Math 261 — Exam 1

October 4, 2017

The use of calculators, notes, and books is **NOT** allowed.

Exercise 1: Since today is October 4th... (10 pts)

1. (4 pts) Compute the factorization of 104 into primes.
2. (6 pts) Deduce the number of divisors of 104, the sum of these divisors, and the value of $\phi(104)$.

Exercise 2: Consecutive composites (16 pts)

1. (4 pts) Find 5 consecutive composite (i.e. not prime) integers $\leqslant 100$.
2. (12 pts) Find 2017 consecutive composite integers.

   Hint: consider numbers of the form $n! + m$, where $n, m \in \mathbb{N}$, $m \leqslant n$, and $n! = 1 \times 2 \times 3 \times \cdots \times n$.

Exercise 3: Making change (11 pts)

1. (8 pts) Find all integers $x, y \in \mathbb{Z}$ such that $20x + 50y = 10000$.
2. (3 pts) Deduce how many different ways there are to pay \$10000 using only banknotes of \$20 and \$50.

Exercise 4: Only 2 (20 pts)

Find all $n \in \mathbb{N}$ such that $\phi(n) = 2$.

Exercise 5: A system of congruences (15 pts)

Find all $x \in \mathbb{Z}$ satisfying both

$$\begin{cases} 4x \equiv 5 \pmod 7 \\ 5x \equiv 3 \pmod 8 \end{cases}$$

Exercise 6: Irreducible polynomials over $\mathbb{Z}/2\mathbb{Z}$ (28 pts)

1. (6 pts) Find all irreducible polynomials of degree 2 over $\mathbb{Z}/2\mathbb{Z}$.
2. (12 pts) Use the previous question and a Euclidian division to deduce that the polynomial $x^4 + x + 1$ is irreducible over $\mathbb{Z}/2\mathbb{Z}$.
3. (10 pts) Find all irreducible polynomials of degree 3 over $\mathbb{Z}/2\mathbb{Z}$.

END