

American University of Beirut

STAT 233

*Advanced Probability and Random Variables*

*Spring 2005*

quiz # 1

- Exercise 1**
- a. (2 points) Let  $A$  and  $B$  two mutually exclusive events. Can they be independent?
- b. (2 points) Let  $A$  and  $B$  be two independent events such that  $P(A) = P(B) = 1/2$ . Find  $P((A' \cap B) \cup (A \cap B'))$ .
- c. (3 points) If  $P(A) > 0$ , show that  $P(A \cap B|A) \geq P(A \cap B|A \cup B)$ .
- d. (3 points) Let  $A$  and  $B$  be two events such that  $P(A|B) = 1$ . Find  $P(B'|A')$ .

**Exercise 2** (5 points) Sam and Jad are involved in a duel. The rules of the duel are that they are to pick up their guns and shoot at each other simultaneously. If one or both are hit, then the duel is over. If both shots miss, then they repeat this process. Suppose that the result of the shots are independent and that each shot of Sam will hit Jad with probability  $1/2$ , and each shot of Jad will hit Sam with probability  $1/3$ . Find the probability that the duel ends at the third round of shots.

**Exercise 3** (5 points) The door of a building is locked 40% of the time. The building attendant has 8 keys of which only one open the door. The attendant picks three keys randomly and goes to the building. Given that the attendant entered the building, what is the probability that the door was open?

**Exercise 4** (5 points) Balls are drawn at random, one at a time, and without replacement from an urn that contains 3 red balls and 3 blue balls. Let  $Y$  be the number of draws needed until the first blue ball appears.

- a) Find the pdf of  $Y$ .
- b) How many draw would you expect to do?