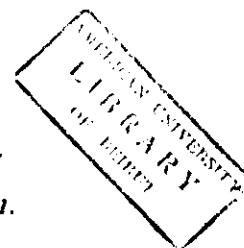


Prof. F.Abi-Khuzam

Math 223
Final

January 29, 1997
2 hrs. and 30 min.



1. Let f be a real-valued continuous function defined on a closed and bounded interval $[a, b]$. State four important properties of f and prove one of them.
2. Suppose f is a real-valued function continuous on $(0, \infty)$ and differentiable on $(0, \infty)$. Suppose also that $f(0) = 0$ and f' is monotonically decreasing. Prove that the function g defined by $g(x) = f(x)/x$ is monotonically decreasing. What can you say about the function $\frac{(1 - \cos x)}{\frac{1}{2}x^2}$ when x is in $(0, \pi/2)$? Justify your answer.
3. Let f be a twice-differentiable function on (c, ∞) and let M, N , and P be the least upper bounds of $|f(x)|$, $|f'(x)|$, $|f''(x)|$ respectively on (c, ∞) . Prove that the square of N can never exceed $4MP$.
4. (a) Suppose f and g are real differentiable functions on $(-\infty, \infty)$. If $f'(t)$ is not equal to $g'(t)$ for every real t how many points x satisfying $f(x) = g(x)$ can there be? Justify your answer.
(b) Suppose that there is a constant $A < 1$ such that $|f'(t)| < A$ for all real t , prove that there is a point x such that $f(x) = x$.
5. Prove the inequalities $2/(2x+1) < \ln|(x+1)/x| < (2x+1)/2x(x+1)$ for $x > 0$.
6. Let $f(x) = (\cos x)/(1+x)$ and $g(x) = (\sin x)/(1+x)^2$. Prove that the integral over $[0, \infty)$ of f equals that of g and that one of these integrals converges absolutely but not the other.
7. (a) Give an example of a bounded differentiable function whose derivative is not bounded on $(0, \infty)$.
(b) If f is a continuous function between two metric spaces X and Y , does f map Cauchy sequences in X to Cauchy sequences in Y . Justify.
(c) Prove that, in a metric space, closed subsets of compact sets are compact.
(d) If α is monotonically increasing on $[0, b]$ what is the value of the Stieltjes integral $\int d\alpha$ over $[0, x]$ for $0 < x < b$. Justify.
(e) State and prove the fundamental theorem of Calculus.

