Practice Final Exam – Math 201 K. Yaghi

Problem 1

1) Write a power series expansion for the function $\frac{1}{2}(e^{r^2} + e^{-r^2})$. (You don't have to derive it).
2) Let R be the region bounded below by the plane $z = 0$, above by the plane $z = 2$ and between cylinders $0 \leq x^2 + y^2 \leq \frac{1}{4}$.

a) Write the volume of R as an iterated triple integral in cylinderical coordinates. Then evaluate the integral.
b) Set up, but donot evaluate, the iterated triple integral in rectangular coordinates that gives the volume of R in the order $dzdydx$.
c) Set up, but donot evaluate, the iterated triple integral in spherical coordinates that gives the volume of R in the ordes (i) $d\rho d\emptyset d\theta$ and (ii)$d\emptyset d\rho d\theta$
d) Evaluate the following triple integral:

$$\iiint_R \frac{1}{2}\left(e^{(x^2+y^2)} + e^{-(x^2+y^2)}\right)dV$$

e) Estimate

$$\iiint_R \frac{1}{2}\frac{\left(e^{(x^2+y^2)} + e^{-(x^2+y^2)}\right)}{10\sqrt{x^2+y^2}}dV$$

with an error of magnitude no more than 0.001.

Problem 2

Evaluate the triple integral $\iiint_R 3x dV$, where R is the region that lies above the xy-plane, under the plane z = 5 + y, bounded by the cylinder $x^2 + y^2 = 4$ and **inside the first octant**.

Problem 3

Which of the following series converge and which diverge? When possible find the sum of the series. For those which converge, do they converge absolutely or conditionally?

(i) $\sum_{n=1}^{\infty} \frac{(-3)^n}{\pi^{n+1}} + \frac{3^n}{n!}$

(ii) $\sum_{n=1}^{\infty} e^{\frac{1}{n}} - 1 - \frac{1}{n}$

(iii) $\sum_{n=1}^{\infty} (\cos[ln(1 + \frac{\pi}{2n})^n])^n$

(iv) $\sum_{n=2}^{\infty}(-1)^n \frac{n}{\ln n}(\sqrt[n]{n} - 1)$

Problem 4

a) A flat circular plate has the shape of the region $x^2 + y^2 \leq 1$. The plate, including the boundary where $x^2 + y^2 = 1$, is heated so that the temperature at the point (x,y) is $T(x, y) = x^2 + 2y^2 - x$. Find the temperatures at the hottest and coldest points of the plate.

b) Consider the function $f(x, y) = 12x^2 + 12y^2 + (x + y)^3$
   - (i) Find the critical points of f and classify them as local maxima, local minima or saddle point.
   - (ii) In what direction(s) does f increases most rapidly at the point $P(1,1)$?
   - (iii) Find the directions of zero change in f at $P(1,2)$.

Problem 5

a) Sketch the region of integration and evaluate the double integral

$$\int_1^{15} \int_0^{1/y} ye^{xy} dxdy$$

b) Evaluate the double integral

$$\int_0^{32} \int_{x^{1/5}}^{2} \frac{1}{y^6 + 1} dydx$$

c) Evaluate the integral

$$\int_0^{\infty} e^{-x^2} dx$$

Problem 6

Integrate the function $f(x, y, z) = \frac{1}{\sqrt[3]{x^2+y^2+z^2}}$ over the region R bounded below by the sphere $x^2 + y^2 + z^2 = 4$ and above by the cone $z = \sqrt{x^2 + y^2}$.