AMERICAN UNIVERSITY OF BEIRUT
DEPARTMENT OF MATHEMATICS
MATH 213, Final Examination
June 18, 2002

Answer The Following Questions:

1. (a) Define the Gauss map of a regular surface. (5 points)
(b) Describe the region of the unit sphere covered by the Gauss map of the surface $z = y^2 - x^2$. (10 points)

2. (a) Define the second fundamental form II_p of a regular surface S at a point $p \in S$. (5 points)
(b) State and prove a geometric interpretation of II_p. (10 points)

3. (a) Prove Euler's formula for the normal curvature of a surface S at a point p in the direction of a unit vector $\mathbf{v}$. (5 points)
(b) Let S be a regular surface with a Gauss map $\mathbf{N}$, $p \in S$ and $H \equiv 0$. Show that if $\mathbf{v}, \mathbf{w} \in T_p(S)$, then

$$< d\mathbf{N}_p(\mathbf{v}), d\mathbf{N}_p(\mathbf{w}) >= -K_p < \mathbf{v}, \mathbf{w} > .$$

Conclude that the Gauss map preserves angles and orientation; that is, an angle between two intersecting regular arcs in S map to an angle of equal size between the image arcs under $\mathbf{N}$.

(15 points)

4. (a) Define the asymptotic curves and lines of curvature of a surface S.
(5 points)
(b) Determine the asymptotic curves and lines of curvature of the surface $S : z = xy$, $-\infty < x, y < \infty$. (10 points)

5. (a) Define the Gaussian and Mean curvature of a regular surface. (5 points)
(b) Find the principle curvatures of the one-sheeted hyperboloid given by the parametrization $\mathbf{x}(u, v) = (\cosh v \cos u, \cosh v \sin u, v)$, where $0 < u < 2\pi$ and $\infty < v < \infty$.

Good Luck