Physics Department | Physics 217 Final Exam | Jan. 30, 1997 Time: 3 hours

Name:______________________________

I.D. No. ______________________

Information:

- No make up of this exam without legal reason
- Try to work out all questions
- This exam will have a total grade of 200

Content	Grade
1. One-dimensional motion	
2. Central force	
3. Rotating frame	
4. Motion of rigid body	
5. Moment of Inertia	
6. Lagrangian Mechanics	

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Problems:

(1) A car of mass m is moving with constant speed v_o along a horizontal track. A force of air resistance is acting given by

$$f(v) = (\alpha + \beta v^2)$$

where α and β are positive constants, and v is the velocity of the car.

(a) How long will it take the car to come to rest after the engine is turned off?

(b) What is the distance traveled by the car?

Hint: $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right), \ a \neq 0$

(2) A particle of mass m is moving under the influence of a central force given by

$$F(r) = -\frac{k}{r^2} + \frac{k'}{r^3}, \ (k, k' > 0$$

(a) Find the Lagrnagian for the particle in terms of the coordinate r and θ

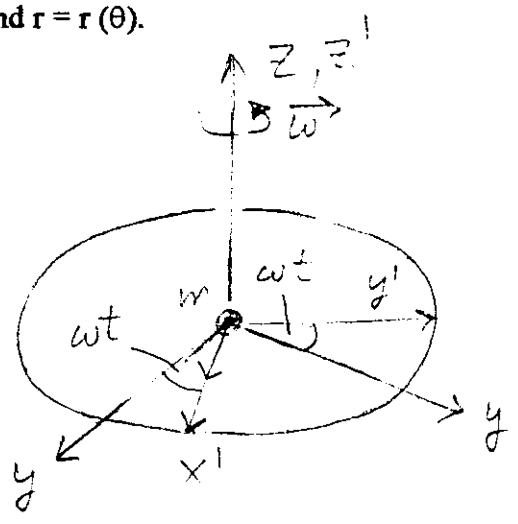
(b) Set-up the equations of motion for r and θ, and show that the angular momentum is conserved.

(c) Use the differential equation for the orbit, to find $r = r(\theta)$.

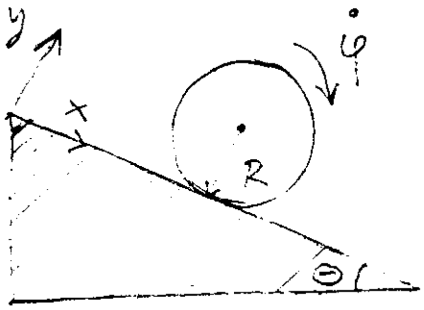
(3) A carousel lies in the xy-plane, and rotates with a constant angular velocity. A bug of mass m starts at the origin, and moves outward without slipping with a speed v_o (see Figure).

(a) Find the force $\bar{F}_b$ exerted by the carousel on the bug.

(b) Write the force $\bar{F}_b$ in terms of the coordinates (x,y,z) and write down the transformation matrix between the two coordinate frames.

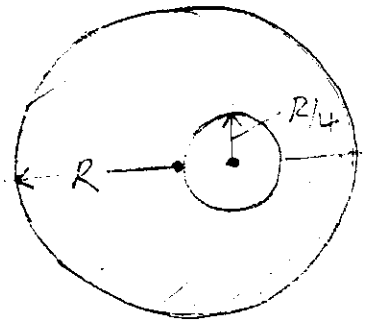


(4) A thin circular ring of radius R and mass on rolls down a plane inclined at angle θ from the horizontal (see Figure).



(a) Calculate the minimum coefficient of friction μ necessary to keep the ring rolling without sliding. (Moment of inertia of the ring: mR^2)

(5) A uniform disk of radius R has a mass M. Suppose that a hole of radius R/4 is drilled into the disk with the edge at the disk center (see Figure).

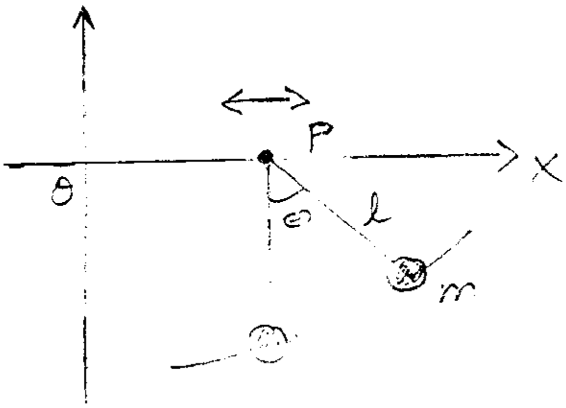


(a) Find the moment of inertia of this system about the central axis of the disk.

Hint: Moment of inertia of the full disk is $1/2\ MR^2$.
It is useful to remember the parallel axis theorem.

(6) A Pendulum of mass m is suspended by a string of length ℓ (negligible mass) from a point of support P.
The point of support performs a harmonic motion according to:

$$x(t) = A \sin \omega t$$



where A and ω are given.
Assume that the pendulum swings in the xy-plane.

(a) Find the Lagrangian for this pendulum.

(b) Obtain the equation of motion of the pendulum.

(c) Solve the equation of motion to obtain $\theta = \theta(t)$.
for small oscillation ($\sin\theta \approx \theta$, $\cos\theta \approx 1$), imposing initital conditions: $\theta = 0$, $\theta(0) = 0$ at $t = 0$.

Hint: complex notation may be useful to solve the equation of motion.