You are allowed to use a calculator and one formula sheet!
(1.)

Let the events $A$ and $B$ have $P(A)=0.5$ and $P(A \cup B)=0.7$.
(a) Find $P(B)$ if $A$ and $B$ are independent. [5 pts]
(b) Find $P(B)$ if $P(A \mid B)=0.5$. [5 pts]
2. Let the events $A$ and $B$ have $P(A \mid B)=P\left(A^{*} \mid B^{*}\right)=P\left(B^{*}\right)=0.95$. Can you determine $P(B \mid A)$ ? [10 pts]
(3.) A fair coin is tossed four independent times. Let $X$ denote the number of times a head is followed immediately by a tail. Find the probability density function, $f(x)$, the cumulative distribution function, $F(x)$, and the moment generating function, $M_{X}(t) .[10 \mathrm{pts}]$
(4. A single die is tossed; then $k$ coins are tossed, where $k$ is the number shown on the die. What is the probability of getting exactly 2 heads? [10 pts]
5. Does there exists a continuous random variable $X$ with mean $\mu_{X}$ and standard deviation $\sigma_{X}$ such that $P\left(\mu_{x}-2 \sigma_{X} \leq X \leq \mu_{X}+2 \sigma_{X}\right)=0.6$ ? [ pts]

