EECE 412/612
Midterm I Solutions

Problem 1

(a) 0.9V
(b) 1.2V
(c) 1.2V
(d) 0.9V
(e) 0.9V
(f) 1.2V
(g) 0V
(h) 0.6V
(i) 0.3V

Problem 2

(a) $V_{T0} = \Phi_{GC} - 2\varphi_f - \frac{Q_{B0}}{C_{ox}} - \frac{Q_{ox}}{C_{ox}}$

$$\varphi_f = \frac{kT}{q} \ln \frac{n_i}{N_A} = 0.026 \ln \frac{1.45\times10^{10}}{2\times10^{15}} = -0.308V$$

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = 1.758\times10^{-7}\ F/cm^2$$

$$Q_{B0} = -\sqrt{2q N_A \varepsilon_{si} |-2\varphi_f|}$$

$$= -\sqrt{2\times1.6\times10^{-19}\times2\times10^{15}\times11.7\times8.854\times10^{-14}\times|-2\times0.308|}$$

$$= -2.01\times10^{-8}$$

$$V_{T0} = -0.85 + 0.615 + \frac{2\times10^{-8}}{1.758\times10^{-7}} - \frac{1.6\times10^{-19}\times2\times10^{11}}{1.758\times10^{-7}} = -0.303$$

(b) P-type implanted needed in the amount of:

$$\Delta V = 0.8 - V_{T0} = 1.303 = \frac{qN_I}{C_{ox}}$$

$$\Rightarrow N_I = 1.21\times10^{12}\ cm^{-2}$$

Problem 3 : Use $C_j(V) = A \times \sqrt{\frac{\varepsilon_{si} q}{2} \left(\frac{N_A N_D}{N_A + N_D} \right)} \times \frac{1}{\sqrt{\varphi_0 - V}}$

, Do not use the formula for K_{eq}

- $\varphi_0 = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2} = 0.939$ V

, for area use four sides and bottom plate:

$$A = \left[5 \times 10 + 2(5 \times 0.5) + 2(10 \times 0.5) \right] = 65 \times 10^{-8}$$

μm^2

$$\Rightarrow C_j(V) = 65 \times 10^{-8} \sqrt{\frac{11.7 \times 8.854 \times 10^{-14} \times 1.6 \times 10^{-19}}{2} \left(\frac{1 \times 10^{36}}{1 \times 10^{20} + 1 \times 10^{16}} \right)}$$

$$\times \frac{1}{\sqrt{0.939 + 5}}$$

$$= 7.678 \times 10^{-15} \text{ F}.$$

Problem 4:

(a) V_{TO}: take row 1 & row 2 $\quad I_{D(SAT)} = \frac{k_n}{2}(V_{GS} - V_{TO})^2$

$$\frac{I_{D1}}{I_{D2}} = \frac{(4 - V_{TO})^2}{(5 - V_{TO})^2} = \frac{256}{441} \Rightarrow \frac{4 - V_{TO}}{5 - V_{TO}} = \pm \frac{16}{21}$$

$\rightarrow V_{TO} = 0.8V$

(b) mobility: $C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = 1.0 \times 10^{-7}$ F/cm

$$256 \times 10^{-6} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (4 - V_{TO})^2 \Rightarrow$$

$$\mu_n = \frac{2 \times 256 \times 10^{-6}}{1.0 \times 10^{-7} \times 1.0 \times 3.2^2} = 500 \text{ cm}^2/V.s$$

(c) γ: Use row 3 or row 4 $\Rightarrow 144 \times 10^{-6} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (4 - V_T)^2$

find $V_T \Rightarrow V_T = 1.6V$

$$\Rightarrow \gamma = \frac{V_T(V_{SB}) - V_{TO}}{\sqrt{|2\phi_F| + V_{SB}} - \sqrt{|2\phi_F|}} = 0.8 \ V^{1/2}$$

Problem 5:

V_{OH}: load is in SAT $\Rightarrow \frac{k_{load}}{2}(V_{DD}-V_{OH}-V_T)^2 = 0$

$\Rightarrow V_{OH} = V_{DD} - V_T = 4.2V$

V_{OL}: load in SAT, driver in linear, input is V_{OH}, output is V_{OL}

$\Rightarrow \frac{k_{load}}{2}(V_{DD}-V_{OL}-V_{T0})^2 = \frac{k_{driver}}{2}\left(2(V_{OH}-V_{T0})V_{OL}-V_{OL}^2\right)$

$\Rightarrow 11V_{OL}^2 - 76.4V_{OL} + 17.64 = 0 \Rightarrow \boxed{V_{OL} = 0.239}$

V_{IL}: load in sat
driver in sat $\quad \frac{k_{load}}{2}(V_{DD}-V_{out}-V_{T0})^2 = \frac{k_{driver}}{2}(V_{in}-V_{T0})^2$

$\Rightarrow (5-V_{out}-0.8)^2 = 10(V_{in}-0.8)^2$

take derivative wrt V_{in}, set $\frac{dV_{out}}{dV_{in}} = -1 \Rightarrow$

$\boxed{V_{out} = 12.2 - 10V_{IL}}$ when $V_{in} = V_{IL}$

replace $\Rightarrow (4.2 - 12.2 + 10V_{IL})^2 = 10(V_{IL}-0.8)^2$

$\Rightarrow \boxed{V_{IL} = 0.8V}$

$\Rightarrow NM_L = V_{IL} - V_{OL} = 0.8 - 0.239 = 0.561V$

Problem 6 :

(a) $V_{out}(t=\infty) = V_{DD} - V_{Tn} = 2.5 - 0.43 = 2.07\,V$

(b) $V_M = \frac{V_{out}(t=\infty)}{2} = 1.04\,V$

low to high when $V_{out} = 0 \Rightarrow I_1 = k_n' \frac{W}{L} \left[(V_{GS} - V_{Tn}) V_{DSATN} - \frac{V_{DSAT}^2}{2} \right]$

$= 1.91 \times 10^{-4}\,A$

$\Rightarrow R_1 = \frac{2.5}{1.91 \times 10^{-4}} = 13.1\,k\Omega.$

∘ when $V_{out} = V_M \Rightarrow I_2 = k_n' \frac{W}{L} \left((V_{GS} - V_{Tn}) V_{DSAT} - \frac{V_{DSAT}^2}{2} \right)$

$= 7.78 \times 10^{-5}\,A$

$\Rightarrow R_2 = \frac{2.5 - 1.04}{7.78 \times 10^{-5}} = 18.8\,k\Omega$

$\Rightarrow R_{eq} = \frac{R_1 + R_2}{2} = 16.0\,k\Omega$

$\Rightarrow t_{PLH} = 0.69\, R_{eq} C_{L2} = 110\,ns$

(c) Repeat for t_{PHL}, we get

$\left.\begin{matrix} I_1 = 1.91 \times 10^{-4}\,A \\ V_{out} = 2.07\,V \end{matrix}\right\} \Rightarrow R_1 = \frac{2.07}{1.91 \times 10^{-4}} = 10.8\,k\Omega$

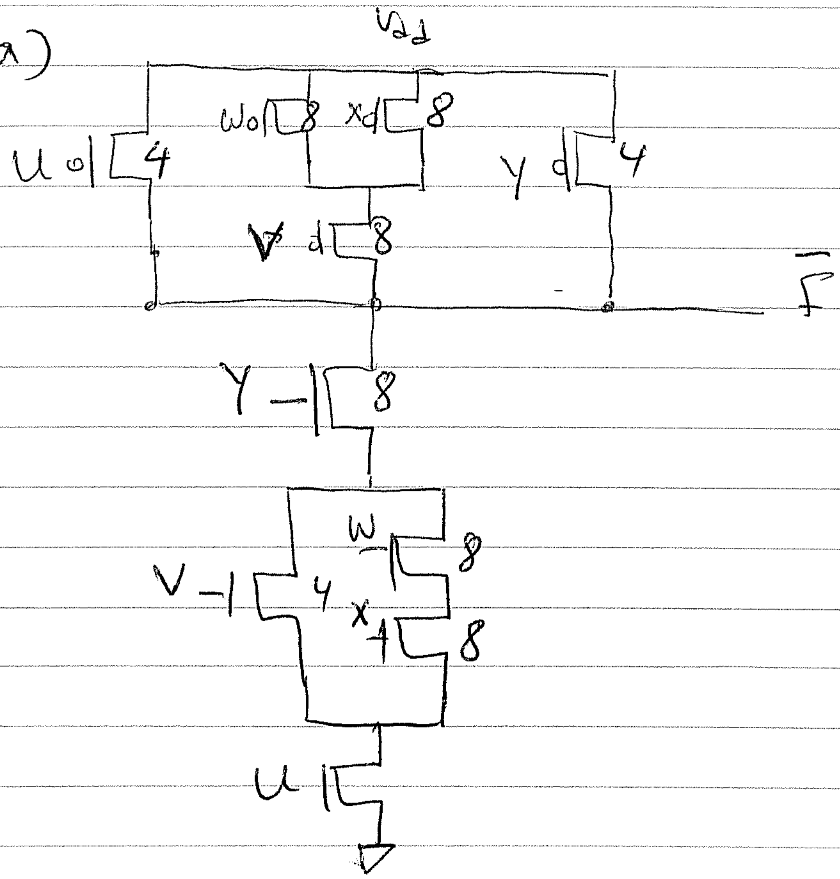
$\left.\begin{matrix} I_2 = I_1 = 1.91 \times 10^{-4}\,A \\ V_{out} = 1.04\,V \end{matrix}\right\} \Rightarrow R_2 = \frac{1.04}{1.91 \times 10^{-4}} = 5.4\,k\Omega$

$\Rightarrow R_{eq} = 8$

$\Rightarrow t_{PHL} = 0.69\, R_{eq} C_{L2} = 56\,ns$

Problem 7 $\overline{F} = UY \cdot (V + WX)$

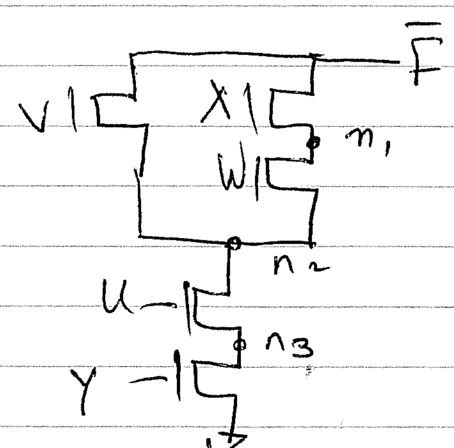
(a)



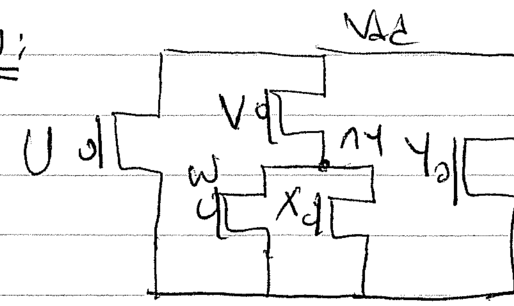
(b) Redraw (a), putting U closest to input

(c) Put X closest to input

PDN:



PUN:



Path: Y → U → V → X → W