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AUB | **Physics 217** | **Jan. 29, 1996**
Physics Department | **Final Exam** | **Time: 2 hours**

Name:_____

ID. No :_____

General Infromation :

(1) No make up of this exam without legal reason

(2) Try to work out all questions, unless you are given a choice

(3) The time given above cannot be exceeded.

(4) The exam will have a total grade of 200

Content	Grade
1. Particle in a central field	_____
2. Rotation	_____
3. Collisions	_____
4. Oscillation of rigid bodies	_____
5. Pendulum with Lagrangian	_____
6. Particle on a wire with Lagrangian (This problem has a bonus)	_____
	Total :

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(1) Particle in a central field *(40 points)*

A particle of mass m moves in a central potential given by $V(r) = kr$, wher $k > 0$ is a given constant

(a) For what angular momentum L and energy E will the orbit be a circle with radius a?

(b) What is the angular frequency ω of this circular motion ?

(c) If the particle is slightly disturbed from its orbit, what will be the frequency ω_s of small oscillation ?

Hint: remember to work with V_{eff}.

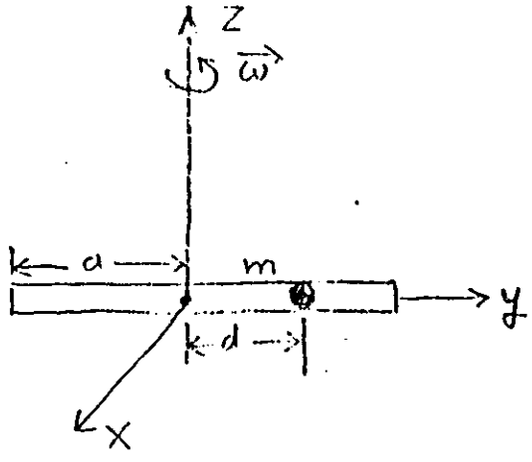
(2) Rotation *(40 points)*

A hollow cylinder tube of length (2 a) rotates with constant angular speed ω about the z-axis (see figure). A particle is initally at rest in the tube at a distance d from the origin O. Neglect frictional effects.

(a) Find the position r(t) and velocity v(t) of the particle at any time.

(b) Show that the time needed for the particle to come out of the tube is

$$T = \frac{1}{\omega} \ln\left(\frac{a + \sqrt{a^2 - d^2}}{d}\right)$$



Hint : you need

$\cos h x = \frac{1}{2}(e^x + e^{-x})$

$\sin h x = \frac{1}{2}(e^x - e^{-x})$

$\cos h^{-1} x = \ln[x + \sqrt{x^2 - 1}]$, $(x > 1)$

(3) Collision *(30 points)*

A neutron moving with initial speed v_1 = 100 m/s collides elastically with a deutron that is initally at rest. If the neutron is deflected by an angle $\theta = 20°$,

(a) What are the final speeds of the neutron and deutron after impact ?

Assume the mass of the deutron $m_d = 2\, m_n$, where m_n is the mass of the neutron.

(4) Oscillation of rigid bodies *(40 points)*

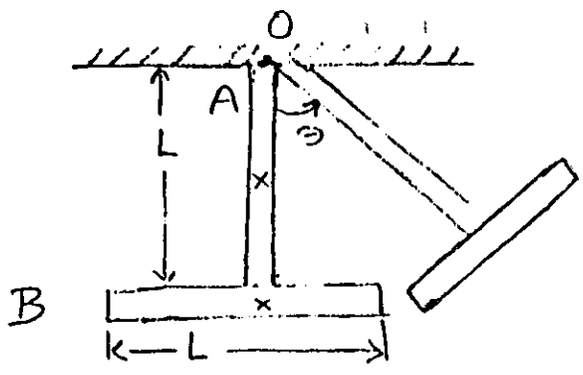
A pendulum is constructed from two identical uniform rods A and B each of length L and mass m. The rods are connected; such that rod A is fixed at the center of rod B (see figure). The system is free to swing about an axis through point O, and it starts from rest.

(a) Calculte the total moment of inertia of this pendulum (about the axis through O).

(b) Find the kinetic energy and the potential energy of the pendulum (be very careful by calculating the potential energy).

(c) By using energy conservation, obtain the equation of motion, and give the angular frequency of small oscillation.

Hint: Moment of inertia of the rod about the center of mass is : $I_{CM} = (1/12)\, M\, L^2$.
• All you need can be expressed in terms of θ and $\dot{\theta}$.



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Use the Lagrange's method to solve one the following problems (5) or (6).

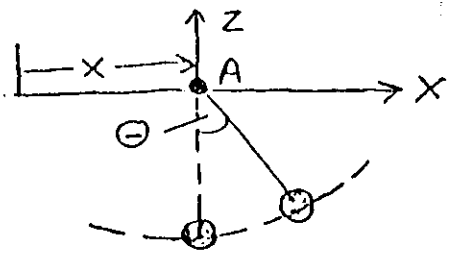
(5) *(50 points)*

A pendulum bob of mass m is suspended by a massless string of length L from a point of support (point A in the figure). The point A moves back and forth according to the law $x = a \cos \omega t$. Assume that the pendulum swings in the vertical xz-plane, and its position is described by the angle θ.

(a) Give the Lagrangian of this system, and find the equation of motion.

(b) Show that for small oscillation, the equation of motion is that of a forced harmonic oscillator.

(c) Find the steady-state motion θ (t), and express the amplitude in terms of (L, a, ω).



(6) *(50 points and bonus)*

A frictionless wire ($\overline{AB}$ in the figure) is fixed at point A, such that it can rotate about the z-axis at fixed angle α. A particle of mass m is constrainted to move on the wire (friction is neglected). The distance of the particle from A is r, and the height $\overline{OA}$ is h.

(a) Give the Lagrangian of this system.

(b) Find the equation of motion, and obtain r(t).

Hint: use spherical coordinates (r, α, φ) to express x, y and z.