

The topic of this problem set is writing I/O Automata.

**1 (20 points).**

(a) **(10 points)** Give an I/O automaton  $A$  such that:

1.  $A$  has a single start state.
2.  $A$  has two output actions  $x, y$ .
3.  $A$  has no states without an outgoing transition, i.e., from every state, either  $x$  or  $y$  or both can be executed.
4. In any finite execution of  $A$ , the number of occurrences of  $x$  and the number of occurrences of  $y$  differ by at most 1.

(b) **(10 points)** Now modify the I/O automaton for (a) so that the number of occurrences of  $x$  and the number of occurrences of  $y$  differ by at most  $n$ , where  $n$  is some constant.

**2 (15 points).** Give an I/O automaton  $A$  such that:

1.  $A$  has a single start state
2.  $A$  has no states without an outgoing transition, i.e., from every state, at least one action can be executed
3.  $A$  has one input action  $x$  and one output action  $y$
4. In any infinite execution of  $A$ , every occurrence of  $x$  is eventually followed by an occurrence of  $y$  (hint: recall that I/O automata are *input-enabled*, so in this case, the input  $x$  must be executable from every state of  $A$ ).

**3 (15 points).** Give an I/O automaton  $A$  such that:

1.  $A$  has a single start state.
2.  $A$  has no states without an outgoing transition, i.e., from every state, at least one action can be executed.
3.  $A$  has one input action  $b$  and two output actions  $a, x$ .
4. In any execution  $\alpha$  of  $A$ , if  $a$  is executed as some point, then  $x$  is not executed unless  $b$  is first executed. In other words, every “interval” in  $\alpha$  from some occurrence of  $a$  until the first subsequent occurrence of  $b$ , does not contain any occurrence of  $x$ .