Midterm Mech 435 Spring 2012

1)

a) system is type one in open loop $\rightarrow$ no ss error

b) Open loop system: $G(s) = \dfrac{K_p K_v K_m}{\tau_m s + K_v K_m + 1}$

$$K_{vel} = \lim_{s \to 0} G(s)$$

$$= \frac{K_p K_v K_m}{K_v K_m + 1}$$

e_{ss} for ramp input should be 0,01

$\Rightarrow \quad e_{ss} = \dfrac{A}{K_v}$ $\qquad A = 1$ because unit ramp input

$\rightarrow \quad 0{,}01 = \dfrac{K_p K_v K_m}{K_v K_m + 1}$ $\qquad K_m = 1$ given

$\Rightarrow \quad 0{,}01 K_v \quad + 0{,}01 = K_p K_v$

$\Rightarrow \quad K_v (K_p - 0{,}01) = 0{,}01$ $\qquad \leftarrow$ relationship between K_p and K_v

c) T_s of ¢ required to be 2 seconds in addition to above requirements.

$$G_{cl} = \frac{K_p K_v K_m / \tau_m}{s^2 + \frac{1 + K_v K_m}{\tau_m} + \frac{1 + K_p K_v K_m}{\tau_m}}$$

$$T_s = 2 \Leftrightarrow \frac{4}{\sigma} = 2 \Leftrightarrow \sigma = 2$$

$\rightarrow \quad \dfrac{1 + K_v K_m}{2 \tau_m} = 2$ $\qquad$ with $K_m = 1$, $\tau_m = 0{,}5$

$\Rightarrow \quad 1 + K_v = 2$

$\Rightarrow \quad K_v = 2$

with problem b) we get $K_p = 0{,}02$

d) if K_v is negative, no clear statement, whether system is still stable or not. For $1 + K_v K_m < 0$, the system will be unstable

Problem 2

Root locus

$$G(s) = \frac{s^2 - s + 1}{s\,(s+1)}$$

a) locate poles / zeros

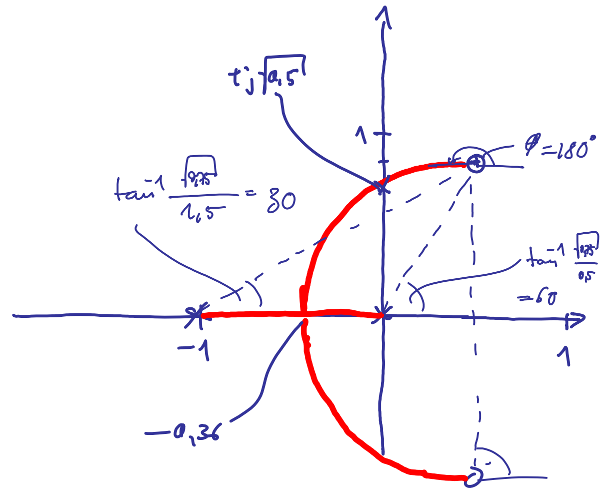
zeros: $s^2 - s + 1 = 0$

$\Leftrightarrow (s - 0{,}5)^2 = -0{,}75$

$\Rightarrow z_{1,2} = +0{,}5 \pm j\sqrt{0{,}75}$

poles: $p_1 = 0$

$p_2 = -1$



b) RL on real axis

c) Asymptotes: none

d) breakaway point expected between −1 and 0

Found by $\frac{dK}{ds} = 0 \Leftrightarrow$ $D'(s)N(s) - N'(s)D(s) = 0$ with $D(s) = s(s+1)$, $N(s) = s^2 - s + 1$

$\Leftrightarrow (2s+1)(s^2 - s + 1) - (2s - 1)(s^2 + s) = 0$

$\Leftrightarrow 2s^3 + s^2 - 2s^2 - s + 2s + 1 - [2s^3 - s^2 + 2s^2 - s] = 0$

$\Leftrightarrow 0 + s^2(1 - 2 + 1 - 2) + s(-1 + 2 + 1) + 1 = 0$

$\Leftrightarrow -2s^2 + 2s + 1 = 0$

$\Leftrightarrow s^2 - s - 0{,}5 = 0$

$\Leftrightarrow (s - 0{,}5)^2 = 0{,}5 + 0{,}25$

$\Leftrightarrow s_{1,2} = 0{,}5 \pm \sqrt{0{,}75}$

$s_1 = 1{,}36$

$s_2 = -0{,}36$

e) Angle of arrival in zeros:

Apply angle criterion in point infinitely near zero $s \approx z_1$

$$\Rightarrow \underbrace{\angle s - z_1}_{\phi\,?} + \underbrace{\angle s - z_2}_{90} - \underbrace{\angle s - p_1}_{60} - \underbrace{\angle s - p_2}_{30} = 180° \pm k360°$$

$$\Rightarrow \phi = 180 \pm k360 + 30 + 60 - 90$$

$$= 180$$

f) K for marginal stability with Routh:

$$1+kG(s) = 0$$

$$\Leftrightarrow 1 + k\frac{s^2 - s + 1}{s(s+1)} = 0$$

$$\Leftrightarrow s^2 + s + ks^2 - ks + k = 0$$

$$\Leftrightarrow s^2(1+k) + s(1-k) + k = 0$$

s^2	$1+k$	k
s	$1-k$	
1	k	

$\Rightarrow$ marginal stable for $k=1$

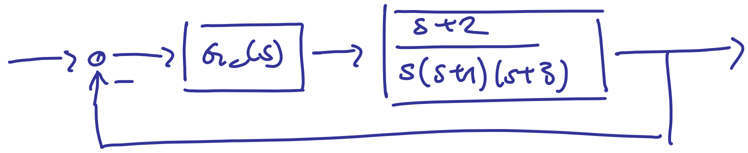
g) $\Rightarrow$ auxiliary polynomial to find pair of critically stable poles: $k=1$

$$\Rightarrow s^2 \cdot 2 + 1 = 0$$

$$\Leftrightarrow s^2 = -0{,}5$$

$$\Leftrightarrow s_{1,2} = \pm j\sqrt{0{,}5}$$

Problem 3

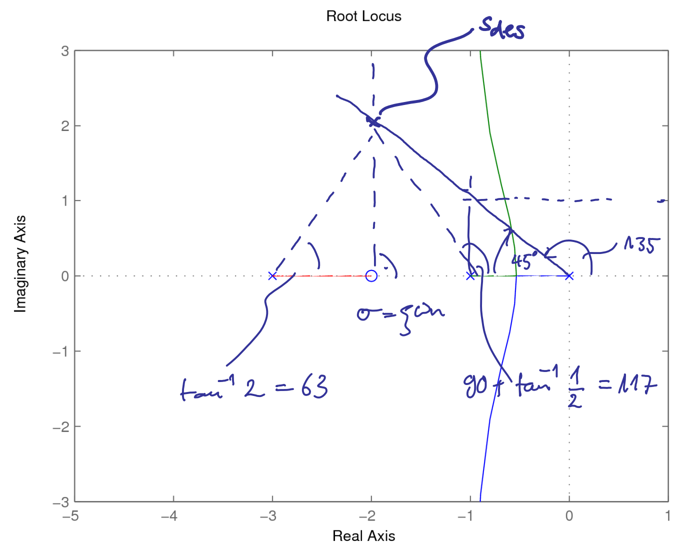


a) Design control such that $T_s = 2s$ and ξ of closed loop pole pair is $\xi = \frac{\sqrt{2}}{2}$

1) $\cos^{-1}\frac{\sqrt{2}}{2} = 45°$

2) $T_s = 2 = \frac{4}{\sigma} \Rightarrow \sigma = 2$

→ ∢ poles are not on RL!



b) Lead design:

$$G_c = K_c\alpha\frac{Ts+1}{\alpha Ts+1}$$

$$= K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$$

3 parameters to determine. First determine angle contribution for RL to go through s_{des}.

$$\angle G_c(s) + \angle s_{des}z_1 - \angle s_{des}p_1 - \angle s_{des}p_2 - \angle s_{des}p_3 = 180 \pm k360°$$

$$\angle G_c(s) = 180 \pm k360° - 90° + 117° + 135° + 63°$$

$$= 45°$$

Next we have to place the pole and zero of the compensator

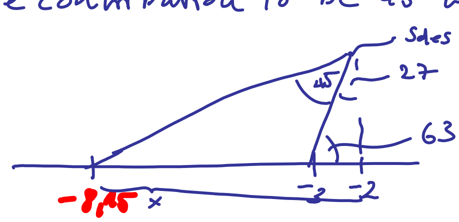
I) Bisector method

II) pole cancellation

→ try pole cancellation ⇒ zero on pole at −3

$\rightarrow \frac{1}{T} = 3 \Rightarrow T = \frac{1}{3}$

For angle contribution to be 45 we chose pole at:



$\tan(45+27) = \frac{x}{2}$

$\Rightarrow 2\cdot\tan(45+27) = x = 6{,}15$

$$\Rightarrow \frac{1}{\alpha T} = +8{,}15$$

$$\Rightarrow \alpha = \frac{1}{\frac{1}{3}\cdot 8{,}15} = 0{,}368$$

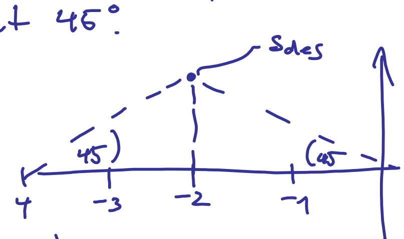
At last, we determine K_c using the angle criterion in s_{des}.

$$K_c \frac{|(s_{des}+2)| \cdot |\cancel{(s_{des}+3)}|}{|s_{des}||(s_{des}+1)||\cancel{(s_{des}+3)}||(s_{des}+8{,}15)|} = 1$$

$$\Leftrightarrow K_c = \frac{\sqrt{8}\cdot\sqrt{5}\sqrt{10{,}15}}{2} = 10{,}25 \Rightarrow G_c = 10{,}25\cdot\frac{s+3}{s+6{,}15}$$

Design PD control:

Angle contribution of PD has to be $45°$.
We can simply draw a line from s_{des} which intersects the Real axis at $45°$.



$$\Rightarrow G_c = K_p(s-\alpha) \qquad \alpha = -4$$

We get k_p through the mag. criterion:

$$K_p \frac{|(s_{des}+2)||s_{des}+4|}{|s_{des}||s_{des}-3||s_{des}-1|} = 1$$

$$\Leftrightarrow K_p = \frac{\sqrt{\cancel{8}\cdot 5\cdot 10{,}15}}{2\cdot\sqrt{\cancel{8}}} = 3{,}56 \Rightarrow G_c = 3{,}56\cdot(s+4)$$