



Mathematics 227, Second Semester 1996-97
Final Examination, June 18, 1997, 8:00 - 10:30

Problem 1. 15 pts.

Write everything you know about the exponential function $f(z) = e^z$.

Problem 2. 15 pts.

Write what you know about the classification of isolated singular points of an analytic function.

Problem 3. 9 pts.

Let $z = x + iy$. Find the "if and only if" condition in terms of x and y for z^2 to be an imaginary number.

Problem 4. 9 pts.

Suppose that $f(z)$ is differentiable at z_0 . Write the proof that f satisfies the Cauchy-Riemann equations at z_0 .

Problem 5. 9 pts.

Find the harmonic conjugate of $f(x, y) = x^3 - 3xy^2 - 3x$.

Problem 6. 9 pts.

Evaluate the following integral: $\int_0^{\frac{\pi}{2}+i} z^2 \cdot \cos z \, dz$.

Problem 7. 9 pts.

Prove that the series $\sum_{n=1}^{\infty} \frac{z^n}{1-z^n}$ converges absolutely for each z such that $|z| < 1$, and diverges when $|z| > 1$.

Problem 8. 9 pts.

Write the Maclaurin series of $f(z) = \frac{z}{(z-2)(z+i)}$. What is its radius of convergence?

Problem 9. 9 pts.

Suppose that $f(z)$ is analytic in a domain D such that $0 \in D$. Let $\sum_{n=0}^{\infty} c_n z^n$ be the Maclaurin series of f . Prove that if $f(z) = -f(-z)$ for each $z \in D$, then $c_{2k} = 0$ for each $k = 0, 1, 2, \dots$

Problem 10. 9 pts.

Suppose that $a, b, c \in \mathbb{R}$ and $ac - b^2 > 0$. Use residues to find the value of the following integral: $\int_{-\infty}^{\infty} \frac{dx}{ax^2 + 2bx + c}$.



Good Luck!