Math 214

Final Exam

February 1, 1997

1. Let D be a dense subset of a space X and U an open subset of X. Prove that
$$\overline{D \cap U} = \overline{U}.$$

2. Let X and Y be two topological spaces, A a subset of X and B a subset of Y. Show that
$$\overline{A \times B} = \overline{A} \times \overline{B}.$$

3. Let $f: X \to Y$ be a map from a space X to a space Y and $G_f = \{(x, f(x)) \in X \times Y \mid x \in X\}$ the graph of f. Prove that the mapping $g : X \to G_f$ given by $g(x) = (x, f(x))$ is a homeomorphism from the space X to the subspace G_f of $X \times Y$ if and only if the map f is continuous.

4. Let $(A_i)_{i \in I}$ be a family of connected subsets of a space X such that $A_i \cap A_j \neq \varnothing$ for any $i, j \in I$. Show that $\bigcup_i A_i$ is a connected subset of the space X.

5. Prove that a space X is regular if and only if the following condition holds :
For every closed subset A of X and every point $x \notin A$, there exists a nbhd. U of x such that $\overline{U} \cap A = \varnothing$.
Formulate an analogous statement that is equivalent to normality.