January 21, 2003

Mathematics 214, First Semester 2002-2003

Final Examination.

Problem 1. 10 pts.

What do you know about metrizable spaces?

Remember to write the definition and quote as many theorems as you can. Also, give some examples of spaces that are metrizable and of spaces that are not metrizable.

Comment: No proofs are required.

Warning: You will lose 1 pt. for each false claim.

Problem 2. 4 pts.

Give a detailed proof that each metrizable space is a regular space.

Problem 3. 4 pts.

Let (X, S) and (Y, T) be topological spaces and take $X \times Y$ with the product topology. Show that the projection $\pi_1 : X \times Y \to X$ is an open function.

Comment: You do not have to prove that π_1 is continuous.

Problem 4. 5 pts.

Let (X, S) be a topological space and take $\mathbb{R}$ with its usual topology. Suppose that $f : X \to \mathbb{R}$ is a continuous function such that $f(x) \neq 0$ for all $x \in X$.

Let $g(x) = \frac{1}{f(x)}$ for each $x \in X$. Prove that this function $g : X \to \mathbb{R}$ is continuous, too.

Warning: The grade for a "proof" that uses sequences will be 0 pts.

Problem 5. 5 pts.

Let A and B be disjoint closed subsets of a space (X, T). Prove that $\text{bd}(A \cup B) = \text{bd}(A) \cup \text{bd}(B)$.

Problem 6. 5 pts.

Prove that the subspaces of $\mathbb{R}^2$ representing the figures A and P are not homeomorphic.

Problem 7. $4 + 3 + 3$ pts.

Let (X, T) be $\mathbb{R}$ with the countable complement topology.

(a) Prove that (X, T) is a T_1-space but not a T_2-space.

(b) Prove that (X, T) is connected.

(c) Prove that (X, T) is not compact.

Bonus. 3 pts.

Give an example of a continuous surjection $f : \mathbb{R}^2 \to [-1, \infty)$.

Good Luck!

Time: 150 minutes.

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