February 2, 2002

Mathematics 214, First Semester 2001-2002

Final Examination.

Problem 1. 10 pts.

What do you know about metrizable spaces?

Problem 2. 6 pts.

Prove that each compact metrizable space is separable.

Problem 3. 6 pts.

Prove that if X and Y are regular spaces, then $X \times Y$ is a regular space, too.

Problem 4. $3 + 6$ pts.

Consider subspaces of $\mathbb{R}^2$ represented by geometric shapes of the capital letters A, E, P and H.

(a) Are any two of them homeomorphic? If "yes", which ones?

(b) Choose 3 non-homeomorphic pairs among them and prove that each of your choices is correct.

Problem 5. $6 + 3$ pts.

Let X be a Hausdorff space. Suppose that $f : X \to X$ is a continuous function such that $f \circ f = f$. Let $A = f(X)$.

(a) Prove that $A = \{x \in X : f(x) = x\}$.

(b) Show that A is a closed subset of X.

Bonus. 5 pts.

Let X be a countable metrizable space. Prove that X is totally disconnected. Does it have to be zero-dimensional?

Good Luck!

Time: 120 minutes.