AMERICAN UNIVERSITY LIBRARY OF BEIRUT

AMERICAN UNIVERSITY of BEIRUT

STAT 233, Final Exam

June 5, 2004

1. Let A_1, A_2, A_3 be independent events with probabilities 1/2, 1/3, 1/4, respectively. Compute $P(A_1 \cup A_2 \cup A_3)$. [exercise number 1.120]

2. Person A tosses a coin and person B rolls a die. This is repeated independently until one head or one of the numbers 1, 2, 3, 4 appears, at which time the game is stopped. Person A wins with the head and B wins with one of the numbers 1, 2, 3, 4. Compute the probability that A wins the game. [exercise number 1.123]

3. The probabilities that the independent events A, B, and C will occur are 3/4, 1/2, and 1/4. What is the probability that at least one will occur. [exercise number 1.133]

4. Let X have the p.d.f. $f(x) = (1/3)(2/3)^x$, if $x = 0, 1, 2, \ldots$, zero elsewhere. Find the conditional p.d.f. of X, given that $X \geq 3$. [exercise number 3.13]

5. Let X and Y be two independent binomial random variables with parameters n and $p = 1/2$. show that

$$P(X = Y) = \frac{(2n)!}{n!n!2^{2n}} \qquad (1)$$

[exercise number 3.82]

You may use the identity $\sum_{k=0}^{m} \binom{n_1}{k}\binom{n_2}{m-k} = \binom{n_1+n_2}{m}$

6. Let the random variables X_1 and X_2 be a random sample of $n = 2$ from the p.d.f. $f(x) = 1$ if $0 < x < 1$, zero elsewhere. Find the distribution of the random variable $Y_1 = X_1 + X_2$ and the distribution of the random variable $Y_2 = X_1 - X_2$. [example 4 of chapter 4]

7. Let X_1 and X_2 be a random sample of size 2 from a distribution which has the p.d.f. $f(x) = 1$ if $0 < x < 1$, 0 elsewhere. Find the p.d.f. of the random variable $Y = X_1/X_2$ [exercise 4.14]
Hint: Consider two cases when $0 < y < 1$ and when $1 < y < \infty$