hx)$	
		$\therefore (\exists x)\ (Ix \bullet \sim Gx)$	
	3.	$Ia \bullet Ha$	2, E.I.
	4.	$Ga \supset \sim Ha$	1, U.I.
	5.	Ia	3, Simp.
	6.	$Ha \bullet Ia$	3, Com.
	7.	Ha	6, Simp.
	8.	$\sim\sim Ha$	7, D.N.
	9.	$\sim Ga$	4, 8, M.T.
	10.	$Ia \bullet \sim Ga$	5, 9, Conj.
	11.	$(\exists x)\ (Ix \bullet \sim Gx)$	10, E.G.
4.	1.	$(x)\ (Jx \supset Kx)$	
	2.	$(x)\ (Kx \supset \sim Lx)$	
		$\therefore (x)\ (Jx \supset \sim Lx)$	
	3.	$Jy \supset Ky$	1, U.I.
	4.	$Ky \supset \sim Ly$	2, U.I.
	5.	$Jy \supset \sim Ly$	3, 4, H.S.
	6.	$(x)\ (Jx \supset \sim Lx)$	5, U.G.
5.	1.	$(x)\ (Mx \supset Nx)$	
	2.	$(\exists x)\ (Ox \bullet Mx)$	
		$\therefore (\exists x)\ (Ox \bullet Nx)$	
	3.	$Oa \bullet Ma$	2, E.I.
	4.	$Ma \supset Na$	1, U.I.
	5.	Oa	3, Simp.
	6.	$Ma \bullet Oa$	3, Com.
	7.	Ma	6, Simp.
	8.	Na	4, 7, M.P.
	9.	$Oa \bullet Na$	5, 8, Conj.
	10.	$(\exists x)\ (Ox \bullet Nx)$	9, E.G.

6.	1.	$(x)\ (Qx \supset Px)$	
	2.	$(\exists x)\ (Qx \bullet Rx)$	
		$\therefore\ (\exists x)\ (Px \bullet Rx)$	
	3.	$Qa \bullet Ra$	2, E.I.
	4.	$Qa \supset Pa$	1, U.I.
	5.	Qa	3, Simp.
	6.	Pa	4, 5, M.P.
	7.	$Ra \bullet Qa$	3, Com.
	8.	Ra	7, Simp.
	9.	$Pa \bullet Ra$	6, 8, Conj.
	10.	$(\exists x)\ (Px \bullet Rx)$	9, E.G.

7.	1.	$(x)\ (Sx \supset Tx)$	
	2.	$(x)\ (Tx \supset Ux)$	
		$\therefore\ (x)\ (Sx \supset Ux)$	
	3.	$Sy \supset Ty$	1, U.I.
	4.	$Ty \supset Uy$	2, U.I.
	5.	$Sy \supset Uy$	3, 4, H.S.
	6.	$(x)\ (Sx \supset Ux)$	5, U.G.

8.	1.	$(x)\ (Vx \supset Wx)$	
	2.	$(x)\ (Xx \supset {\sim}Wx)$	
		$\therefore\ (x)\ (Vx \supset {\sim}Xx)$	
	3.	$Vy \supset Wy$	1, U.I.
	4.	$Xy \supset {\sim}Wy$	2, U.I.
	5.	${\sim}{\sim}Wy \supset {\sim}Xy$	4, Trans.
	6.	$Wy \supset {\sim}Xy$	5, D.N.
	7.	$Vy \supset {\sim}Xy$	3, 6, H.S.
	8.	$(x)\ (Vx \supset {\sim}Xx)$	7, U.G.

9.	1.	$(x)\ (Dx \supset Bx)$	
	2.	$(x)\ (Bx \supset Sx)$	
		$\therefore\ (x)\ (Dx \supset Sx)$	
	3.	$Dy \supset By$	1, U.I.
	4.	$By \supset Sy$	2, U.I.
	5.	$Dy \supset Sy$	3, 4, H.S.
	6.	$(x)\ (Dx \supset Sx)$	5, U.G.

10. 1. $(x)(Ax \supset Rx)$
2. $\sim Rs$
$\therefore \sim As$
3. $As \supset Rs$ 1, U.I.
4. $\sim As$ 3, 2, M.T.

Section 10.6–A
Exercises on pages 427–428

1.
$$\left.\begin{array}{l}(\exists x)(Ax \bullet Bx)\\(\exists x)(Cx \bullet Bx)\\\therefore (x)(Cx \supset \sim Ax)\end{array}\right\}\quad \begin{array}{c}\text{logically}\\\text{equivalent}\\\text{in } \boxed{a} \text{ to}\end{array}\quad \left\{\begin{array}{l}Aa \bullet Ba\\Aa \bullet Ba\\\therefore (Ca \supset \sim Aa)\end{array}\right.$$

We may prove the argument invalid in this model by assigning truth values as follows:

Aa	Ba	Ca
T	T	T

Since the argument has been proved invalid in this model, the argument has been proved invalid.

2. logically equivalent in $\boxed{a}$ to
and proved invalid by

$Da \supset \sim Ea$
$Ea \supset Fa$
$\therefore Fa \supset \sim Da$

Da	Ea	Fa
T	F	T

3. logically equivalent in $\boxed{a}$ to
and proved invalid by

$Ga \supset Ha$
$Ga \supset Ia$
$\therefore Ia \supset Ha$

Ga	Ha	Ia
F	F	T

4. logically equivalent in $\boxed{a,b}$ to and proved invalid by

$(Ja \bullet Ka) \vee (Jb \bullet Kb)$
$(Ka \bullet La) \vee (Kb \bullet Lb)$
$\therefore (La \bullet Ja) \vee (Lb \bullet Jb)$

	Ja	*Jb*	*Ka*	*Kb*	*La*	*Lb*
	T	F	T	T	F	T
or	F	T	T	T	T	F

5. $$\left.\begin{array}{l}(\exists x)\,(Mx \bullet Nx)\\(\exists x)\,(Mx \bullet Ox)\\\therefore (X)\,(Ox \supset Nx)\end{array}\right\}\ \text{logically equivalent in } \boxed{a,b} \text{ to }\ \left\{\begin{array}{l}(Ma \bullet Na) \vee (Mb \bullet Nb)\\(Ma \bullet Oa) \vee (Mb \bullet Ob)\\\therefore\ (Oa \supset Na) \bullet (Ob \supset Nb)\end{array}\right.$$

proved invalid by

Ma	*Mb*	*Na*	*Nb*	*Oa*	*Ob*
T	T	T	F	T	T

or any of several other truth-value assignments.

6. logically equivalent in $\boxed{a}$ to and proved invalid by

$Pa \supset {\sim}Qa$
$Pa \supset {\sim}Ra$
$\therefore\ Ra \supset {\sim}Qa$

Pa	*Qa*	*Ra*
F	T	T

7. logically equivalent in $\boxed{a}$ to and proved invalid by

$Sa \supset {\sim}Ta$
$Ta \supset Ua$
$\therefore\ Ua \bullet {\sim}Sa$

Sa	*Ta*	*Ua*
T	F	T

8. logically equivalent in $\boxed{a,b}$ to and proved invalid by

$(Va \bullet {\sim}Wa) \vee (Vb \bullet {\sim}Wb)$
$(Wa \bullet {\sim}Xa) \vee (Wb \bullet {\sim}Xb)$
$\therefore\ (Xa \bullet {\sim}Va) \vee (Xb \bullet {\sim}Vb)$

Va	*Vb*	*Wa*	*Wb*	*Xa*	*Xb*
T	T	F	T	T	F

or any of several other truth-value assignments.

9. logically equivalent in $\boxed{a}$ to and proved invalid by

$$\begin{array}{l} Ya \bullet Za \\ Aa \bullet Za \\ \therefore Aa \bullet \sim Ya \end{array}$$

Ya	*Za*	*Aa*
T	T	T

10.
$$\left.\begin{array}{l} (\exists x)\ (Bx \bullet \sim Cx) \\ (x)\ (Dx \supset \sim Cx) \\ \therefore (x)\ (Dx \supset Bx) \end{array}\right\} \text{ logically equivalent in } \boxed{a, b} \text{ to } \left\{\begin{array}{l} (Ba \bullet \sim Ca) \vee (Bb \bullet \sim Cb) \\ (Da \supset \sim Ca) \bullet (Db \supset \sim Cb) \\ \therefore (Da \supset Ba) \bullet (Db \supset Bb) \end{array}\right.$$

proved invalid by

Ba	*Bb*	*Ca*	*Cb*	*Da*	*Db*
F	T	F	F	T	T

Section 10.6–B
Exercises on pages 428–429

1.
$$\left.\begin{array}{l} (x)\ (Ax \supset Bx) \\ (x)\ (Cx \supset Bx) \\ \therefore (x)\ (Ax \supset Cx) \end{array}\right\} \text{ logically equivalent in } \boxed{a} \text{ to } \left\{\begin{array}{l} Aa \supset Ba \\ Ca \supset Ba \\ \therefore Aa \supset Ca \end{array}\right.$$

proved invalid by

Aa	*Ba*	*Ca*
T	T	F

2.
$$\begin{array}{l} (x)\ (Dx \supset \sim Ex) \\ (\exists x)\ (Fx \bullet Ex) \\ (\exists x)\ (Dx \bullet \sim Fx) \end{array} \qquad \begin{array}{l} Da \supset \sim Ea \\ Fa \bullet Ea \\ \therefore Da \bullet \sim Fa \end{array}$$

Da	*Ea*	*Fa*
F	T	T

3.
$$\begin{array}{l} (x)\ (Gx \supset Hx) \\ (\exists x)\ (Ix \bullet Hx) \\ \therefore (\exists x)\ (Gx \bullet Ix) \end{array} \qquad \begin{array}{l} Ga \supset Ha \\ Ia \bullet Ha \\ \therefore Ga \bullet Ia \end{array}$$

Ga	*Ha*	*Ia*
F	T	T

4.
$$\begin{array}{l} (\exists x)\ (Jx \bullet \sim Kx) \\ (\exists x)\ (Kx \bullet \sim Lx) \\ \therefore (\exists x)\ (Jx \bullet \sim Lx) \end{array} \qquad \begin{array}{l} (Ja \bullet \sim Ka) \vee (Jb \bullet \sim Kb) \\ (Ka \bullet \sim La) \vee (Kb \bullet \sim Lb) \\ \therefore (Ja \bullet \sim La) \vee (Jb \bullet \sim Lb) \end{array}$$

Ja	*Jb*	*Ka*	*Kb*	*La*	*Lb*
T	F	F	T	T	F

5. $\left.\begin{array}{l}(\exists x)\,(Mx \bullet Nx)\\ (\exists x)\,(Ox \bullet \sim Nx)\\ \therefore (x)\,(Ox \supset \sim Mx)\end{array}\right\}$ logically equivalent in $\boxed{a, b}$ to $\left\{\begin{array}{l}(Ma \bullet Na) \vee (Mb \bullet Nb)\\ (Oa \bullet \sim Na) \vee (Ob \bullet \sim Nb)\\ \therefore\ (Oa \supset \sim Ma) \bullet (Ob \supset \sim Mb)\end{array}\right.$

proved invalid by

Ma	*Mb*	*Na*	*Nb*	*Oa*	*Ob*
T	T	T	F	T	T

or any of several other truth-value assignments.

6.
$(\exists x)\,(Px \bullet Qx)$
$(\exists x)\,(Qx \bullet \sim Rx)$
$(\exists x)\,(Px \bullet \sim Rx)$

$(Pa \bullet Qa) \vee (Pb \bullet Qb)$
$(Qa \bullet \sim Ra) \vee (Qb \bullet \sim Rb)$
$\therefore (Pa \bullet \sim Ra) \vee (Pb \bullet \sim Rb)$

Pa	*Pb*	*Qa*	*Qb*	*Ra*	*Rb*
T	F	T	T	T	F

7. $(\exists x)\,(Px \cdot Lx)$ $\quad$ $Pa \cdot La$
$(\exists x)\,(Lx \cdot \sim Ox)$ $\quad$ $La \cdot \sim Oa$
$\therefore\ (\exists x)\,(Ox \cdot \sim Px)$ $\quad$ $\therefore\ Oa \cdot \sim Pa$

Pa	*La*	*Oa*
T	T	F

8. $(x)\,(Dx \supset Bx)$ $\quad$ $Da \supset Ba$
$(x)\,(Sx \supset Bx)$ $\quad$ $Sa \supset Ba$
$\therefore\ (x)\,(Dx \supset Sx)$ $\quad$ $\therefore\ Da \supset Sa$

Da	*Ba*	*Sa*
T	T	F

9. $(x)\,(Mx \supset Bx)$ $\quad$ $Ma \supset Ba$
$(\exists x)\,(Bx \bullet Ox)$ $\quad$ $Ba \bullet Oa$
$\therefore\ (\exists x)\,(Mx \bullet Ox)$ $\quad$ $\therefore\ Ma \bullet Oa$

Ma	*Ba*	*Oa*
F	T	T

10. $\left.\begin{array}{l}(x)\,(Mx \supset Sx)\\ (x)\,(Wx \supset Mx)\\ \therefore (x)\,(Sx \supset Wx)\end{array}\right\}$ logically equivalent in $\boxed{a}$ to $\left\{\begin{array}{l}Ma \supset Sa\\ Wa \supset Ma\\ \therefore\ Sa \supset Wa\end{array}\right.$

proved invalid by

Ma	*Sa*	*Wa*
T	T	F

Section 10.7–A
Exercises on pages 432–433

1. $(x)\ [(Ax \vee Ox) \supset (Dx \bullet Nx)]$
2. $(\exists x)\ [Fx \bullet (Ex \supset Cx)]$
3. $(x)\ [Cx \supset ({\sim}Sx \vee Bx)]$
4. $(x)\ \{(Tx \bullet Mx) \supset [(Dx \bullet Hx) \supset Ax]\}$
5. $(x)\ Gx \supset (Wx \equiv Lx)$
6. $(x)\ \{[Bx \bullet (Wx \equiv Lx)] \supset {\sim}Sx\}$
7. $(\exists x)\ [(Px \bullet Wx) \bullet {\sim}(Ex \bullet Cx)]$
8. $(\exists x)\ [(Tx \bullet Cx) \bullet {\sim}(Sx \vee Bx)]$
9. $(x)\ [(Px \bullet Dx) \supset Cx]$
10. $(x)\ \{Ax \supset [(Bx \supset Wx) \bullet (Px \supset Sx)]\}$
11. $(x)\ \{[Ax \supset ({\sim}Nx \supset Px)] \bullet [Gx \supset ({\sim}Px \supset Nx)] \bullet [Fx \supset (Nx \supset Px)] \bullet [Rx \supset (Px \supset Nx)]\}$

Section 10.7–B
Exercises on pages 433–434

1.

1.	$(x)\ [(Ax \vee Bx) \supset (Cx \bullet Dx)]$	
	$\therefore\ (x)\ (Bx \supset Cx)$	
2.	$(Ay \vee By) \supset (Cy \bullet Dy)$	1, U.I.
3.	${\sim}(Ay \vee By) \vee (Cy \bullet Dy)$	2, Impl.
4.	$[{\sim}(Ay \vee By) \vee Cy] \bullet [{\sim}(Ay \vee By) \vee Dy]$	3, Dist.
5.	${\sim}(Ay \vee By) \vee Cy$	4, Simp.
6.	$Cy \vee {\sim}(Ay \vee By)$	5, Com.
7.	$Cy \vee ({\sim}Ay \bullet {\sim}By)$	6, De M.
8.	$(Cy \vee {\sim}Ay) \bullet (Cy \vee {\sim}By)$	7, Dist.
9.	$(Cy \vee {\sim}By) \bullet (Cy \vee {\sim}Ay)$	8, Com.
10.	$Cy \vee {\sim}By$	9, Simp.
11.	${\sim}By \vee Cy$	10, Com.
12.	$By \supset Cy$	11, Impl.
13.	$(x)\ (Bx \supset Cx)$	12, U.G.

2. Logically equivalent in $\boxed{a, b}$ to

$\{(Ea \cdot Fa) \bullet [(Ea \vee Fa) \supset (Ga \bullet Ha)]\} \vee \{(Eb \bullet Fb) \cdot [(Eb \vee Fb) \supset (Gb \bullet Hb)]\}$

$\therefore (Ea \supset Ha) \bullet (Eb \supset Hb)$

proved invalid by

Ea	*Eb*	*Fa*	*Fb*	*Ga*	*Gb*	*Ha*	*Hb*
T	T	T	T	T	T	T	F

or any of several other truth-value assignments.

3.

1.	$(x) \{[Ix \supset (Jx \bullet \sim Kx)] \cdot [Jx \supset (Ix \supset Kx)]\}$	
2.	$(\exists x) [(Ix \bullet Jx) \bullet \sim Lx]$	
	$\therefore (\exists x) (Kx \cdot Lx)$	
3.	$(Ia \bullet Ja) \bullet \sim La$	2, E.I.
4.	$[Ia \supset (Ja \bullet \sim Ka)] \bullet [Ja \supset (Ia \supset Ka)]$	1, U.I.
5.	$Ia \bullet Ja$	3, Simp.
6.	$Ia \supset (Ja \bullet \sim Ka)$	4, Simp.
7.	Ia	5, Simp.
8.	$Ja \bullet \sim Ka$	6, 7, M.P.
9.	$[Ja \supset (Ia \supset Ka)] \bullet [Ia \supset (Ja \bullet \sim Ka)]$	4, Com.
10.	$Ja \supset (Ia \supset Ka)$	9, Simp.
11.	Ja	8, Simp.
12.	$Ia \supset Ka$	10, 11, M.P.
13.	Ka	12, 7, M.P.
14.	$Ka \vee (\exists x) (Kx \bullet Lx)$	13, Add.
15.	$\sim Ka \bullet Ja$	8, Com.
16.	$\sim Ka$	15, Simp.
17.	$(\exists x) (Kx \bullet Lx)$	14, 16, D.S.

4. Logically equivalent in $\boxed{a}$ to

$(Ma \cdot Na) \supset (Oa \vee Pa)$

$(Oa \cdot Pa) \supset (Qa \vee Ra)$

$\therefore (Ma \vee Oa) \supset Ra$

and proved invalid by

Ma	*Na*	*Oa*	*Pa*	*Qa*	*Ra*
T	F	F	F	F	F

or any of several other truth-value assignments.

5. $\left.\begin{array}{l} (\exists x)\ (Sx \bullet Tx) \\ (\exists x)\ (Ux \bullet \sim Sx) \\ (\exists x)\ (Vx \bullet \sim Tx) \\ \therefore (\exists x)\ (Ux \bullet Vx) \end{array}\right\}$ logically equivalent in $\boxed{a, b, c}$ to $\left\{\begin{array}{l} (Sa \bullet Ta) \vee (Sb \bullet Tb) \vee (Sc \bullet Tc) \\ (Ua \bullet \sim Sa) \vee (Ub \bullet \sim Sb) \vee (Uc \bullet \sim Sc) \\ (Va \bullet \sim Ta) \vee (Vb \bullet \sim Tb) \vee (Vc \bullet \sim Tc) \\ \therefore\ (Ua \bullet Va) \vee (Ub \bullet Vb) \vee (Uc \bullet Vc) \end{array}\right.$

proved invalid by

Sa	*Sb*	*Sc*	*Ta*	*Tb*	*Tc*	*Ua*	*Ub*	*Uc*	*Va*	*Vb*	*Vc*
T	F	T	T	T	F	F	T	F	T	F	T

or any of several other truth-value assignments.

6.

1.	$(x)\ [Wx \supset (Xx \supset Yx)]$	
2.	$(\exists x)\ [Xx \bullet (Zx \bullet \sim Ax)]$	
3.	$(x)\ [(Wx \supset Yx) \supset (Bx \supset Ax)$	
	$\therefore\ (\exists x)\ (Zx \bullet \sim Bx)$	
4.	$Xa \bullet (Za \bullet \sim Aa)$	2, E.I.
5.	$Wa \supset (Xa \supset Ya)$	1, U.I.
6.	$(Wa \supset Ya) \supset (Ba \supset Aa)$	3, U.I.
7.	$(Wa \bullet Xa) \supset Ya$	5, Exp.
8.	$(Xa \bullet Wa) \supset Ya$	7, Com.
9.	$Xa \supset (Wa \supset Ya)$	8, Exp.
10.	Xa	4, Simp.
11.	$Wa \supset Ya$	9, 10, M.P.
12.	$Ba \supset Aa$	6, 11, M.P.
13.	$(Za \bullet \sim Aa) \bullet Xa$	4, Com.
14.	$Za \bullet \sim Aa$	13, Simp.
15.	Za	14, Simp.
16.	$\sim Aa \bullet Za$	14, Com.
17.	$\sim Aa$	16, Simp.
18.	$\sim Ba$	12, 17, M.T.
19.	$Za \bullet \sim Ba$	15, 18, Conj.
20.	$(\exists x)\ (Zx \bullet \sim Bx)$	19, E.G.

7. Logically equivalent in $\boxed{a, b}$ to

$[Ca \bullet \sim(Da \supset Ea)] \vee [Cb \bullet \sim(Db \supset Eb)]$

$[(Ca \bullet Da) \supset Fa] \bullet [(Cb \bullet Db) \supset Fb]$

$[Ea \bullet \sim(Da \supset Ca)] \vee [Eb \bullet \sim(Db \supset Cb)]$

$(Ga \supset Ca) \bullet (Gb \supset Cb)$

$\therefore\ (Ga \bullet \sim Fa) \vee (Gb \bullet \sim Fb)$

and proved invalid by

Ca	*Cb*	*Da*	*Db*	*Ea*	*Eb*	*Fa*	*Fb*	*Ga*	*Gb*
T	F	T	T	F	T	T	T	T	F

or any of several other truth-value assignments.

8.

1.	$(x)\ (Hx \supset Ix)$	
2.	$(x)\ [(Hx \bullet Ix) \supset Jx]$	
3.	$(x)\ [{\sim}Kx \supset (Hx \vee Ix)]$	
4.	$(x)\ [(Jx \vee {\sim}Jx) \supset (Ix \supset Hx)]$	
	$\therefore\ (x)\ (Jx \vee Kx)$	
5.	$(Hy \bullet Iy) \supset Jy$	2, U.I.
6.	${\sim}Jy \supset {\sim}(Hy \bullet Iy)$	5, Trans.
7.	${\sim}Jy \supset [{\sim}Jy \bullet {\sim}(Hy \bullet Iy)]$	6, Abs.
8.	${\sim\sim}Jy \vee [{\sim}Jy \bullet {\sim}(Hy \bullet Iy)]$	7, Impl.
9.	$({\sim\sim}Jy \vee {\sim}Jy) \bullet [{\sim\sim}Jy \vee {\sim}(Hy \bullet Iy)]$	8, Dist.
10.	${\sim\sim}Jy \vee {\sim}Jy$	9, Simp.
11.	$Jy \vee {\sim}Jy$	10, D.N.
12.	$(Jy \vee {\sim}Jy) \supset (Iy \supset Hy)$	4, U.I.
13.	$Iy \supset Hy$	12, 11, M.P.
14.	$Hy \supset Iy$	1, U.I.
15.	$(Hy \supset Iy) \bullet (Iy \supset Hy)$	14, 13, Conj.
16.	$Hy \equiv Iy$	15, Equiv.
17.	$(Hy \bullet Iy) \vee ({\sim}Hy \bullet {\sim}Iy)$	16, Equiv.
18.	${\sim}Ky \supset (Hy \vee Iy)$	3, U.I.
19.	${\sim}(Hy \vee Iy) \vee {\sim\sim}Ky$	18, Trans.
20.	${\sim}(Hy \vee Iy) \supset Ky$	19, D.N.
21.	$({\sim}Hy \bullet {\sim}Iy) \supset Ky$	20, De M.
22.	$[(Hy \bullet Iy) \supset Jy] \bullet [({\sim}Hy \bullet {\sim}Iy) \supset Ky]$	5, 21, Conj.
23.	$Jy \vee Ky$	22, 17, C.D.
24.	$(x)\ (Jx \vee Kx)$	23, U.G.

9.

1.	$(x)\ \{(Lx \vee Mx) \supset \{[(Nx \bullet Ox) \vee Px] \supset Qx\}\}$	
2.	$(\exists x)\ (Mx \bullet {\sim}Lx)$	
3.	$(x)\ \{[(Ox \supset Qx) \bullet {\sim}Rx] \supset Mx\}$	
4.	$(\exists x)\ (Lx \bullet {\sim}Mx)$	
	$\therefore\ (\exists x)\ (Nx \supset Rx)$	
5.	$La \bullet {\sim}Ma$	4, E.I.

6.	La	5, Simp.
7.	$La \vee Ma$	6, Add.
8.	$(La \vee Ma) \supset \{[(Na \bullet Oa) \vee Pa] \supset Qa\}$	1, U.I.
9.	$[(Na \bullet Oa) \vee Pa] \supset Qa$	8, 7, M.P.
10.	$\sim[(Na \bullet Oa) \vee Pa] \vee Qa$	9, Impl.
11.	$Qa \vee \sim[(Na \bullet Oa) \vee Pa]$	10, Com.
12.	$Qa \vee [\sim(Na \bullet Oa) \bullet \sim Pa]$	11, De M.
13.	$[Qa \vee \sim(Na \bullet Oa)] \bullet (Qa \vee \sim Pa)$	12, Dist.
14.	$Qa \vee \sim(Na \bullet Oa)$	13, Simp.
15.	$\sim(Na \bullet Oa) \vee Qa$	14, Com.
16.	$(Na \bullet Oa) \supset Qa$	15, Impl.
17.	$Na \supset (Oa \supset Qa)$	16, Exp.
18.	$[(Oa \supset Qa) \bullet \sim Ra] \supset Ma$	3, U.I.
19.	$\sim Ma \bullet La$	5, Com.
20.	$\sim Ma$	19, Simp.
21.	$\sim[(Oa \supset Qa) \bullet \sim Ra]$	18, 20, M.T.
22.	$\sim(Oa \supset Qa) \vee \sim\sim Ra$	21, De M.
23.	$\sim(Oa \supset Qa) \vee Ra$	22, D.N.
24.	$(Oa \supset Qa) \supset Ra$	23, Impl.
25.	$Na \supset Ra$	17, 24, H.S.
26.	$(\exists x)\,(Nx \supset Rx)$	25, E.G.

10.
$$\left.\begin{array}{l} (x)\,[(Sx \vee Tx) \supset \\ \quad \sim(Ux \vee Vx)] \\ (\exists x)\,(Sx \bullet \sim Wx) \\ (\exists x)\,(Tx \bullet \sim Xx) \\ (x)\,(\sim Wx \supset Xx) \\ \therefore (\exists x)\,(Ux \bullet \sim Vx) \end{array}\right\} \begin{array}{l} \text{logically} \\ \text{equivalent} \\ \text{in } \boxed{a,b} \text{ to} \end{array} \left\{\begin{array}{l} [(Sa \vee Ta) \supset \sim(Ua \vee Va)] \bullet \\ \quad [(Sb \vee Tb) \supset \sim(Ub \vee Vb)] \\ (Sa \bullet \sim Wa) \vee (Sb \bullet \sim Wb) \\ (Ta \bullet \sim Xa) \vee (Tb \bullet \sim Xb) \\ (\sim Wa \supset Xa) \bullet (\sim Wb \supset Xb) \\ \therefore\ (Ua \bullet \sim Va) \vee (Ub \bullet \sim Vb) \end{array}\right.$$

and proved invalid by

Sa	*Sb*	*Ta*	*Tb*	*Ua*	*Ub*	*Va*	*Vb*	*Wa*	*Wb*	*Xa*	*Xb*
T	T	T	T	F	F	F	F	F	T	T	F

or any of several other truth-value assignments.

Section 10.7–C
Exercises on page 434

1.

1. $(x)\ [(Ax \lor Bx) \supset Cx]$	
2. $(x)\ (Vx \supset Ax)$	
$\therefore\ (x)\ (Vx \supset Cx)$	
3. $(Ay \lor By) \supset Cy$	1, U.I.
4. $Vy \supset Ay$	2, U.I.
5. $\sim Vy \lor Ay$	4, Impl.
6. $(\sim Vy \lor Ay) \lor By$	5, Add.
7. $\sim Vy \lor (Ay \lor By)$	6, Assoc.
8. $Vy \supset (Ay \lor By)$	7, Impl.
9. $Vy \supset Cy$	8, 3, H.S.
10. $(x)\ (Vx \supset Cx)$	9, U.G.

2.

1. $(x)\ [Tx \supset (Ex \lor Ux)]$	
2. $(\exists x)\ (Tx \bullet \sim Ux)$	
$\therefore\ (\exists x)\ (Ex \bullet Tx)$	
3. $Ta \bullet \sim Ua$	2, E.I.
4. $Ta \supset (Ea \lor Ua)$	1, U.I.
5. Ta	3, Simp.
6. $Ea \lor Ua$	4, 5, M.P.
7. $\sim Ua \bullet Ta$	3, Com.
8. $\sim Ua$	7, Simp.
9. $Ua \lor Ea$	6, Com.
10. Ea	9, 8, D.S.
11. $Ea \bullet Ta$	10, 5, Conj.
12. $(\exists x)\ (Ex \bullet Tx)$	11, E.G.

3.

$(x)\ [(Ax \lor Sx) \supset (Ox \lor Vx)]$	$(Aa \lor Sa) \supset (Oa \lor Va)$
$(\exists x)\ (Sx \bullet \sim Ox)$	$Sa \bullet \sim Oa$
$\therefore\ (\exists x)\ (Ax \bullet Vx)$	$\therefore\ Aa \bullet Va$

Aa	*Sa*	*Oa*	*Va*
F	T	F	T

4.

1. $(x)\ \{[Ex \bullet (Sx \lor Dx)] \supset \sim Px\}$	
$\therefore\ (x)\ [(Dx \bullet Ex) \supset \sim Px]$	
2. $[Ey \bullet (Sy \lor Dy)] \supset \sim Py$	1, U.I.

3. ~[*Ey* • (*Sy* ∨ *Dy*)] ∨ ~*Py*	2, Impl.
4. ~[*Ey* • (*Dy* ∨ *Sy*)] ∨ ~*Py*	3, Com.
5. ~[(*Ey* • *Dy*) ∨ (*Ey* • *Sy*)] ∨ ~*Py*	4, Dist.
6. ~*Py* ∨ ~[(*Ey* • *Dy*) ∨ (*Ey* • *Sy*)]	5, Com.
7. ~*Py* ∨ [~(*Ey* • *Dy*) • ~(*Ey* • *Sy*)]	6, De M.
8. [~*Py* ∨ ~(*Ey* • *Dy*)] • [~*Py* ∨ ~(*Ey* • *Sy*)]	7, Dist.
9. ~*Py* ∨ ~(*Ey* • *Dy*)	8, Simp.
10. ~(*Ey* • *Dy*) ∨ ~*Py*	9, Com.
11. ~(*Dy* • *Ey*) ∨ ~*Py*	10, Com.
12. (*Dy* • *Ey*) ⊃ ~*Py*	11, Impl.
13. (*x*) [(*Dx* • *Ex*) ⊃ ~*Px*]	12, U.G.

5. (*x*) {[*Ex* • (*Ix* ∨ *Tx*)] ⊃ ~*Sx*}
(∃*x*) (*Ex* • *Ix*)
(∃*x*) (*Ex* • *Tx*)
∴ (*x*) (*Ex* ⊃ ~*Sx*)

This argument is logically equivalent in $\boxed{a, b}$ to

{[*Ea* • (*Ia* ∨ *Ta*)] ⊃ ~*Sa*} • {[*Eb* • (*Ib* ∨ *Tb*)] ⊃ ~*Sb*}
(*Ea* • *Ia*) ∨ (*Eb* • *Ib*)
(*Ea* • *Ta*) ∨ (*Eb* • *Tb*)
∴ (*Ea* ⊃ ~*Sa*) • (*Eb* ⊃ ~*Sb*)

which is proved invalid by

	Ea	*Eb*	*Ia*	*Ib*	*Ta*	*Tb*	*Sa*	*Sb*
	T	T	T	F	T	F	F	T
or	T	T	F	T	F	T	T	F

6.

1.	(*x*) (*Gx* ⊃ *Ex*)	
2.	(*x*) (*Wx* ⊃ ~*Sx*)	
3.	(∃*x*) (*Wx* • ~*Ex*)	
	∴ (∃*x*) [~(*Gx* ∨ *Sx*)]	
4.	*Wa* • ~*Ea*	3, E.I.
5.	*Wa* ⊃ ~*Sa*	2, U.I.
6.	*Wa*	4, Simp.
7.	~*Sa*	5, 6, M.P.
8.	~*Ea* • *Wa*	4, Com.

9.	$\sim Ea$	8, Simp.
10.	$Ga \supset Ea$	1, U.I.
11.	$\sim Ga$	10, 9, M.T.
12.	$\sim Ga \bullet \sim Sa$	11, 7, Conj.
13.	$\sim(Ga \vee Sa)$	12, De M.
14.	$(\exists x)\ [\sim(Gx \vee Sx)]$	13, E.G.

7. $(x)\ (Tx \supset Cx)$ — $Ta \supset Ca$
$(x)\ (Rx \supset \sim Lx)$ — $Ra \supset \sim La$
$(\exists x)\ [\sim(Tx \vee Lx)]$ — $\sim(Ta \vee La)$
$\therefore\ (\exists x)\ (Rx \bullet \sim Cx)$ — $\therefore\ Ra \bullet \sim Ca$

Ta	*Ca*	*Ra*	*La*
F	T	T	F

8. $(\exists x)\ [Px \bullet (Ax \bullet \sim Ix)]$ — $[Pa \bullet (Aa \bullet \sim Ia)] \vee [Pb \bullet (Ab \bullet \sim Ib)]$
$(x)\ (Px \supset Gx)$ — $(Pa \supset Ga) \bullet (Pb \supset Gb)$
$(\exists x)\ (Px \bullet \sim Ax)$ — $(Pa \bullet \sim Aa) \vee (Pb \bullet \sim Ab)$
$(x)\ (Sx \supset Ax)$ — $(Sa \supset Aa) \bullet (Sb \supset Ab)$
$\therefore\ (\exists x)\ (Sx \bullet \sim Gx)$ — $\therefore\ (Sa \bullet \sim Ga) \vee (Sb \bullet \sim Gb)$

Pa	*Pb*	*Aa*	*Ab*	*Ia*	*Ib*	*Ga*	*Gb*	*Sa*	*Sb*
T	T	T	F	F	T	T	T	T	F

9.

1.	$(\exists x)\ [Px \bullet (Sx \bullet \sim Ix)]$	
2.	$(x)\ (Px \supset Ax)$	
3.	$(\exists x)\ (Px \bullet \sim Sx)$	
4.	$(x)\ (Jx \supset Sx)$	
	$\therefore\ (\exists x)\ (Ax \bullet \sim Jx)$	
5.	$Pa \bullet \sim Sa$	3, E.I.
6.	Pa	5, Simp.
7.	$Pa \supset Aa$	2, U.I.
8.	Aa	7, 6, M.P.
9.	$\sim Sa \bullet Pa$	5, Com.
10.	$\sim Sa$	9, Simp.
11.	$Ja \supset Sa$	4, U.I.
12.	$\sim Ja$	11, 10, M.T.
13.	$Aa \bullet \sim Ja$	8, 12, Conj.
14.	$(\exists x)\ (Ax \bullet \sim Jx)$	13, E.G.

10.	1.	$(x)\ [Bx \supset (Ix \supset Wx)]$	
	2.	$(x)\ [Bx \supset (Wx \supset Ix)]$	
		$\therefore\ (x)\ \{Bx \supset [(Ix \vee Wx) \supset (Ix \bullet Wx)]\}$	
	3.	$By \supset (Iy \supset Wy)$	1, U.I.
	4.	$By \supset (Wy \supset Iy)$	2, U.I.
	5.	$[By \supset (Iy \supset Wy)] \bullet [By \supset (Wy \supset Iy)]$	3, 4, Conj.
	6.	$[{\sim}By \vee (Iy \supset Wy)] \bullet [{\sim}By \vee (Wy \supset Iy)]$	5, Impl.
	7.	${\sim}By \vee [(Iy \supset Wy) \bullet (Wy \supset Iy)]$	6, Dist.
	8.	${\sim}By \vee (Iy \equiv Wy)$	7, Equiv.
	9.	${\sim}By \vee [(Iy \bullet Wy) \vee ({\sim}Iy \bullet {\sim}Wy)]$	8, Equiv.
	10.	${\sim}By \vee [({\sim}Iy \bullet {\sim}Wy) \vee (Iy \bullet Wy)]$	9, Com.
	11.	${\sim}By \vee [{\sim}(Iy \vee Wy) \vee (Iy \bullet Wy)]$	10, De M.
	12.	$By \supset [{\sim}(Iy \vee Wy) \vee (Iy \bullet Wy)]$	11, Impl.
	13.	$By \supset [(Iy \vee Wy) \supset (Iy \bullet Wy)]$	12, Impl.
	14.	$(x)\ \{Bx \supset [(Ix \vee Wx) \supset (Ix \bullet Wx)]\}$	13, U.G.

Section 10.7 – D
Exercises on pages 435–436

1.	1.	$(x)\ [(Cx \bullet {\sim}Tx) \supset Px]$	
	2.	$(x)\ (Ox \supset Cx)$	
	3.	$(\exists x)\ (Ox \bullet {\sim}Px)$	
		$\therefore\ (\exists x)\ (Tx)$	
	4.	$Oa \bullet {\sim}Pa$	3, E.I.
	5.	$Oa \supset Ca$	2, U.I.
	6.	$(Ca \bullet {\sim}Ta) \supset Pa$	1, U.I.
	7.	Oa	4, Simp.
	8.	Ca	5, 7, M.P.
	9.	${\sim}Pa \bullet Oa$	4, Com.
	10.	${\sim}Pa$	9, Simp.
	11.	$Ca \supset ({\sim}Ta \supset Pa)$	6, Exp.
	12.	${\sim}Ta \supset Pa$	11, 8, M.P.
	13.	${\sim}{\sim}Ta$	12, 10, M.T.
	14.	Ta	13, D.N.
	15.	$(\exists x)\ (Tx)$	14, E.G.

2.	1.	$(x)\ [(Dx \vee Lx) \supset Px]$	
	2.	$(x)\ [(Px \vee Ex) \supset Rx]$	
		$\therefore\ (x)\ (Dx \supset Rx)$	
	3.	$(Dy \vee Ly) \supset Py$	1, U.I.

4.	$\sim(Dy \vee Ly) \vee Py$	3, Impl.
5.	$(\sim Dy \bullet \sim Ly) \vee Py$	4, De M.
6.	$Py \vee (\sim Dy \bullet \sim Ly)$	5, Com.
7.	$(Py \vee \sim Dy) \bullet (Py \vee \sim Ly)$	6, Dist.
8.	$Py \vee \sim Dy$	7, Simp.
9.	$\sim Dy \vee Py$	8, Com.
10.	$(\sim Dy \vee Py) \vee Ey$	9, Add.
11.	$\sim Dy \vee (Py \vee Ey)$	10, Assoc.
12.	$Dy \supset (Py \vee Ey)$	11, Impl.
13.	$(Py \vee Ey) \supset Ry$	2, U.I.
14.	$Dy \supset Ry$	12, 13, H.S.
15.	$(x)\ (Dx \supset Rx)$	14, U.G.

3. $(x)\ [Mx \supset (Lx \vee Px)]$ $\quad$ $[Ma \supset (La \vee Pa)]$
$(\exists x)\ (Mx \bullet \sim Cx)$ $\quad$ $(Ma \bullet \sim Ca)$
$\therefore (\exists x)\ (Lx \bullet \sim Cx)$ $\quad$ $\therefore (La \bullet \sim Ca)$

Ma	*La*	*Pa*	*Ca*
T	F	T	F

4.

1.	$(x)\ [Cx \supset (Sx \vee Ox)]$	
2.	$(x)\ (Sx \supset \sim Wx)$	
3.	$(\exists x)\ (Cx \bullet Wx)$	
	$\therefore (x)\ (Cx \bullet Ox)$	
4.	$Ca \bullet Wa$	3, E.I.
5.	$Wa \bullet Ca$	4, Com.
6.	Wa	5, Simp.
7.	$Sa \supset \sim Wa$	2, U.I.
8.	$\sim\sim Wa$	6, D.N.
9.	$\sim Sa$	7, 8, M.T.
10.	$Ca \supset (Sa \vee Oa)$	1, U.I.
11.	Ca	4, Simp.
12.	$Sa \vee Oa$	10, 11, M.P.
13.	Oa	12, 9, D.S.
14.	$Ca \bullet Oa$	11, 13, Conj.
15.	$(\exists x)\ (Cx \bullet Ox)$	14, E.G.

5. $(\exists x)\ (Dx \bullet Ax)$
$(x)\ [Ax \supset (Jx \vee Cx)]$
$(x)\ (Dx \supset \sim Cx)$

$(x)\ [(Jx \bullet Ix) \supset {\sim}Px]$
$(\exists x)\ (Dx \bullet Ix)$
$\therefore\ (\exists x)\ (Dx \bullet {\sim}Px)$

This argument is logically equivalent in $\boxed{a, b}$ to

$(Da \bullet Aa) \vee (Db \bullet Ab)$
$[Aa \supset (Ja \vee Ca)] \bullet [Ab \supset (Jb \vee Cb)]$
$(Da \supset {\sim}Ca) \bullet (Db \supset {\sim}Cb)$
$[(Ja \bullet Ia) \supset {\sim}Pa] \bullet [(Jb \bullet Ib) \supset {\sim}Pb]$
$(Da \bullet Ia) \vee (Db \bullet Ib)$
$\therefore\ (Da \bullet {\sim}Pa) \vee (Db \bullet {\sim}Pb)$

Proved invalid by

	Da	*Db*	*Aa*	*Ab*	*Ja*	*Jb*	*Ca*	*Cb*	*Ia*	*Ib*	*Pa*	*Pb*
	T	T	T	F	T	F	F	F	F	T	T	T
or	T	T	F	T	F	T	F	F	T	F	T	T

6.

1.	$(x)\ \{[Cx \bullet (Lx \vee Ox)] \supset {\sim}Fx\}$	
2.	$(x)\ ({\sim}Fx \supset {\sim}Ex)$	
	$\therefore\ (x)\ [(Cx \bullet Lx) \supset {\sim}Ex]$	
3.	$[Cy \bullet (Ly \vee Oy)] \supset {\sim}Fy$	1, U.I.
4.	${\sim}Fy \supset {\sim}Ey$	2, U.I.
5.	$[Cy \bullet (Ly \vee Oy)] \supset {\sim}Ey$	3, 4, H.S.
6.	$[(Cy \bullet Ly) \vee (Cy \bullet Oy)] \supset {\sim}Ey$	5, Dist.
7.	${\sim}[(Cy \bullet Ly) \vee (Cy \bullet Oy)] \vee {\sim}Ey$	6, Impl.
8.	$[{\sim}(Cy \bullet Ly) \bullet {\sim}(Cy \bullet Oy)] \vee {\sim}Ey$	7, De M.
9.	${\sim}Ey \vee [{\sim}(Cy \bullet Ly) \bullet {\sim}(Cy \bullet Oy)]$	8, Com.
10.	$[{\sim}Ey \vee {\sim}(Cy \bullet Ly)] \bullet [{\sim}Ey \vee {\sim}(Cy \bullet Oy)]$	9, Dist.
11.	${\sim}Ey \vee {\sim}(Cy \bullet Ly)$	10, Simp.
12.	${\sim}(Cy \bullet Ly) \vee {\sim}Ey$	11, Com.
13.	$(Cy \bullet Ly) \supset {\sim}Ey$	12, Impl.
14.	$(x)\ [(Cx \bullet Lx) \supset {\sim}Ex]$	13, U.G.

7.

$(x)\ [(Mx \bullet Tx) \supset {\sim}Fx]$	$[(Ma \bullet Ta) \supset {\sim}Fa]$
$(x)\ [(Bx \bullet Tx) \supset Ox]$	$[(Ba \bullet Ta) \supset Oa]$
$(\exists x)\ [(Ax \bullet Sx) \bullet Bx]$	$[(Aa \bullet Sa) \bullet Ba]$

$(x)\ (Sx \supset Fx)$	$(Sa \supset Fa)$
$(x)\ (Bx \supset Mx)$	$(Ba \supset Ma)$
$\therefore\ (\exists x)\ (Ax \bullet {\sim}Ox)$	$\therefore\ (Aa \bullet {\sim}Oa)$

Ma	*Ta*	*Fa*	*Ba*	*Oa*	*Aa*	*Sa*
T	F	T	T	T	T	T

8.

1.	$(x)\ [(Cx \bullet Kx) \supset (Fx \supset Vx)]$	
2.	$(x)\ \{Cx \supset [Fx \equiv {\sim}(Ix \vee Px)]\}$*	
3.	$(x)\ (Cx \supset Kx)$	
4.	$(x)\ (Kx \supset {\sim}Px)$	
5.	$(\exists x)\ (Cx \bullet {\sim}Vx)$	
	$\therefore\ (\exists x)\ (Cx \bullet Ix)$	
6.	$Ca \bullet {\sim}Va$	5, E.I.
7.	$Ca \supset Ka$	3, U.I.
8.	$Ka \supset {\sim}Pa$	4, U.I.
9.	Ca	6, Simp.
10.	Ka	7, 9, M.P.
11.	$Ca \bullet Ka$	9, 10, Conj.
12.	$(Ca \bullet Ka) \supset (Fa \supset Va)$	1, U.I.
13.	$Fa \supset Va$	12, 11, M.P.
14.	${\sim}Va \bullet Ca$	6, Com.
15.	${\sim}Va$	14, Simp.
16.	${\sim}Fa$	13, 15, M.T.
17.	$Ca \supset [Fa \equiv {\sim}(Ia \vee Pa)]$	2, U.I.
18.	$Fa \equiv {\sim}(Ia \vee Pa)$	17, 9, M.P.
19.	$[Fa \supset {\sim}(Ia \vee Pa)] \bullet [{\sim}(Ia \vee Pa) \supset Fa]$	18, Equiv.
20.	$[{\sim}(Ia \vee Pa) \supset Fa] \bullet [Fa \supset {\sim}(Ia \vee Pa)]$	19, Com.
21.	${\sim}(Ia \vee Pa) \supset Fa$	20, Simp.
22.	${\sim}{\sim}(Ia \vee Pa)$	21, 16, M.T.
23.	$Ia \vee Pa$	22, D.N.
24.	$Pa \vee Ia$	23, Com.
25.	${\sim}Pa$	8, 10, M.P.
26.	Ia	24, 25, D.S.
27.	$Ca \bullet Ia$	9, 26, Conj.
28.	$(\exists x)\ (Cx \bullet Ix)$	27, E.G.

* If the second premise is interpreted differently as $(x)\ \{Cx \supset [{\sim}(Ix \vee Px) \supset Fx]\}$, then the proof of validity is two steps shorter. This alternative interpretation is also correct.

9.	$(x)\ [Lx \supset (Dx \bullet Wx)]$	$La \supset (Da \bullet Wa)$
	$(x)\ \{Wx \supset [(Gx \supset Ex) \bullet (Tx \supset Cx)]\}$	$Wa \supset [(Ga \supset Ea) \bullet (Ta \supset Ca)]$
	$(x)\ [(Dx \bullet Ax) \supset \sim Tx]$	$(Da \bullet Aa) \supset \sim Ta$
	$(\exists x)\ [Lx \bullet (Cx \bullet \sim Ex)]$	$La \bullet (Ca \bullet \sim Ea)$
	$\therefore (\exists x)\ (Lx \bullet \sim Ax)$	$\therefore La \bullet \sim Aa$

La	*Da*	*Wa*	*Ga*	*Ea*	*Ta*	*Ca*	*Aa*
T	T	T	F	F	F	T	T

10.	1.	$(\exists x)\ (Cx \bullet Rx)$	
	2.	$(x)\ [Rx \supset (Sx \vee Bx)]$	
	3.	$(x)\ [Bx \supset (Dx \vee Px)]$	
	4.	$(x)\ (Px \supset Lx)$	
	5.	$(x)\ (Dx \supset Hx)$	
	6.	$(x)\ (\sim Hx)$	
	7.	$(x)\ \{[(Cx \bullet Rx) \bullet Fx] \supset Ax\}$	
	8.	$(x)\ (Rx \supset Fx)$	
	9.	$(x)\ [Cx \supset \sim(Lx \bullet Ax)]$	
		$\therefore (\exists x)\ (Cx \bullet Sx)$	
	10.	$Ca \bullet Ra$	1, E.I.
	11.	$Ra \bullet Ca$	10, Com.
	12.	Ra	11, Simp.
	13.	$Ra \supset Fa$	8, U.I.
	14.	Fa	13, 12, M.P.
	15.	$(Ca \bullet Ra) \bullet Fa$	10, 14, Conj.
	16.	$[(Ca \bullet Ra) \bullet Fa] \supset Aa$	7, U.I.
	17.	Aa	16, 15, M.P.
	18.	$Ca \supset \sim(La \bullet Aa)$	9, U.I.
	19.	Ca	10, Simp.
	20.	$\sim(La \bullet Aa)$	18, 19, M.P.
	21.	$\sim La \vee \sim Aa$	20, De M.
	22.	$\sim Aa \vee \sim La$	21, Com.
	23.	$Aa \supset \sim La$	22, Impl.
	24.	$\sim La$	23, 17, M.P.
	25.	$Pa \supset La$	4, U.I.
	26.	$\sim Pa$	25, 24, M.T.
	27.	$Da \supset Ha$	5, U.I.
	28.	$\sim Ha$	6, U.I.
	29.	$\sim Da$	27, 28, M.T.

	30.	$\sim Da \bullet \sim Pa$	29, 26, Conj.
	31.	$\sim(Da \vee Pa)$	30, De M.
	32.	$Ba \supset (Da \vee Pa)$	3, U.I.
	33.	$\sim Ba$	32, 31, M.T.
	34.	$Ra \supset (Sa \vee Ba)$	2, U.I.
	35.	$Sa \vee Ba$	34, 12, M.P.
	36.	$Ba \vee Sa$	35, Com.
	37.	Sa	36, 33, D.S.
	38.	$Ca \bullet Sa$	19, 37, Conj.
	39.	$(\exists x)\ (Cx \bullet Sx)$	38, E.G.

11.	1.	$(x)\ [Ex \supset (Vx \bullet Rx)]$	
	2.	$(x)\ [Vx \supset (Dx \bullet Ex)]$	
		$\therefore\ (x)\ [(Vx \vee Ex) \supset (Vx \bullet Ex)]$	
	3.	$Ey \supset (Vy \bullet Ry)$	1, U.I.
	4.	$\sim Ey \vee (Vy \bullet Ry)$	3, Impl.
	5.	$(\sim Ey \vee Vy) \bullet (\sim Ey \vee Ry)$	4, Dist.
	6.	$\sim Ey \vee Vy$	5, Simp.
	7.	$Ey \supset Vy$	6, Impl.
	8.	$Vy \supset (Dy \bullet Ey)$	2, U.I.
	9.	$Vy \supset (Ey \bullet Dy)$	8, Com.
	10.	$\sim Vy \vee (Ey \bullet Dy)$	9, Impl.
	11.	$(\sim Vy \vee Ey) \bullet (\sim Vy \vee Dy)$	10, Dist.
	12.	$\sim Vy \vee Ey$	11, Simp.
	13.	$Vy \supset Ey$	12, Impl.
	14.	$(Vy \supset Ey) \bullet (Ey \supset Vy)$	13, 7, Conj.
	15.	$Vy \equiv Ey$	14, Equiv.
	16.	$(Vy \bullet Ey) \vee (\sim Vy \bullet \sim Ey)$	15, Equiv.
	17.	$(\sim Vy \bullet \sim Ey) \vee (Vy \bullet Ey)$	16, Com.
	18.	$\sim(Vy \vee Ey) \vee (Vy \bullet Ey)$	17, De M.
	19.	$(Vy \vee Ey) \supset (Vy \bullet Ey)$	18, Impl.
	20.	$(x)\ [(Vx \vee Ex) \supset (Vx \bullet Ex)]$	19, U.G.

12.	1.	$(x)\ [(Fx \vee Gx) \supset Hx]$	
	2.	$(x)\ [Hx \supset \sim(Ix \vee Jx)]$	
	3.	$(\exists x)\ [Gx \bullet (Jx \bullet Kx)]$	
	4.	$(\exists x)\ (Fx \bullet \sim Kx)$	
		$\therefore\ (\exists x)\ (Fx \bullet Ix)$	
	5.	$Ga \bullet (Ja \bullet Ka)$	3, E.I.

6.	Ga	5, Simp.
7.	$Ga \vee Fa$	6, Add.
8.	$Fa \vee Ga$	7, Com.
9.	$(Fa \vee Ga) \supset Ha$	1, U.I.
10.	Ha	9, 8, M.P.
11.	$Ha \supset \sim(Ia \vee Ja)$	2, U.I.
12.	$\sim(Ia \vee Ja)$	11, 10, M.P.
13.	$\sim Ia \bullet \sim Ja$	12, De M.
14.	$\sim Ja \bullet \sim Ia$	13, Com.
15.	$\sim Ja$	14, Simp.
16.	$(Ja \bullet Ka) \bullet Ga$	5, Com.
17.	$Ja \bullet Ka$	16, Simp.
18.	Ja	17, Simp.
19.	$Ja \vee (\exists x)\ (Fx \bullet Ix)$	18, Add.
20.	$(\exists x)\ (Fx \bullet Ix)$	19, 15, D.S.

13.

$(x)\ [(Fx \vee Gx) \supset Hx]$	$[(Fa \vee Ga) \supset Ha] \bullet [(Fb \vee Gb) \supset Hb]$
$(x)\ [Hx \supset \sim(Ix \bullet Jx)]$	$[Ha \supset \sim(Ia \bullet Ja)] \bullet [(Hb \supset \sim(Ib \bullet Jb)]$
$(\exists x)\ [Gx \bullet (Jx \bullet Kx)]$	$[Ga \bullet (Ja \bullet Ka)] \vee [Gb \bullet (Jb \bullet Kb)]$
$(\exists x)\ (Fx \bullet \sim Kx)]$	$(Fa \bullet \sim Ka) \vee (Fb \bullet \sim Kb)$
$\therefore\ (\exists x)\ (Fx \bullet \sim Ix)$	$\therefore (Fa \bullet \sim Ia) \vee (Fb \bullet \sim Ib)$

Fa	*Fb*	*Ga*	*Gb*	*Ha*	*Hb*	*Ia*	*Ib*	*Ja*	*Jb*	*Ka*	*Kb*
T	F	T	T	T	T	T	F	F	T	F	T

or numerous other truth-value assignments.

14.

1.	$(x)\ (Gx \supset Vx)$	
2.	$(x)\ (Rx \supset Ox)$	
	$\therefore\ (x)\ [(Gx \bullet Rx) \supset (Vx \bullet Ox)]$	
3.	$Gy \supset Vy$	1, U.I.
4.	$Ry \supset Oy$	2, U.I.
5.	$\sim Gy \vee Vy$	3, Impl.
6.	$(\sim Gy \vee Vy) \vee \sim Ry$	5, Add.
7.	$\sim Gy \vee (Vy \vee \sim Ry)$	6, Assoc.
8.	$\sim Gy \vee (\sim Ry \vee Vy)$	7, Com.
9.	$(\sim Gy \vee \sim Ry) \vee Vy$	8, Assoc.
10.	$\sim Ry \vee Oy$	4, Impl.
11.	$(\sim Ry \vee Oy) \vee \sim Gy$	10, Add.
12.	$\sim Gy \vee (\sim Ry \vee Oy)$	11, Com.

	13.	$({\sim}Gy \lor {\sim}Ry) \lor Oy$	12, Assoc.
	14.	$[({\sim}Gy \lor {\sim}Ry) \lor Vy] \bullet [({\sim}Gy \lor {\sim}Ry) \lor Oy]$	9, 13, Conj.
	15.	$({\sim}Gy \lor {\sim}Ry) \lor (Vy \bullet Oy)$	14, Dist.
	16.	${\sim}(Gy \bullet Ry) \lor (Vy \bullet Oy)$	15, De M.
	17.	$(Gy \bullet Ry) \supset (Vy \bullet Oy)$	16, Impl.
	18.	$(x)\ [(Gx \bullet Rx) \supset (Vx \bullet Ox)]$	17, U.G.

15.	1.	$(x)\ (Ox \supset Sx)$	
	2.	$(\mathrm{x})\ (Lx \supset Tx)$	
		$\therefore (x)\ [(Ox \lor Lx) \supset (Sx \lor Tx)]$	
	3.	$Oy \supset Sy$	1, U.I.
	4.	$Ly \supset Ty$	2, U.I.
	5.	${\sim}Oy \lor Sy$	3, Impl.
	6.	$({\sim}Oy \lor Sy) \lor Ty$	5, Add.
	7.	${\sim}Oy \lor (Sy \lor Ty)$	6, Assoc.
	8.	$(Sy \lor Ty) \lor {\sim}Oy$	7, Com.
	9.	${\sim}Ly \lor Ty$	4, Impl.
	10.	$({\sim}Ly \lor Ty) \lor Sy$	9, Add.
	11.	${\sim}Ly \lor (Ty \lor Sy)$	10, Assoc.
	12.	${\sim}Ly \lor (Sy \lor Ty)$	11, Com.
	13.	$(Sy \lor Ty) \lor {\sim}Ly$	12, Com.
	14.	$[(Sy \lor Ty) \lor {\sim}Oy] \bullet [(Sy \lor Ty) \lor {\sim}Ly]$	8, 13, Conj.
	15.	$(Sy \lor Ty) \lor ({\sim}Oy \bullet {\sim}Ly)$	14, Dist.
	16.	$({\sim}Oy \bullet {\sim}Ly) \lor (Sy \lor Ty)$	15, Com.
	17.	${\sim}(Oy \lor Ly) \lor (Sy \lor Ty)$	16, De M.
	18.	$(Oy \lor Ly) \supset (Sy \lor Ty)$	17, Impl.
	19.	$(x)\ [(Ox \lor Lx) \supset (Sx \lor Tx)]$	18, U.G.

16.	1.	Ms	
		$\therefore (x)\ (Mx \lor {\sim}Mx)$	
	2.	$Ms \lor {\sim}My$	1, Add.
	3.	${\sim}My \lor Ms$	2, Com.
	4.	$My \supset Ms$	3, Imp.
	5.	$My \supset (My \bullet Ms)$	4, Abs.
	6.	${\sim}My \lor (My \bullet Ms)$	5, Impl.
	7.	$({\sim}My \lor My) \bullet ({\sim}My \lor Ms)$	6, Dist.
	8.	${\sim}My \lor My$	7, Simp.
	9.	$My \lor {\sim}My$	8, Com.
	10.	$(x)\ (Mx \lor {\sim}Mx)$	9, U.G.

Chapter 11

Section 11.2
Exercises on pages 448–452

1. This is an analogical argument. The analogy drawn here is between beating a man when his hands are bound and being wiser than a woman as a consequence of a better education, one party having an enormous advantage in both cases. In the first case, it is plain that one with such an advantage ought not to boast of his courage; in the second case (this argument concludes), it is equally inappropriate for one with such an advantage to boast of his relative wisdom.

2. This is an analogical argument. The author is plainly criticizing the inconsistency of those who oppose Zionism and yet say they are not anti-Semitic.

3. Non-argumentative use of analogy.

4. Non-argumentative use of analogy.

5. Analogical argument.

6. This is an analogical argument. A child may think that the racing of the waters of the Rhine, at Mainz, is caused by the wheels of the water mills there, when we know, of course, that the wheels turn because of the racing waters. So, by analogy, von Liebig thought it a mistake to attribute fermentation to microbes when the microbes (he believed) were in fact caused by the fermentation. Justus von Liebig was a great chemist and biologist, but in this matter his analogical argument led to the wrong conclusion.

7. This short passage need not be serving as an analogical argument, but very likely it is doing so—depending upon context, of course. If it is an argument, the unstated conclusion is that when one talks about Christianity one cannot avoid saying something about sin.

8. Non-argumentative use of analogy.

9. Plainly an argument—whose conclusion is that Australians ought not eat kangaroos!

10. Analogical argument.

11. Non-argumentative use of analogy.

12. This is an analogical argument whose conclusion is that the Elgin Marbles ought to be returned to Greece.

13. This is an analogical argument whose conclusion is that marriage oppresses women.

14. Non-argumentative use of analogy.

15. Non-argumentative use of analogy.

16. This is a non-argumentative use of analogy, aiming to convey the great difficulties encountered in the quest for usable energy derived from fusion.

17. This passage serves chiefly as an argument, whose conclusion is that the very limited effects of definition in mathematics ought be fully understood, and respected.

18. Non-argumentative use of analogy.

19. Non-argumentative—and very touching!—use of analogy.

20. Analogical argument.

Section 11.3 – A
Exercises on pages 457–459

1. a. More probable. **Number of similar respects**. The change provides an additional respect in which the instance in the conclusion is the same as those in the premises.

   b. More probable. **Number of entities**. With this change the number of entities in the premises is substantially increased.

   c. More probable. **Claim made by the conclusion**. With this change in the premises, the conclusion, although unchanged, is now, relatively speaking, substantially more modest.

   d. More probable. **Variety among the premises**. With this change, the dissimilarity among the instances in the premises is clearly established.

   e. Less probable. **Disanalogy**. With this change in the premises, a significant difference between the instance in the conclusion and the instances in the premises is introduced.

   f. Neither. **Relevance**. It is unlikely that the dividends paid by tobacco companies would have any impact on the profitability of oil companies or the price of their shares.

2. a. less; criterion 5; an important disanalogy has arisen.
   b. more; criterion 2; more dissimilarity among the premises.
   c. neither; criterion 4; the additional premise is not relevant.
   d. more; criterion 1; number of entities increased.
   e. more; criterion 3; number of respects increased.
   f. more; criterion 5; with this premise added the conclusion becomes relatively more modest.

3. a. more; criterion 1; number of entities increased.
   b. neither; criterion 4; this additional premise is not relevant.
   c. more; criterion 3; number of respects increased.
   d. more; criterion 6; with this premise added the conclusion becomes relatively more modest.
   e. more; criterion 2; more dissimilarity among the premises.
   f. less; criterion 5; the premise introduces a significant disanalogy.

4. a. more; criterion 2; more dissimilarity among the premises.
   b. more; criterion 3; another similar respect has been added.
   c. less; criterion 5; an important disanalogy has been introduced.
   d. more; criterion 6; the added premise renders the conclusion relatively more modest.
   e. neither. (But, although the hour is not relevant to the substance of the course, Bill may know that he is more alert and/or appreciative early in the morning, in which case another significant respect may have been added, and the conclusion becomes more probable on criterion 3.)
   f. more; criterion 1; since all those courses mentioned are social sciences, the number of entities has been increased by the addition of the premise.

5. (a) more, criterion 2; (b) less, criterion 5; (c) more, criterion 3; (d) neither, criterion 4; (e) more, criterion 6; (f) more, criterion 1.

Section 11.3 – B (Four analyses are provided here, as models)
Exercises on pages 459–462

1. Large diamonds, armies, great intellects all have the attributes of greatness [of value for diamonds, of military strength for armies, of mental superiority for intellects], and of divisibility [through cutting for diamonds; dispersion for armies; interruption, disturbance, and distraction for intellects].

   Large diamonds and armies all have the attribute of having their greatness diminish when they are divided. Therefore great intellects also have the attribute of having their greatness diminish when they are divided.

   (1) Only two kinds of entities (armies and large diamonds) are given as analogus examples, which is not very many. On the other hand, there are many, many instances of both kinds of entities. By our first criterion the argument is fairly cogent.

   (2) Although there are only two kinds of entities in the premises, armies and large diamonds are quite dissimilar to each other, so from the point of view of our second criterion, the argument is moderately cogent.

   (3) There are only two respects in which the things involved are said to be analogous. This is not many and the argument is accordingly rather weak.

   (4) Schopenhauer recognizes that the question of relevance is important, for he introduces a separate little discussion on this point. He urges that the superiority (the "greatness") of a great intellect "depends upon" its concentration or undividedness. Here he invokes the illustrative or explanatory (nonargumentative) analogy of the concave mirror, which focuses all its available light upon one point. There is indeed some merit in this claim, and by our fourth criterion the argument has a fairly high degree of cogency.

   (5) The instances with which the conclusion deals are enormously different from the instances mentioned in the premises. There are so many disanalogies between intellects, on the one hand, and large diamonds and armies, on the other, that by our fifth criterion Schopenhauer's argument is almost totally lacking in probative force.

   (6) The conclusion states only that, when "divided," a great intellect will sink to the level of an ordinary one. This is not a terribly bold conclusion relative to the premises, and so by our sixth criterion the argument is fairly cogent.

   Finally, it must be admitted that the whole passage might plausibly be analyzed as invoking large diamonds and armies for illustrative and explanatory rather than argumentative purposes. The plausibility of this alternative analysis, however, derives more from the weakness of the analogical argument than from what is explicitly stated in the passage in question.

5. This passage can be analyzed in two different ways. In both ways the analogical argument is presented primarily as an illustration of the biologist's reasoning.

First analysis:

Porpoises and men have lungs, warm blood, and hair. Men are mammals.
Therefore porpoises also are mammals.

(1) There are many instances examined, which makes the conclusion probable.

(2) There are very few dissimilarities among men—biologically speaking—and by our second criterion this tends to weaken the argument.

(3) The premises mention only three respects in which porpoises and men resemble each other. This is not a large number—at least not enough to make the argument plausible.

(4) In terms of relevance the argument is superlatively good, because biologists have found the three attributes noted in the premises to be remarkably dependable indicators of other mammalian characteristics.

(5) There are many disanalogies between men and porpoises: porpoises are aquatic, men are terrestrial; porpoises have tails, men do not; porpoises do not have the well-developed, highly differentiated limbs characteristic of men; and so on. These tend to weaken the argument.

(6) The conclusion is very bold relative to the premises, because so many attributes are summarized in the term "mammal" (as evidenced by the variety of additional attributes confidently predicted by the zoologist). This tends, of course, to weaken the argument.

Alternative analysis:

Porpoises and humans have lungs, warm blood, and hair. Humans also nurse their young with milk, have a four-chambered heart, bones of a particular type, a certain general pattern of nerves and blood vessels, and red blood cells that lack nuclei. Therefore porpoises also nurse their young with milk, have a four-chambered heart, bones of a same particular type, the same general pattern of nerves and blood vessels, and red blood cells that lack nuclei.

This version of the analogical argument contained in the given passage is evaluated in much the same way as the first one discussed. It is a somewhat stronger argument than the first one according to the sixth criterion, because in spite of the apparently greater detail in the second version's conclusion, it is more modest than that of the first version, since being a mammal entails all of these anatomical details plus many more.

[Nature constantly reminds us that such arguments are only probable. The platypus, for instance, resembles other mammals in having lungs, hair, warm blood, milk glands, and so on. Yet while other mammals are viviparous (bearing their young alive), the platypus lays eggs.]

10. This is an example of a very strong analogical argument. It is not likely to be found deficient under any of the six criteria for appraisal. The number of instances (my past visits to the dentist) probably is considerable. The variety of work done on my teeth during these visits (variety within the cases used in the premises) is likely to be substantial. The respects in which my dental visits and the dental visit in question are similar are likely to be many and significant: the same kind of treatment, on the same bodily organs, using the same kind of dental instruments, and so on. This is a case very much like those with which I have direct experience. The causal relevance of the treatments is undoubted. The claim made in the conclusion (merely that the extraction was painful) is modest and entirely reasonable. If the argument proves in some degree vulnerable, that is likely to be because the person whose treatment is in question differs importantly from me with respect to his or her tolerance for pain. On this fifth criterion—the identification of disanalogies—the argument may be attacked, but unless the disanalogies are very significant, the attack is not likely to succeed in persuading me that his tooth extraction without anesthetic did not hurt him.

15. A watch and other human artifacts display an intricacy that justifies our inference that they have been designed by their maker. Natural mechanisms also are intricate, as are the processes of the universe; hence we are justified in concluding that they also are designed by some maker.

    (1) There are huge numbers of manufactured mechanisms that we know to have been designed and made. On the first criterion, the argument has much support.

    (2) There are many great dissimilarities among the cases in the premises, which strengthens the argument, but these dissimilarities do not outweigh the major disanalogies noted under (5), so the argument cannot be said to gain very much from its strength on this criterion.

(3) There is only one respect in which the products of human design are claimed to be like the products of the divine Maker, namely, the intricacy and complexity of the designs encountered—the "curious adapting of means to ends," as Hume put it in the *Dialogues Concerning Natural Religion.* Although this is only one respect, it is a respect of great importance if established. This single (but disputable) respect leaves the argument in problematic circumstances.

(4) Whether the analogy is relevant is difficult to say. For those who doubt the applicability of cause-and-effect reasoning beyond the range of experienced phenomena, it would not be relevant. For those who accept the universal applicability of causal analysis, going beyond human experience to the universe itself, the analogy is indeed relevant.

(5) There are many great disanalogies between the human artifacts mentioned in the premises and the natural mechanisms we encounter. The size, duration, and general character of the universe render it different from any watch or other humanly designed machine, in many fundamental ways. From this point of view, the conclusion has only little probability.

(6) How modest the conclusion is, relative to the premises, depends on what is included in the claim that there is a divine Maker of the natural universe. If implicit in this conclusion is the singularity, perfection, infinity, and incorporeality of a supernatural maker (as commonly intended by such arguments), the conclusion is very bold relative to the premises, rendering the argument weak. If the qualities normally attributed to God are not part of the conclusion, the mere claim that there is a "Maker" may be modest enough to be well supported by the premises. All things considered, the argument is neither worthless nor compelling. The degree of probability with which it warrants its conclusion decreases, however, as the similarity of the Maker in that conclusion to the God of traditional Western theism increases. The truth or falsehood of such theism, of course, is not affected by the weakness of an argument designed to establish it.

Section 11.4
Exercises on pages 465–468

1. The argument being refuted has the conclusion that TV cameras should not be allowed in the courtroom because they make the justice system look bad. The refuting argument has the conclusion that allowing journalists to cover the Vietnam War made American conduct in that war look bad.

The two arguments are indeed similar in form; this analogical argument has much merit. If, in the case of the war, what we learn from the coverage is upsetting, that is not to be blamed on the coverage but on the conduct itself. If what we learn from TV coverage of the court is upsetting (although this may prove not to be so), that is not to be blamed upon the TV cameras but upon the faults of the system itself.

2. The argument being refuted has the conclusion that the Bolsheviks, pursuing the communist revolution that supposedly will ultimately overthrow the bourgeoisie, should never compromise. The refuting argument has a form that is similar but not identical; the ascent of an unexplored mountain may indeed require one to move in zigzags and to retrace one's steps—but there is an important disanalogy between the circumstances of the explorer and those of the communist revolutionary, in that the latter may be bound by principles—moral principles or metaphysical commitments—that preclude compromises of some kinds, or compromises with some parties. Lenin's famous refuting analogy has some argumentative force, but it is not compelling.

3. The argument being refuted is the claim that there can be no characteristics of the human mind attributable to nonphysical causes, because the brain (a physical organ) has been so thoroughly explored that no sites in it remain as a possible locus for such causes. The refuting analogy, although far-fetched and jocular, has some merit, in ridiculing the claim that God does not exist because an astronaut, searching space, did not find him. The disanalogies are very great, of course; the astronaut (for example) certainly did not search the whole of space. But the refuting analogy, by exhibiting the logical inadequacy of drawing conclusions about the existence of nonphysical entities from the exploration of physical sites, has substantial bite.

4. The target argument opposes the construction of new highways as a long-term solution to traffic, on the ground that new highways induce more traffic. The attacking analogical argument concludes that it would be equally foolish to refrain from building more grocery stores on the grounds that they would attract more customers. The customers are there, and so are the cars; neither the highways nor the stores are their causes. The form of the two arguments is indeed similar, but the attacking analogy remains problematic because, while more customers attracted to a given grocery store may be a good thing, more traffic attracted to a given highway is generally thought not to be so.

5. The argument being refuted is the following:
Trees are cut down in very great numbers to make paper.
Using recycled paper would make it unnecessary to cut down many of those trees.
Therefore, we ought to use recycled paper to reduce the destruction of trees.
The refuting analogy is:

Cornstalks are cut down in very great number to harvest corn.
Cutting back on corn consumption would make it unnecessary to cut down many of those cornstalks.
Therefore we ought to cut down on corn consumption in order to reduce the destrution of cornstalks.
The refuting analogy does have the same form as the argument under attack. Moreover, its premises are true and its conclusion surely is false. These considerations make this an effective counterargument. However, the refuting analogy supposes that the environmental status of cornstalks is essentially akin to that of trees. That plainly is disputable, and if a substantial disanalogy can be exhibited here, that would greatly weaken the purportedly refuting analogical argument.

6. The argument being refuted is the following: Since almost every immigrant who passed through Ellis Island was bound for New York, and not New Jersey, the island ought to be considered a part of New York rather than of New Jersey. The refuting argument, with similar form, is that most passengers arriving at Newark International Airport are bound for New York and not New Jersey—but we do not for a moment suppose that this supports the claim that Newark airport is in New York! (The undisputed Kentucky location of Cincinnati International Airport is offered as an additional example.)

   The arguments do have similar form, and the refuting analogy has much merit. There is however, an important disanalogy: there is no dispute over the fact that Newark International Airport and Cincinnati International Airport are located in New Jersey and Kentucky, respectively. The question of which state the airports are located in is settled. In contrast, the ownership of Ellis Island is historically uncertain, and for this reason it may well be argued that the destination of those who arrived there is a relevant consideration.

   [This dispute has recently been settled,through arbitration, by a division of Ellis Island; the arbitrator's decision gave part of it to New York, but (based upon historical records) the major portion of the island went to New Jersey.]

7. The largest argument is the claim that Islamic terrorism cannot be caused by poverty and injustice, because one of its most powerful leaders is a multimillionaire. The attacking analogy suggests that if the circumstance of the leader were indeed a determining factor, the circumstances of Abraham Lincoln, who was not a slave, would show that slavery couldn't have been a cause of the American civil war. There is some merit in the refuting analogy, which has essentially the same form as its target. The refuting analogy is farfetched, but then the original claim, that the wealth of Osama bin Laden is ground for a judgment about the causes of Islamic terrorism, is also farfetched.

8. The target argument concludes that it is reasonable to suppose that the universe we live in was designed just because it has properties that make the development of intelligent life possible. The refuting analogy points out that a given set of results in a lottery does not show that that outcome was foreordained. Some number in the lottery must win; when we learn which number it is we cannot conclude that it was chosen by design. Some set of laws appears to govern the universe; when we know which ones they are we cannot conclude that they were chosen by design.

9. The target argument concludes that the physical material investigated is such that the object (artificial intelligence) can never be developed; the attacking analogy exhibits the similar mistake made long ago when that kind of reason (the "wrong" physical material) was thought to show that automobiles could never replace horses. Artificial intelligence may indeed prove elusive—but this analogy is a meritorious attack on one weak ground for supposing that the objective is unobtainable.

10. The argument being refuted concludes that some policy ought to be adopted in the United States because it is adopted in every European democracy, and because Europeans are amazed that we do not adopt it. The refuting analogy is that Europeans are also amazed that we bathe as frequently as we do—and of course we certainly would not change our bathing habits for that reason. The refuting analogy has essentially the same form as its target, which is appropriately ridiculed; but of course the weakness of that argument leaves open the question of whether or not we would be well advised to adopt the policy the argument was intended to support.

Chapter 12

Section 12.4 – A (Two models are provided)
Exercises on pages 479–481

1. The method of agreement is plainly the chief analytical tool in this investigation. In a sharply limited time period and over a limited area, all of those who became sick with hepatitis A were found to have eaten at a particular Chi-Chi's restaurant.

   Let *C* be the circumstance of having eaten at the Chi-Chi's restaurant, *D* through *O* be other circumstances of the diners who ate at Chi-Chi's, *s* be the phenomenon of contracting hepatitis A, and *t* through *z* be other outcomes for the diners. Then, we can schematize the argument as follows:

   C DEFG occurs with *s tuv*
   C HIJK occurs with *s xyz*
   C LMNO occurs with *s wyt*

   We can reasonably conclude that *C* is the cause of *s*.

   That restaurant was quickly closed, of course; but the method of agreement could only go so far as to identify the food in which the virus was carried. The more remote and eliminable cause of the outbreak, the particular source of the contamination of the virus-carrying scallions, could not be identified by the use of this method alone.

5. This is a straightforward use of the method of agreement. Of all the pairs of brothers, who were gay, the one feature that was common—not to all of them, but to a very high percentage of them—was that they shared certain DNA sequences on their X chromosome. This analysis has some merit—but it falls far short of proving that the brothers' homosexuality was caused by those sequences. We note in the first place that those sequences were *not* shared by *all* the gay brothers, which immediately casts some doubt on the alleged causal relation; agreement is not universal. And we note in the second place that there may very well be *other* characteristics shared by those pairs of brothers which led to their homosexuality—characteristics perhaps not yet identified, and certainly not discussed in this research. Although this use of the method of agreement does not serve as proof, it does point to a range of considerations worthy of further investigation in seeking causal connections pertaining to homosexuality.

Section 12.4 – B (Two models are provided)
Exercises on pages 484–488

1. This is a very good illustration of the method of difference, because, to the extent reasonably possible for the investigators, there were no other significant differences between the groups of subjects, except the one being tested: the presence or absence of intervening sleep. Those subjects who had slept performed markedly better in recall; those who had not slept performed markedly worse. The method of difference is plainly the chief tool here. But much depends upon the size of the subject pool, and on the ability of researchers to eliminate other factors. If previously unrecognized differences had later been discovered between the subjects in the two groups, this effort to apply the method of difference would not have succeeded.

5. This is a splendid illustration of the method of difference, and one that is typical of successful investigations in the world of medical research. A test to determine the causal impact of the gene *Ras-GRF* was devised by breeding two populations of mice, populations that did not differ in any important respects aside from the presence or absence of that gene. All those mice that lack the gene, but none of those in which the gene is present, exhibit the failure to learn from experience, and repeat behaviors (returning to dark corners) that cost them an electric shock. We may reflect upon these results with the help of the schematic representation of the method of difference:

   ABCD occur together with *w x y z*
   BCD occur together with *x y z*

   Here, the absence of the gene *Ras-GRF* is represented by *A*; other circumstances of the subject mice in both the experimental and control groups are represented by *B, C*, and *D*; failure to avoid shocks is represented by *w*; and other responses of the mice during the experiment are represented by *x, y*, and *z*.

   We may conclude, as Mill would have put it, that *A* (the absence of that gene) is the cause, or an indispensable part of the cause, of *w*, the incautious response leading to repeated shocks.

Section 12.4 – C (One model is provided)
Exercises on pages 489–491

4. This research used the method of difference to narrow down some causal relations already suggested by a previous application of the method of agreement—namely, that animals that share the circumstance of being on low-calorie diets also share the circumstances of having longer life spans and an abnormally cool body temperature. Without the subsequent application of the method of difference, it would be tempting to conclude that low body temperatures and extended life spans are directly caused by a low-calorie diet. The "difference" part of the research casts serious doubt on this simplistic conclusion by removing the low-calorie diet circumstance while keeping (with the help of genetic engineering) the low-body-temperature circumstance. A body temperature below the normal range is thus isolated as a relevant contributing factor to longevity.

Section 12.4 – D (Two models are provided)
Exercises on pages 493–495

1. This is a case in which the method of residues does not confirm any particular hypothesis about the cause of the slowing of objects moving away from or around the sun, but it gives good reason to search for some cause (of the slowing phenomenon) not heretofore recognized or understood. Calculations based upon the many known factors that enter into the determination of the trajectories or orbits of such moving bodies yield results that do not accord with observational data. Those data present a puzzling discrepancy, a "residue" needing further explanation. The natural suggestion that this discrepancy is merely the result of some error in measurement is put in serious doubt when investigations repeatedly yield the same results after carefully accounting for possible errors. Something theoretically new—but presently unknown—appears to be operative. If that is the case, it is likely to be identified before long, and when it is identified that discovery will be attributable in part to the provocation of this application of the method of residues.

5. *A B* occur together with *a b*.
   B is known to be the cause of *b*.
   Therefore *A* is the cause of *a*.

   Here, *B* is the balloon by itself, uninflated; *A* is the air with which the balloon is inflated; *b* is the reading of the weight of the balloon when not inflated; *a b* is the reading of the weight of the balloon when inflated. The conclusion is that *a* is the reading of the weight of the air with which the balloon is inflated, and that the air with which the balloon is inflated must therefore be the cause of the residual weight reading.

Section 12.4 – E (Three models are provided)

Exercises on pages 498–500

1. The variations examined in this study are indeed concomitant: as the family incomes rose above the poverty line due to the casino payments, the incidence of psychiatric symptoms among the children of those families diminished, while there appeared to be no impact if the casino payments did not bring the family income above that poverty line. That psychiatric problems and poverty vary concomitantly would have been more solidly shown if there were evidence that children in families that had once been above the poverty line and then dropped below it began, after that drop, to manifest increased psychiatric symptoms.

4. The concomitant variations in this pair of studies are straightforward: there appears to be an inverse relationship between the number of hours of sleep and the number of accidents on the following day. In the first study, the *reduction* of sleep by one hour (because of the shift to daylight time) resulted in a marked *increase* in accidents on the day following. In the second study, the *increase* in the hours of sleep (because of the shift back from daylight to standard time) resulted in a marked *decrease* in accidents on the day following. Other causal factors may enter in, of course, but it would be hard to deny that this concomitance of variation does tend to confirm that accidents are in some degree caused by sleep deficiency.

7. The concomitant variations noted in this passage by a very distinguished economist, James Surowieki, are important as well as interesting. As the price of oil varies so does the strength of reformers and radicals in Iran: the lower the price of oil, the greater the power of more peaceable reformers; the higher the price, the greater the power of more bellicose radicals. Low-key foreign-policy pronouncements by the United States that do not threaten Iran, and consequently do not result in higher oil prices, may therefore actually have substantial benefit (as compared to more aggressive statements) in advancing long-term American objectives.

Section 12.5 (Five models are provided)
Exercises on pages 505–511

1. The method of concomitant variation is the chief tool in this case, where the phenomena that vary concomitantly are destructive and criminal behaviors on the one side, and the timing of the loss of virginity on the other side. But this timing is relative, not absolute: the factor of interest is not just an early loss of virginity—it is *the loss of virginity earlier than the average of one's peers*. This departure from the norm when it comes to sexual codes seems to go hand in hand with a departure from the norm when it comes to other kinds of codes.

5. The joint method of agreement and difference is used very cleverly in this investigation. First the investigator collected the data that show agreement in ages of death; only afterward, by further investigation, did he determine that there is (apparently) only one factor (left-handedness) that accounts for (or helps to account for) the patterns of difference in the age at which death takes place.

12. The method of difference is here put in sharp relief. Everything stays the same for the run of people who use the copying machine—except the dime "found" in the machine by some of them. Those who find the dime see the world through rosier glasses, at least for a very short while. Attitudes and outlook seem to be perceptibly changed by a happy windfall, even when that windfall is very small.

14. This is a powerful and very instructive application of the method of concomitant variation: the faster a car is driven the more likely it is that its driver or passengers will die in an accident. Because the deaths do not precede the greater speed, but follow it, we may reasonably conclude that greater speed causes more death.

16. This investigation achieves its result by using the method of difference, but doing so in a negative way. The assumption of the investigators, reasonably made, was that if noetic intervention (prayer, and the like) were at all efficacious there would be a difference in primary medical outcomes between those being treated for coronary heart disease who were the recipients of such intervention, and those who were not recipients of such intervention. The recipients, one might have thought, would have fared somewhat better. But there was no such difference—from which it may be inferred, by *modus tollens,* that prayer and related forms of non-physical intervention are without medical efficacy.

Chapter 13

Section 13.4 (Four models are provided)
Exercises on pages 532–537

1. The data to be explained here are the reports of the mapping of radiation in the entire cosmos. Two conflicting theories about the size and shape of the universe are offered to account for the resultant maps: one of these involves the hypothesis that the cosmos has ascertainable limits and a finite specifiable shape, that of a dodecahedron; the competing hypothesis makes the claim that no such shape is detectable from the radio maps at hand and therefore the known data support the conclusion that, so far as we can tell, the universe is infinite.

   While both hypotheses can be formulated in ways compatible with already-established theory, the peculiarity of the dodecahedron description of the universe renders it somewhat less easy to assimilate; yet its relative simplicity and clarity also render it in some ways more attractive. But in deciding between these competing hypotheses the critical consideration will be their predictive or explanatory power. The dodecahedron theory entails very specific predictions: it predicts that among the mapping data collected there will appear matching circular patterns where the surfaces of the envisaged dodecahedron are intersected by radiation. This is falsifiable, and therefore one or the other of the competing hypotheses is likely to be disconfirmed when further analyses of the radio maps of the universe are completed.

5. The data to be explained in this investigation are the well-established reports that, in general, boy babies are heavier than girl babies at birth. This is not in doubt. To account for this difference two different hypotheses are put forward: (a) that mothers of boys simply take in more energy than mothers of girls, or (b) that the mothers of boys for some reason use the energy they take in more efficiently. Before the study, both of these hypotheses were compatible with what was known about human gestation. But after measuring the food intake of mothers with babies of both sexes, it became clear that the first hypothesis is more compatible with the new data than the second hypothesis. Moreover, because it is far easier to measure actual caloric intake than the efficiency in the use of that intake, hypothesis (a) is also more usefully predictive. And for the same reason—ease of measurement—it is also simpler. The hypothesis that the increased weight of baby boys is caused by the greater amounts of food their mothers eat during pregnancy is quite well confirmed. Of course, that leads to further questions about the impact of the sex of fetus upon its mother, questions that other studies might be able to answer.

6. Some animals, including humans of course, are aware of themselves as individuals. But it is not known how deep into the animal kingdom this capacity for self-awareness extends. This experiment was designed to determine whether elephants are among the animals that can recognize themselves. The hypothesis explored is that elephants, looking at themselves in a mirror, would act in a way that would indicate that they grasped the fact that the images before them were images of *themselves*. To test this hypothesis the elephants were *marked* in a way that they could not see directly—a white X placed above one eye. If the hypothesis were correct then the elephant might act in ways that would confirm it. If the elephant did not know directly that the white X was over its own eye, and subsequently saw its own image in a mirror and saw a white X over the eye in that image, it would reach for that X on its own body *only if it believed that the elephant seen in the mirror was the image of her own body*. One elephant in this experiment did act in a way that made this self-recognition clear: seeing the white X in the mirror, she repeatedly touched with her trunk the white X on her body, the X she could not see directly. That repeated touching could only be explained by the fact that she believed that she had seen *herself*. At least some elephants, the investigators conclude, do have the capacity for self-awareness.

10. The data to be explained here are the remarkably steep fluctuations in the populations of lemmings in northern Europe. Hypotheses to explain these fluctuations have been many and various (even including self-annihilation) but none (at least until publication of this study) had given a fully adequate explanation of the four-year boom-and-bust cycles of lemming populations. The explanation proposed and confirmed by the study reported here relies only upon the behavior and fluctuation of the populations of four predator species. This study is a model of good science: 1) Its leading hypothesis is perfectly compatible with what is known about lemmings and related species, and avoids introducing the notion of suicide, which goes against our general understanding of the habits of wildlife; 2) Its predictive power is very great because, without relying upon any other factors, this theory can predict, retrospectively and prospectively, the steep rise and fall of lemming populations; and 3) It is attractively simple, in the sense that one and only one powerful causal factor, the known conduct of lemming predators, serves to provide the full explanation of the previously puzzling fluctuations.

Chapter 14

Section 14.2 – A
Exercises on pages 545–547

1. a. If each card drawn *is replaced* before the next drawing is made, the component events have absolutely no effect on one another and are therefore *independent*. In this case, $P(a \text{ and } b \text{ and } c) = P(a) \times P(b) \times P(c)$. There are 52 cards in the deck, of which four are aces. So the probability of drawing the first ace, $P(a)$, is 4/52, or 1/13. The probability of drawing the second ace, $P(b)$, is likewise 1/13, as is the probability of drawing the third ace, $P(c)$. So the probability of the joint occurrence of *a* and *b* and *c* is 1/13 × 1/13 × 1/13, or 1/2,197.

   b. If the cards drawn are *not replaced,* the component events are *dependent*, not independent. The formula is $P(a \text{ and } b \text{ and } c) = P(a) \times P(b \text{ if } a) \times P(c \text{ if } a \text{ and } b)$. In this case, the probability of drawing the first ace, $P(a)$, remains 4/52, or 1/13. But the probability of drawing a second ace if the first card drawn was an ace, $P(b \text{ if } a)$, is 3/51, or 1/17. And the probability of drawing a third ace if the first two cards drawn were aces, $P(c \text{ if } a \text{ and } b)$, is 2/50, or 1/25. The probability of the joint occurrence of these three dependent events is therefore 1/13 × 1/17 × 1/25, or 1/5,525.

   The probability of getting three successive aces in the second case is much lower than in the first, as one might expect, because without replacement the chances of getting an ace in each successive drawing are reduced by success in the preceding drawing.

2. 1/2 × 1/2 × 1/2 = 1/8

3. a. 40/67 × 40/67 × 40/67 × 40/67= 2,560,000/20,151,121

   b. 40/67 × 39/66 × 38/65 × 37/64 = 703/5,896

4. (1/6 × 1/6 × 1/6) × (1/6 × 1/6 × 1/6) × (1/6 × 1/6 × 1/6) = 1/10,077,696

5. 1/4 × 1/3 × 1/2 × 1/1 = 1/24

   The component events here are not independent, but in this case each success (in reaching the right house) *increases* rather than decreases the probability of the next success, because the number of available houses is fixed. After three men reach the correct house, the fourth (having to go to a different house) *must* succeed!

6. 1/25. Patient A enters through any door, and the probability that he used that door, given that he used it, is 1. The probability that patient B entered through the same door is 1/5, as it is also for patient C. 1 × 1/5 × 1/5 = 1/25.

Using the addition theorem, introduced in the following section (15.4), the same result is obtained: The probability of all three patients entering door #1 is $1/5 \times 1/5 \times 1/5 = 1/125$. That same probability applies to doors #2, #3, #4 and #5. Adding $1/125 + 1/125 + 1/125 + 1/125 + 1/125$ yields $1/25$.

7. No, he did not deserve a million-to-one payoff. The chances of his success, supposing the odds given, were better than 7 in a million. He wins if and only if all of his selected horses win; the probability of that happening is the product of the probabilities of all six winning in turn. If we convert each of the odds fractions given in the problem into decimal form, and then multiply them successively, we learn then that the probability of his winning was approximately .00000074.

8. $1/2 \times 1/3 \times 1/4 \times 3/300 \times 2/299 = 1/358,800$.

9. Flush. In this set of circumstances the probability of a flush is 9/47, while that of a straight is 8/47.

10. The probability that all four students will identify the same tire may be calculated in two different ways—just as the solution to problem 6 in this same set may be reached in two different ways.

    Suppose that the first student, A, names the front left tire. The probability of his doing so, *after having done so*, is 1. Now the probability of the second student, B, naming that tire is 1/4, there being four tires all (from B's point of view) equipossibly the one that A had named. The same is true of student C, and of student D. Therefore, regardless of which tire A does happen to name, (front left, or any other), the probability that all four students will name the same tire is $1 \times 1/4 \times 1/4 \times 1/4 = 1/64$ or .016.

    The same result could be achieved by first specifying a particular tire (say, the front left tire) and asking: What is the probability of all four students naming that specified tire? This would be $1/4 \times 1/4 \times 1/4 \times 1/4 = .004$. But the condition specified in the problem, that all four name the *same* tire, would be satisfied if all named the front left, or if all named the front right, or if all named the rear left, or if all named the rear right tire. So, if we were to approach the problem in this way, we also would need to inquire as to the probability of *either* the one or the other of these four outcomes—a calculation requiring the addition theorem, explained in Section 14.2B, for alternative outcomes. Because the four successful outcomes are mutually exclusive, we can simply sum the four probabilities: $.004 + .004 + .004 + .004 = .016$. The two ways of approaching the problem must yield exactly the same result, of course.

This dual analysis applies likewise to the three patients arriving at a building with five entrances, in Exercise 6. One may calculate $1 \times 1/5 \times 1/5 = 1/25$; or (using the addition theorem discussed in Section 14.2B), one may calculate $1/5 \times 1/5 \times 1/5 = 1/125$ and then add $1/125 + 1/125 + 1/125 + 1/125 + 1/125 = 1/25$.

Section 14.2 – B
Exercises on pages 552–553

1. Probability of losing with a 2, a 3, or a 12 is 4/36 or 1/9
   Probability of throwing a 4, and then a 7 before another 4, is $3/36 \times 6/9 = 1/18$
   Probability of throwing a 10, and then a 7 before another 10, is likewise 1/18
   Probability of throwing a 5, and the a 7 before another 5, is $4/36 \times 6/10 = 1/15$
   Probability of throwing a 9, and then a 7 before another 9, is likewise 1/15
   Probability of throwing a 6, and then a 7 before another 6, is $5/36 \times 6/11 = 5/66$
   Probability of throwing an 8, and then a 7 before another 8, is likewise 5/66
   Sum of the probabilities of the exclusive ways of the shooter's losing is 251/495
   So the shooter's chance of winning is $1 - 251/495 = 244/495$ or .493.

2. (a) $1 - 27/64 = 37/64$

   (b) $1 - 703/1{,}700 = 997/1{,}700$

3. $1 - 1/8 = 7/8$

4. (a) 1/6
   The probability of all three being red is $5/30 \times 5/30 \times 5/30 = 1/216$.
   The probability of all three being white is $10/30 \times 10/30 \times 10/30 = 1/27$.
   The probability of all three being blue is $15/30 \times 15/30 \times 15/30 = 1/8$.
   The probability of all three being the same color is the sum of these fractions, since they are mutually exclusive:

   $$1/216 + 1/27 + 1/8 = 36/216 = 1/6.$$

   (b) $1/406 + 6/203 + 13/116 = 117/812$

5. Yes. You lose the bet only if you throw a 2, or a 3, or a 4, or a 5, on *both* rolls of the die. On each throw, the chance of getting one of those four numbers is 4/6, or 2/3. The chance of losing the bet is therefore $2/3 \times 2/3$, or 4/9. Your chance of winning the bet, therefore, is $1 - 4/9 = 5/9 = .556$.

6. a) The probability that there will *not* be a duplication of birth date (month and day, ignoring year) in a group of thirty students, rounds to about .3.

   The calculation is done as follows: Begin with the birth date of any given person; the probability that he has that birth date is (of course) 1. The probability that the next person does not duplicate this birth date is 364/365. The probability that the birth date of the third person does not duplicate either one of the other two dates is 363/365, and the probability that the fourth does not duplicate any one of the earlier three is 362/365, and so on. To calculate the probability of not getting any duplication at all we must multiply all these fractions, one for each succeeding person: $364/365 \times 363/365 \times 362/365\ldots$ and so on. If there are thirty persons we must multiply the series to include the thirtieth person, for whom the probability of no duplication would be 335/365. (The probability we just calculated can be subtracted from 1 to determine the probability of *getting* a duplication.)

   The probability of a duplication of birth dates approximates .5 when the product of the multiplied series approximates .5; rounding to five decimal places throughout, that product reaches .49266 when the group consists of 23 persons.

7. a) .948658 (The probability that the woman will *not* live at least another 25 years is $1 - .801 = .199$, while the probability that the man will not live at least another 25 years is $1 - .742 = .258$. The product of these two probabilities is .051342, which is the probability that neither will live at least another 25 years. Subtracting this from 1 yields the probability that at least one of them will live that long.)

   b) .354316 (The probability that neither the man nor the woman will live at least another 25 years was already computed to be is .051342. The probability that *both* will live at least another 25 years is $.742 \times .801 = .594342$. The sum of these two probabilities can be subtracted from 1 to yield the probability that *only* one of them lives at least another 25 years.)

8. $(1/2 \times 6/10) + (1/2 \times 10/12) = 43/60$; if all the bottles had been in one case then the probability would have been 8/11.

9. There are three ways in which he might improve his hand on the draw:
   If he draws another jack. One jack remains in the deck of 47 unknown cards, of which he draws two. The probability of this improvement is 2/47.
   If he draws a pair of a number he has not previously held this hand. The probability of this improvement is 60/1,081.

If he draws a pair of a number of one of the two cards he had earlier drawn and discarded. The probability of this is 6/1,081.

The probability of the alternative occurrence of these three mutually exclusive improvements is the sum of their probabilities: 2/47 (= 46/1,081) + 60/1,081 + 6/1,081 = 112/1,081.

10. **CHALLENGE TO THE READER**

This problem, which has been the focus of some controversy, may be analyzed in two different ways:

First analysis:

a. There are 28 possible pairs in the abbreviated deck consisting of four kings and four aces. Of these 28 possible pairs, only seven (equipossible) pairs contain the ace of spades. Of these seven pairs, three contain two aces. If we know that the pair drawn contains the ace of spades, the probability that this pair contains two aces is 3/7.

b. However, if we know only that one of the cards in the pair is an ace, we know only that the pair drawn is one of the 22 (equipossible) pairs that contain at least one ace. Of these 22 pairs, six contain two aces. Therefore, if we know only that the pair contains an ace, the probability that the pair drawn contains two aces is 6/22, or 3/11.

In this first analysis, the probabilities in the two cases are different.

Second analysis:

a. If one of the cards of the pair drawn is known to be the ace of spades, there are seven other possible cards with which the pair may be completed. Of these seven, three are aces. Therefore, if we know that one of the cards drawn is the ace of spades, the probability that this pair contains two aces is 3/7.

b. If we know only that one of the cards drawn is an ace, we know that it is either the ace of spades, or the ace of hearts, or the ace of diamonds, or the ace of clubs. If it is the ace of spades, the analysis immediately preceding applies, and the probability that this pair contains two aces is again 3/7.

   If the ace is the ace of hearts, the same analysis applies; as it does if the card drawn is the ace of diamonds, or the ace of clubs. Therefore, even if we know only that an ace is one of the cards drawn, the probability that the pair contains two aces remains 3/7.

   In this second analysis, the probabilities in the two cases are the same.

How we choose between these two analyses depends upon how the problem is formulated. If the problem is formulated in such a way that the solution turns only on the probability of drawing a *next card* of some description, the second analysis is better. But if one approaches the problem as one in which we must determine the probability of *already dealt pairs of cards* (as it does appear in this exercise) the former analysis is the correct one. Because one is likely to think of this problem as one about the probability of next cards to be dealt, the solution is markedly counter-intuitive.

Section 14.3
Exercises on pages 558–559

1. a) $ 3.82

   b) $ 19,100,000.00

   But note: This was a very unusual set of circumstances!

2. 78 cents. (seven ninths of a dollar)

3. 82 cents. (nine elevenths of a dollar)

4. $15—of course, this supposes that the coin is fair, and that heads and tails are therefore equipossible on each toss.

5. This problem requires only a straightforward use of the product theorem. The probability of selecting, at random, just those two cows out of four, is the probability of selecting one of that pair on the first choosing (1/2), times the probability of selecting the other one of that pair on the second choosing, where the first already had been selected (1/3). So the calculation would be: $1/2 \times 1/3 = 1/6$.

6. The favorite. A one-dollar bet on the favorite purchases an expectation of 92 cents, but a one-dollar bet on the dark horse purchases an expectation of only 90 cents.

7. The common stock. $100 spent on common stock purchases an expectation of $93.80, whereas $100 spent on the preferred stock purchases an expectation value of only $93.50. Only rarely, however, can we calculate the value of purchases of this kind with such precision.

8. Approximately $1.11.

9. There are four mutually exclusive ways of winning:
   First prize: 1/4,000 × $ 1,000 = 25 cents
   Second prize: 1/4,000 × $ 400 = 10 cents
   Third prize: 1/4,000 × $ 250 = 6.3 cents
   Fourth prize: 1/4,000 × $ 100 = 2.5 cents

These are the returns if one holds one of the four winning tickets; all other tickets return 0. The sum of these mutually exclusive returns is 43.8 cents; if we suppose that all the tickets are sold, each $1 ticket has an expectation value of a little less than 44 cents.

10. The calculation of the bettor's chances of winning on the "Don't Pass-Bar3" line is the probability of the player's losing when the game is played according to the normal rules, *with the provision that he does not lose if he gets a 3 on the first roll*. The probability of a 3 on the first roll is 2/36 or .056. The probability of the player losing on the normal rules is .507, as was shown in Section 14.2B. Therefore the probability of the player losing, barring the loss on a first-roll 3, is $.507 - .056 = .451$. Since this is the probability of the player's losing if he cannot lose by getting a three on the first roll, it is the probability of the bettor winning on the "Don't Pass-Bar 3"line. So the expected value of a $100 bet on the "Don't Pass-Bar 3" line is .451 × $200 = $90.20.

    Note that this bet, which the house will gladly accept, is substantially less favorable to the bettor than simply betting on the pass line—that is, simply betting on the player to win. The expected value of such a $100 wager (i.e., on the player to win according to normal rules) is .493 × $200 = $98.60.