Math 201 — Fall 2009–10
Calculus and Analytic Geometry III, sections 1–8, 24–26
Quiz 1, November 2 — Duration: 1 hour

GRADES:

1 (/30)	2 (/10)	3 (/15)	4 (/20)	5 (/25)	TOTAL/100

YOUR NAME:

YOUR AUB ID#:

PLEASE CIRCLE YOUR SECTION:

Section 1 Lecture MWF 3 Professor Makdisi Recitation F 11 Ms. Nassif	Section 2 Lecture MWF 3 Professor Makdisi Recitation F 2 Ms. Nassif	Section 3 Lecture MWF 3 Professor Makdisi Recitation F 4 Ms. Nassif	Section 4 Lecture MWF 3 Professor Makdisi Recitation F 9 Ms. Nassif
Section 5 Lecture MWF 10 Professor Raji Recitation T 11 Professor Raji	Section 6 Lecture MWF 10 Professor Raji Recitation T 3:30 Ms. Itani	Section 7 Lecture MWF 10 Professor Raji Recitation T 8 Ms. Itani	Section 8 Lecture MWF 10 Professor Raji Recitation T 2 Ms. Itani
Section 24 Lecture MWF 2 Professor Tlas Recitation F 11 Dr. Yamani	Section 25 Lecture MWF 2 Professor Tlas Recitation F 12 Dr. Yamani	Section 26 Lecture MWF 2 Professor Tlas Recitation F 3 Professor Tlas	

INSTRUCTIONS:

1. **Write your NAME and AUB ID number, and circle your SECTION above.**
2. **Solve the problems inside the booklet. Explain your steps precisely and clearly to ensure full credit. Partial solutions will receive partial credit.**
3. **You may use the back of each page for scratchwork OR for solutions. There are three extra blank sheets at the end, for extra scratchwork or solutions. If you need to continue a solution on another page, INDICATE CLEARLY WHERE THE GRADER SHOULD CONTINUE READING.**
4. **Closed book and notes. NO CALCULATORS ALLOWED. Turn OFF and put away any cell phones.**

GOOD LUCK!

An overview of the exam problems.
Take a minute to look at all the questions, THEN
solve each problem on its corresponding page INSIDE the booklet.

1. (5 pts each part, 30 pts total) For each of the following series, determine whether it converges or diverges. Justify your answer.

a) $\displaystyle\sum_{n=0}^{\infty} \frac{n+1}{n+2}$ b) $\displaystyle\sum_{n=0}^{\infty} \frac{n!(2n)!}{(3n)!}$ c) $\displaystyle\sum_{n=2}^{\infty} \left(1-\frac{6}{n}\right)^{n^2}$

d) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n}}$ e) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 \sin n}{2^n}$ f) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(1/n^{0.6})}{n^{0.7}}$

2. (5 pts each part, 10 pts total)

a) Show that the following series converges, and find its value (you are **not** required to simplify the expression):

$$\sum_{n=1}^{\infty} \left(\frac{(-5)^{n+1}}{7^n} + \frac{3^{n-1}}{4^{n+2}}\right)$$

Careful! The series starts at $n = 1$.

b) Find the value of the following series (you do not have to show that it converges):

$$\sum_{k=2}^{\infty} \frac{(-1)^k \pi^{2k}}{(2k)!}$$

Careful! The series starts at $k = 2$.

3. (15 pts) For which values of x does the following power series converge? Also, for which values of x is the convergence absolute? (Remember to test the endpoints!)

$$\sum_{n=0}^{\infty} \frac{(x-8)^n}{2^n(n+2)}.$$

4. (10 pts each part, 20 pts total)

a) Find the second-order Taylor polynomial $P_2(x)$ for the function $f(x) = \ln(3x+2)$ **centered at** $x = 1$.

(Your answer will have the form $P_2(x) = c_0 + c_1(x-1) + c_2(x-1)^2$ with specific numbers c_0, c_1, c_2 that you must find. Be careful with taking derivatives.)

b) Use Taylor's theorem to show that $|f(1.1) - P_2(1.1)| \le 10^{-4}$.

Possibly useful numbers: $4^3 = 64$, $5^3 = 125$, $6^3 = 216$.

5. (25 pts total)

a) (9 pts) Express the following integral as a series:

$$L = \int_{x=0}^{0.1} \frac{e^{-x^2} - 1}{x} dx.$$

b) (9 pts) Find a specific partial sum s_N for which you can show that $|s_N - L| \le 10^{-12}$.

c) (7 pts) Challenge: answer the same question as in part (b) for the different integral

$$M = \int_{x=0}^{0.1} \frac{e^{x^2} - 1}{x} dx.$$

This means that you should find a specific partial sum t_N of a different series for which you can show that that $|t_N - M| \le 10^{-12}$.

1. (5 pts each part, 30 pts total) For each of the following series, determine whether it converges or diverges. Justify your answer.

a) $\displaystyle\sum_{n=0}^{\infty} \frac{n+1}{n+2}$

b) $\displaystyle\sum_{n=0}^{\infty} \frac{n!(2n)!}{(3n)!}$

c) $\displaystyle\sum_{n=2}^{\infty} \left(1 - \frac{6}{n}\right)^{n^2}$

1, continued.

d) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n}}$

e) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 \sin n}{2^n}$

f) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(1/n^{0.6})}{n^{0.7}}$

2. (5 pts each part, 10 pts total)

a) Show that the following series converges, and find its value (you are **not** required to simplify the expression):

$$\sum_{n=1}^{\infty}\left(\frac{(-5)^{n+1}}{7^n}+\frac{3^{n-1}}{4^{n+2}}\right)$$

Careful! The series starts at $n = 1$.

b) Find the value of the following series (you do not have to show that it converges):

$$\sum_{k=2}^{\infty}\frac{(-1)^k\pi^{2k}}{(2k)!}$$

Careful! The series starts at $k = 2$.

3. (15 pts) For which values of x does the following power series converge? Also, for which values of x is the convergence absolute? (Remember to test the endpoints!)

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(Your answer will have the form $P_2(x) = c_0 + c_1(x-1) + c_2(x-1)^2$ with specific numbers c_0, c_1, c_2 that you must find. Be careful with taking derivatives.)

b) Use Taylor's theorem to show that $|f(1.1) - P_2(1.1)| \leq 10^{-4}$.
Possibly useful numbers: $4^3 = 64$, $5^3 = 125$, $6^3 = 216$.

5. (25 pts total)

a) (9 pts) Express the following integral as a series:

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b) (9 pts) Find a specific partial sum s_N for which you can show that $|s_N - L| \leq 10^{-12}$.

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$$M = \int_{x=0}^{0.1} \frac{e^{x^2} - 1}{x} dx.$$

This means that you should find a specific partial sum t_N of a different series for which you can show that that $|t_N - M| \leq 10^{-12}$.

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