



MATH 214: FINAL EXAM, JANUARY 28TH 2004

NAME :

One piece of advice: half of the exam checks if you are familiar with basic definitions and examples. Assuming you studied for the exam, this should be very easy. I suggest you go through the exam and answer these easy questions first. If you feel something is not stated clearly, come over and ask me. But please - one by one!!!

* Problem #1: (20 points)

* Problem #2: (15 points)

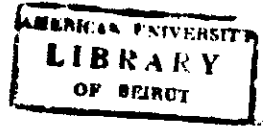
* Problem #3: (25 points)

* Problem #4: (35 points)

* Problem #5: (25 points)

* Problem #6: (30 points)

* Total: (150 points)

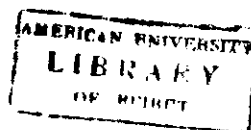


Problem #2 Continuous Maps / Sequences.

(1) (5 points) Let $f : (X, \mathcal{T}) \rightarrow (Y, \mathcal{S})$ be a map between topological spaces. Define what it means for f to be continuous.

(2) (5 points) Let (a_n) be a sequence of elements of a topological space (X, \mathcal{T}) and let $a \in X$. Define what it means $a_n \rightarrow a$ in (X, \mathcal{T}) .

(3) (5 points) Let (a_n) be a sequence of elements of (X, \mathcal{T}) , let $a, b \in X$ and let $a_n \rightarrow a$ and $a_n \rightarrow b$. Is it possible that $a \neq b$? Justify your claim.



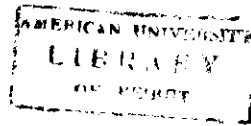
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Problem #3 Quotient Spaces / Product Spaces.

(1) (5 points) Let $f : (X, \mathcal{T}) \rightarrow (Y, \mathcal{S})$ be a map between topological spaces. Define what it means for f to be a quotient map.

(2) (5 points) Give three examples of quotient maps. NO PROOFS please.

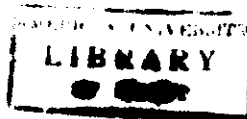
(3) (5 points) Let $\{(X_\lambda, \mathcal{T}_\lambda) | \lambda \in \Lambda\}$ be a family of (non-empty) topological spaces. Define the Tychonoff topology on the product $\prod_{\lambda \in \Lambda} X_\lambda$.



(4) (10 points) Prove the projection maps

$$p_{\lambda_0} : \prod_{\lambda \in \Lambda} X_\lambda \rightarrow X_{\lambda_0}$$

are quotient maps. If you find this hard, you may consider a simpler case when $\Lambda = \{1, 2\}$ - for half the credit.



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Problem #4 Separation Axioms.

(1) (5 points) Define what it means for a topological space (X, T) to be:

i) T_0

ii) T_1

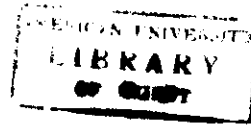
iii) T_2

(2) (5 points) Give an example of a space which is NOT T_2 . Justify BRIEFLY.

(3) (5 points). Define:

i) regular spaces

ii) normal spaces



(4) (10 points) Let (X, \mathcal{T}) be NORMAL and let $U, V \in \mathcal{T}$ satisfy

$$V \subseteq Cl(V) \subseteq U.$$

Prove there exists $W \in \mathcal{T}$ such that:

$$V \subseteq Cl(V) \subseteq W \subseteq Cl(W) \subseteq U.$$

(5) (10 points) Let (X, \mathcal{T}) be NORMAL and let $U, V \in \mathcal{T}$ satisfy

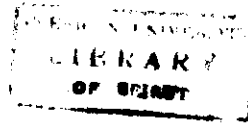
$$V \subseteq Cl(V) \subseteq U.$$

Prove there exists $f : X \rightarrow \mathbb{R}$ such that

i) $f|_V = 1$

ii) $Cl(\{x | f(x) \neq 0\}) \subseteq U.$

Feel free to use part (4) even if you haven't solved it.



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Problem #5 Compactness.

(1) (5 points) Define what it means for a topological space (X, \mathcal{T}) to be:

i) compact

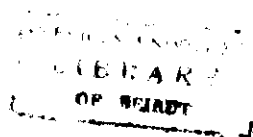
ii) locally compact

iii) Lindelöf.

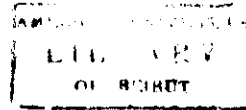
(2) (5 points) Let (X, \mathcal{T}) be a topological space. Define its one point compactification (X^*, \mathcal{T}^*) .

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(3) (5 points) Prove that the topological space (X^*, \mathcal{T}^*) from part (2) is compact.



- (4) (10 points) Prove that the topological space (X^*, T^*) from part (2) is Hausdorff if and only if
- i) (X, T) is Hausdorff and
 - ii) (X, T) is locally compact.



- (4) (5 points) Give a yes or no answer to this: is it true that connected spaces are always:
- i) path connected?
 - ii) locally connected?
- (5) (5 points) Give a BRIEF justification of the claim you made in part ii) of the previous question.