Prof. F.Abi-Khuzam **Math 223 Final** *January 29,1997* **2 hrs. and 30 min.**

1. Let f be a real -valued continuous function defined on a closed and bounded interval $[a,b]$. State four important properties of f and prove one of them.

2.Suppose f is a real-valued function continuous on $[0, \infty)$ and differentiable on $(0,\infty)$. Suppose also that $f(0) = 0$ and f' is monotonically decreasing. Prove that the function g defined by $g(x) = f(x)/x$ is monotonically decreasing. What can you say about the function $\dfrac{(1-\cos x)}{\frac{1}{2}x^2}$ when x is in $(0, \pi/2)$? Justify your answer.

3. Let f be a twice - differentiable function on (c, ∞) and let M, N, and P be the least upper bounds of $|f(x)|$, $|f'(x)|$, $|f''(x)|$ respectively on (c,∞). Prove that the square of N can never exceed $4MP$.

4. (a) Suppose f and g are real differentiable functions on $(-\infty, \infty)$.If $f'(t)$ is not equal to $g'(t)$ for every real t how many points x satisfying $f(x) = g(x)$ can there be? Justify your answer.
(b) Suppose that there is a constant $A < 1$ such that $|f'(t)| < A$ for all real t, prove that there is a point x such that $f(x) = x$.

5. Prove the inequalities $2/(2x+1) < \ln[(x+1)/x] < (2x+1)/2x(x+1)$ for $x > 0$.

6. Let $f(x) = (\cos x)/(1+x)$ and $g(x) = (\sin x)/(1+x)^2$. Prove that the integral over $[0, \infty)$ of f equals that of g and that one of these integrals converges absolutely but not the other.

7.(a) Give an example of a bounded differentiable function whose derivative is not bounded on $(0, \infty)$.
(b) If f is a continuous function between two metric spaces X and Y, does f map cauchy sequences in X to cauchy sequences in Y.Justify.
(c) Prove that, in a metric space, closed subsets of compact sets are compact.
(d) If α is monotonically increasing on $[0,b]$ what is the value of the Stieltjes integral $\int d\alpha$ over $[0,x]$ for $0 < x < b$.Justify.
(e) State and prove the fundamental theorem of Calculus.