

Please draw a horizontal line across the page between the answers to each question

You may refer to the following during the exam:

- the course textbook
- the course lecture notes
- your homework solutions
- any notes that you have taken in class

You may **not** refer to any other materials. Good luck!

1. (40 points) You will receive no credit if you do not show intermediate assertions.

(a, 10 points) Find a nontrivial (i.e., not identical to *false*) precondition P so that the following is valid, where x, y are integer variables. Show all intermediate assertions.

$$\{P\}$$

$$y = x;$$

$$x = y;$$

$$\{x + y = 0\}$$

(b, 10 points) Find a nontrivial (i.e., not identical to *true*) postcondition Q so that the following is valid, where x, y are integer variables. Show all intermediate assertions.

$$\{x + y = 0\}$$

$$x = -x;$$

$$y = x;$$

$$\{Q\}$$

(c, 20 points) Find a nontrivial (i.e., not identical to *false*) precondition P so that the following is valid, where x, y, z are integer variables. Show all intermediate assertions.

$$\{P\}$$

if ($even(x)$)

$$x = x/2;$$

$$y = y * 2;$$

else

$$z = z + y;$$

$$x = x - 1;$$

endif

$$\{z + x * y = C\}$$

2. (60 points) Write an instance method `union` for the `IntSet` data abstraction that satisfies the following specification (where \cup denotes the set union operator):

```
public void union(IntSet s)
    //REQUIRES: s != null
    //EFFECTS: AF(els,top)_post = AF(els,top) U AF(s.els, s.top)
    //MODIFIES: els, top
```

Your method is to be implemented within the `IntSetNoDupSort` implementation.

You earn the 60 points for this question as follows:

(a, 20 points) Write code that is correct.

(b, 20 points) Annotate the code with assertions, and use these to show that the code is correct, i.e., that the postcondition (**EFFECTS** clause) holds upon termination provided that the precondition $s \neq \text{null}$ holds initially.

(c, 20 points) Show that your implementation runs in time $O(\text{top} + s.\text{top})$.