

MATHEMATICS 213, FINAL EXAMINATION

SECOND SEMESTER, 1996-97



Answer the following questions:

I. (a) State the *ruler* postulate. (3 pts.)

(b) Use the ruler postulate to prove that

(i) of any three collinear points exactly one lies between the other two.

(6 pts.)

(ii) if four collinear points  $A, B, C$  and  $D$  satisfy  $A - B - C$  and  $B - C - D$ , then they satisfy  $A - B - D$  and  $A - C - D$ . (6 pts.)

II. Use the *plane-separation* postulate and the postulate of *Pasch* to prove that for any triangle  $\triangle ABC$

(a) the interiors of the segments  $\overline{BC}$ ,  $\overline{CA}$  and  $\overline{AB}$  have no transversal, and that (5 pts.)

(b) the rays  $\overrightarrow{BC} \setminus \overline{BC}$ ,  $\overrightarrow{AC} \setminus \overline{AC}$  and  $\overrightarrow{AB} \setminus \overline{AB}$  always have a transversal. (5 pts.)

III. (a) Define a convex quadrilateral. (3 pts.)

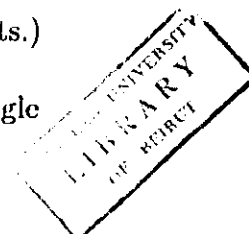
(b) Use the *Crossbar* Theorem to Show that the diagonals of a convex quadrilateral intersect each other. (6 pts.)

(c) Show that no full line can be contained in the interior of a triangle. (6 pts.)

IV. (a) State the *SAS* Postulate. (3 pts.)

(b) Prove the *ASA* Theorem from the *SAS* Postulate. (6 pts.)

(c) Use the *exterior angle* Theorem to prove that for a given triangle



$\triangle ABC$  with a point  $D$  satisfying  $B - D - C$ , either  $AD < AB$  or  $AD < AC$ .

(6 pts.)

V. (a) Define a *parallel projection*. (3 pts.)

(b) Show that parallel projections preserve betweenness. (6 pts.)

(c) Give Euclid's proof of the *Basic Similarity* Theorem which uses the equivalence of areas of two triangles having equal bases and altitudes. (6 pts.)

VI. (a) Show that isometry is an equivalence relation. (3 pts.)

(b) Prove that if a set  $M$  is isometric to the unit interval  $[0,1]$ , then  $M$  is a unit interval. (6 pts.)

(c) Show that a dilation preserve circles. (6 pts.)

VII. (a) Use an unmarked ruler and a collapsible compass to divide a segment into  $n$  congruent segments. (6 pts.)

(b) Define the *Surd field*. (3 pts.)

(c) Is  $3^{1/3}$  a Surd number? Justify your answer. (3 pts.)