DEPARTMENT OF MATHEMATICS
AMERICAN UNIVERSITY OF BEIRUT
MATH 227, FINAL EXAMINATION, SPRING 2005

Answer the following questions:

1. (a) Define the residue of a function at a point z_0 and state the Cauchy residue theorem. (4 pts.)

(b) Use the Cauchy residue theorem to evaluate the integral (6 pts.)

$$\int_C \frac{Logz}{(z+1)^m}\,dz \quad (m = 1, 2, \cdots;\ C : z(t) = 2e^{it}, 0 \le t \le 2\pi).$$

2. (a) Use the Cauchy residue theorem to evaluate the integral (5 pts.)

$$\int_C z^m \exp(1/z)\,dz \quad (m = 1, 2, \cdots;\ C : z(t) = e^{it}, 0 \le t \le 2\pi).$$

(b) Use residues to show that (5 pts.)

$$\int_0^\infty \frac{\cos ax}{x^2+1}dx = \frac{\pi}{2e^a}, \quad (a > 0).$$

3. (a) State the single residue theorem regarding the value of a line integral

$$\int_C f(z)\,dz,$$

where C is a positively-oriented closed contour and f is analytic in the plane except at finitely many points $z_1, \cdots, z_n$ lying within C. (4 pts.)

(b) Use the single residue theorem to evaluate the integral (6 pts.)

$$\int_C \frac{z^2 e^{1/z}}{1+z^3}\,dz \quad (C : z(t) = 2e^{it}, 0 \le t \le 2\pi).$$

4. (a) State Laurent's theorem. (4 pts.)

(b) Find the Laurent's series of the function (6 pts.)

$$f(z) = \frac{z+1}{z(2z-1)}$$

for values (i) $|z| > 1/2$, and (ii) $|z - 1/2| < 1/2$.

5. (a) State the Cauchy integral formula. (4 pts.)

(b) Use the Cauchy integral formula to Compute (6 pts.)

$$\int_C e^z z^{-m}\,dz \quad (m = 0, \pm 1, \pm 2, \cdots;\ C : z(t) = e^{it}, 0 \le t \le 2\pi).$$