American University of Beirut

STAT 233

Advanced Probability and Random Variables

Spring 2005

Final Exam

Exercise 1 Let A, B and C three independent events with probabilities $1/2, 1/6, 1/4$. Find $P((A^c \cap B^c) \cup C)$.

Exercise 2 An urn contains 12 balls, of which 4 are white. Three players A, B and C successively draw ball from the urn, A first, then B, then C, then A and so on. The winner is the first one to draw a white ball. What is the probability that A win the game if the draw is:

a. without replacement

b. with replacement

Exercise 3 A fair coin is tossed until at least two heads and at least one tail have been obtained. Let X denote the number of tosses that are required. Find the pdf of X. Find the mgf of X, and use it to find $E(X)$.

Exercise 4 Let $X \hookrightarrow b(n, \frac{1}{2})$ and $Y \hookrightarrow b(m, \frac{1}{2})$ be two independent binomial distributions. Prove that $Z = X - Y + m$ has a binomial distribution with parameters $n + m$ and $p = 1/2$.

you may use the identity $\sum_{j=0}^{k} \binom{n}{j}\binom{m}{k-j} = \binom{n+m}{k}$

Exercise 5 Let X be a random a random variable with pdf

$$f(x) = 1 \qquad 0 < x < 1$$

Let $Y = -2 \ln X$.

a. Find the pdf of Y.

b. Find $E(Y)$ and $Var(Y)$.

Exercise 6 Let X and Y have joint pdf

$$f(x, y) = 8xy \qquad 0 < x < y < 1$$

Find the joint pdf of $U = X/Y$ and $V = Y$, and show that U and V are independent.

Good luck