January 30, 2001

Mathematics 214, First Semester 2000-2001
Final Examination.

Problem 1. 8 pts.

What do you know about topological invariants?

Write the definition and list as many topological invariants as you know. Then give two different examples of how topological invariants can be used.

Problem 2. 8 pts.

Let X be a set and $f, g : X \to X$ be two functions. Suppose that $f \circ g = \mathrm{id}_X$, that is, that $f(g(x)) = x$ for each $x \in X$.

Prove that f is onto and that g is one-to-one.

Problem 3. 8 pts.

Let X be a metrizable space. Let $A \subset X$ be a non-empty set and consider A with its subspace topology inherited from X.

Prove that if X is separable, then A is separable, too.

Problem 4. 8 pts.

Let X and Y be two topological spaces.

Prove that the product space $X \times Y$ is separable if and only if both X and Y are separable.

Problem 5. 8 pts.

Show that no two of the following three spaces are homeomorphic:

$\mathbb{R}$, $\mathbb{R}^2$ and the unit circle $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ (each space is with its usual topology).

Bonus. 6 pts.

Let X be a topological space. Consider the product space $X \times X$ and its subset $\Delta = \{(x, x) : x \in X\}$ (most often, Δ is called *the diagonal*).

Show that if Δ is an open set in $X \times X$, then X is a discrete space.

Good Luck!

Time: 120 minutes.