AUB
Physics Dept.

Phys 217
Final Exam

Feb. 2, 2006
Time 2 1/2 hours

Name: ______________________________

Id No.: ______________________________

All questions are obligatory

Content:

1. Central force
2. Scattering in a central potential
3. Lagrange Equation
4. Hamilton's Equations
5. Rotating systems

M. Eid

1 Central force

Given is the following orbit of a particle of mass m :

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$$r = a(1+\cos\theta), \quad (a>0 \text{ constant})$$

Find the central force that leads to this orbit.
Show that this force behaves like $\frac{1}{r^4}$.

2 Central Potential and Scattering

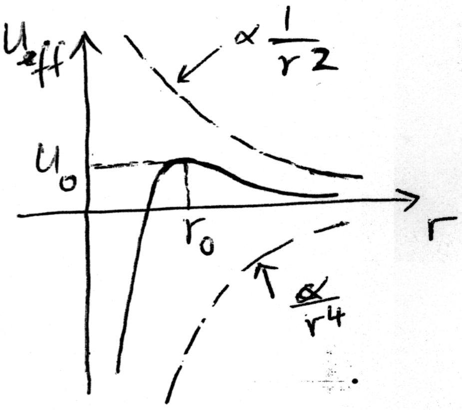
15 A Particle of mass m experiences a force whose potential is given by

$$U(r) = -\frac{\alpha}{r^4}, \quad (\alpha > 0 \text{ is a constant})$$

Consider the effective Potential U_{eff} (see Figure)

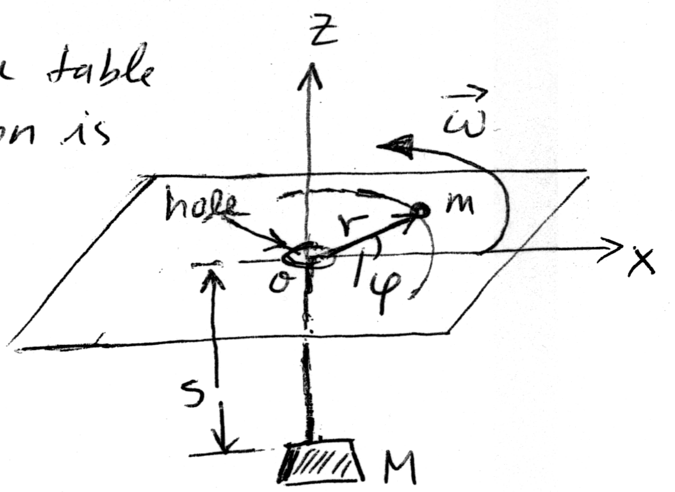
10 a) Find r_0 and $U_0 = (U_{eff})_{max}$ in terms of (l, m, α)

10 b) The Particle is captured by the potential when it has an energy $E \geq U_0$. Suppose that $E = U_0$ and find the total cross section for this capture process



3 Lagrange Equation

30) The mass m rotates on a table assumed to be smooth (friction is neglected).



A string of length $l = r + s$ where s is the portion of the string below the hole through which ~~the~~ the block M is connected

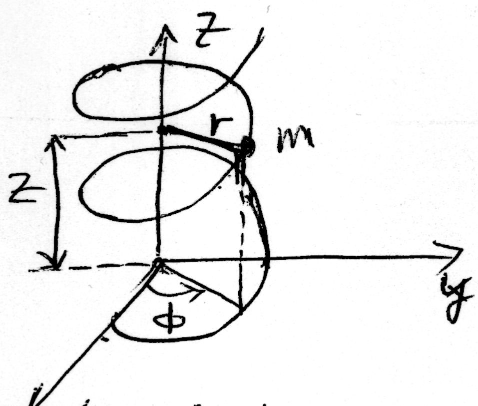
5 a) There are four constraints in this system, which? How many degrees of freedom are involved here?

15 b) The general coordinates are clearly (φ, s). Find the Lagrangian of this system and obtain the equations of motion for r.

5 c) Determine ω_0 such that $\ddot{s} = 0$ ($\ddot{s}$ = acceleration of M)

5 d) suppose that $\omega = 0$, what type of motion is the block M going to make?. In other words, calculate $\ddot{s}$ in this case

4 Hamilton's equations



cylindrical symmetry

(20) A particle of mass m moves under the influence of the force of gravity along a spiral path $z = a\phi$ and $r = $ constant (see Figure)

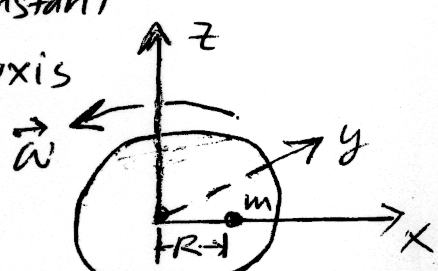
Find the Hamiltonian of this system and obtain the Hamilton's equations. Obtain a differential equation for z (do not solve it)

Hint: since $r =$ constant, you have cylindrical symmetry (r, ϕ, z)

5 Non-inertial systems

(20) A smooth disk is rotating at constant angular velocity ω about a vertical axis passing through its center. A small coin of

is pushed away from the center of the disk at a distance R from that center.



Both origins O and O' coincide

(10a) Set up the equation of motion for the coin in the rotating system. Neglect the weight of the coin.

(10b) You obtain a coupled system of differential equations, which you can solve with the method of complex variable as it was done in the case of the Faucault pendulum.

Use the initial conditions: (at $t=0$) $x(0)=R$, $\dot{x}(0)=v_0$; $y(0)=0$, $\dot{y}(0)=0$