



AMERICAN UNIVERSITY OF BEIRUT

STAT 233 FINAL EXAMINATION

Time = 1 hour 30 minutes

June 12, 2003

1. Let A and B be two events defined on the same sample space.
 - (a) If $P(A) = .3$, $P(B) = .4$, and $P(A|B) = .5$, what is the conditional probability $P(A|B^*)$? (5pts)
 - (b) Prove or disprove: If $P(A|B) = P(A|B^*)$ then A and B are independent. (5 pts)
2. A fair die is tossed 12 times.
 - (a) Let X be the number of dice that show the face 6 and Y be the number of dice that show the face 1. Find the joint probability distribution of (X, Y) . (5 pts)
 - (b) Find $E(XY)$. (5pts)
3. The truncated Geometric distribution is defined as the following:
$$P(X = x) = k(1 - p)^{x-1}p; \text{ if } x = 2, 3, \dots$$
Find constant k , $E(X)$, and $Var(X)$. (10pts)
4. Given that the joint density of (X, Y) is
$$f(x, y) = \frac{k}{(1 + x + y)^3}; x > 0; y > 0$$
 - (a) Find the constant k . (5pts)
 - (b) Find the marginal density $f_1(x)$. (5pts)
 - (c) Find the conditional density $f_{1|2}(x|y)$. (5pts)
5. Suppose that X and Y are independent random variables with the same *p.d.f.* $f(x) = \theta e^{-\theta x}; x > 0$. Show that $X + Y$ and X/Y are independent and compute $E(X/Y)$. (15pts)
6. Let X and Y be independent and identically distributed random variables with common density function, $f(x) = xe^{-x^2/2}$ for $x > 0$. Find the distribution of the random variable $R = \sqrt{X^2 + Y^2}$ (10pts)