Physics 217, Final Exam, January 23 2008
Time: 3 hours

1. A particle of mass m is constrained to move on the frictionless inner surface of a cone of half-angle α. Write the equations of motion of m in spherical coordinates. What are the conditions on the initial velocity v_0 and initial distance r_0 so that the particle moves in a circular orbit about the vertical axis.

2. An object of mass m orbits a central potential $V(r)$ with angular momentum L. Its orbit is $r = ae^{-b\theta}$ where θ is the azimuthal angle measured in the orbital plane. Find $V(r)$.

3. Three masses m_1, m_2 and m_3 are placed at the corners of an equilateral triangle of side s, attract each other according to Newton's law of gravitation. a) Determine the position of the center of mass $\mathbf{R}$. b) Find the resultant force $\mathbf{F}_1$ exerted by m_2 and m_3 on m_1 and show that it is parallel to $\mathbf{R}$. c) Show that m_1 moves in a circle about the center of mass. Evaluate the radius, velocity of this orbit. d) Show that the period of this orbit is given by
$$T = 2\pi s\sqrt{\frac{s}{G\left(m_1 + m_2 + m_3\right)}}$$

4. A perfectly smooth horizontal disk is rotating with an angular velocity ω about a vertical axis passing through its center. A person on the disk at a distance R from the center gives a perfectly smooth coin (negligible size) of mass m a push toward the origin. This push gives it an initial velocity v relative to the disk. a) Write the equations of motion for m in the $x - y$ plane. b) Show that the equation of motion are equivalent to the single complex equation
$$\ddot{z} + 2i\omega\dot{z} - \omega^2 z = 0$$
where $z = x + iy$.c) Solve this differential equation by assuming that $z = e^{\gamma t}$ and by taking care that in case of double root the independent solutions are of the form $e^{\gamma t}$ and $te^{\gamma t}$. d) Determine the arbitrary parameters in the differential equation in terms of initial distance R and velocity V.

5. A block of mass M is rigidly connected to a massless circular track of radius a on a frictionless horizontal table. A particle of mass m is confined to move without friction on the circular track which is vertical. a) Set up the Lagrangian using θ and x as coordinates. b) Find the equations of motion. c) In the limit of small θ solve the equations of motion (by eliminating $\ddot{x}$) to determine θ as function of time.

6. A simple pendulum of length $4l$ and mass m is hung from another simple pendulum of length $3l$ and mass m. a) Write the Lagrangian for this system and determine the equations of motion. b) Solve the system of equations for small oscillations by determining the normal modes and normal coordinates.