Math 303–Final Exam (Fall 00)

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1. Let μ be a measure, $\varphi_1, \varphi_2, \dots$ nonnegative measurable functions, and $A_1, A_2, \dots$ positive numbers. Also let

$$B_l = \int |\varphi_l - A_l| d\mu.$$

Suppose

$$(i) \qquad \sum_{l=1}^{\infty} A_l = \infty$$

$$(ii) \qquad \sum_{l=1}^{\infty} B_l < \infty.$$

Prove that

$$\sum_{l=1}^{\infty} \varphi_l = \infty \qquad \text{almost everywhere.}$$

2. Let μ be a measure on a set X. If f is a complex measurable function, put

$$||f||_{1,\infty} = \sup_{0<t<\infty} t\mu(|f| > t)$$

$$||f||_0 = \int \frac{|f|}{1+|f|} d\mu.$$

Let the complex measurable functions $f_1, f_2, \dots$ be such that

$$\lim_{n\to\infty} ||f_n||_{1,\infty} = 0.$$

If μ is finite, prove that

$$\lim_{n\to\infty} ||f_n||_0 = 0.$$