Spring 2015	**Quiz 1**	**Date: February 21**
K.Yaghi	**Math 201- Section X**	**Duration: 1 hour**

Problem 1 *(answer on page* 1 *of the booklet)*
Which of the following sequences converge, and which diverge? Find the limit of each convergent sequence. (7 *pts each*)

a) $a_n = \sqrt[n]{n + (-1)^n n}$ b) $b_n = \left(\frac{n^{10} - 2n^9}{n^{10} + n^9}\right)^{2+n}$ c) $c_n = \frac{1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}}{3 \ln \sqrt{n}}$

Problem 2 (*answer on pages* 2 *&* 3 *of the booklet*)
Which of the following series converge, and which diverge? When possible find the sum of the series. (8 *pts each*)

a) $\sum_{n=1}^{\infty} \frac{3^{n-2}}{6^{n-1}} + \frac{(-1)^n 3^n}{n!}$ b) $\sum_{n=1}^{\infty} \frac{n^2 \cos n}{3^{n+1}}$ c) $\sum_{n=3}^{\infty} \frac{(-1)^{\frac{n(n+1)}{2}}}{n (\ln n)^2}$ d) $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \arctan \frac{1}{n}\right)^{0.3}$

Problem 3 (*answer on page* 4 *of the booklet*)
Find the interval of convergence of the power series

$$\sum_{n=2}^{\infty} (-1)^n \ln n \left(e^{\frac{1}{n}} - 1\right)^{13} (x-5)^n$$

For what values of x does the series converge absolutely? Conditionally? (20 *pts*)

Problem 4 (*answer on pages* 5, 6 *& the last page of the booklet*)

a) (4 *pts*) Write a power series expansion for the function $f(x) = e^x$ about the point $x = 0$. Also find the taylor polynomials $p1(x)$ and $p2(x)$ generated by $f(x)$ about the point $x = 0$.

b) (6 *pts*) Use the alternating series estimation theorem to prove that $e < 3$. **(Hint estimate first the error resulting from the approximation $e^{-1} \approx \sum_{n=1}^{5} \frac{(-1)^n}{n!}$)**

c) (7 *pts*) Use Taylor's theorem and part (b) to prove that

$$|f(x) - p1(x)| < \frac{3}{2} x^2 \qquad 0 < x < 1$$

And deduce that $e^x > 1 + x$ for all x ∈ R.

d) (4 *pts*) Find a power series expansion for $g(x) = \frac{1}{2}(e^{2x} + e^{-2x})$ about the point $x = 0$. Also find the taylor polynomials $T1(x)$ and $T2(x)$ generated by $g(x)$ about the point $x = 0$.

e) (7 *pts*) Estimate $g(0.5)$ by $T2(0.5)$ and use Taylor's theorem to estimate the resulting error. Does $T2(0.5)$ tend to be too small or too large?

Good Luck & Best Wishes

K. Yaghi