MECH 435L – Control Systems Lab

Lab 4 Report

Ahmad Ghalayini
Caren Serhal
Wahid El-Shaar

Part I:

Material used:

- 1 Resistor
- 1 Capacitor
- A function generator
- An oscilloscope
- Connecting wires
- A breadboard

The circuit used in the first part of the lab was a simple RC circuit. The resistor and the capacitor were connected in series and a we had to input a step voltage to this circuit. Practically, if we apply a unit step voltage, we will be unable to see the response since it will follow the input after a certain time, therefore we apply square input voltages and steady the behavior of the voltage at the terminals of the capacitor. The following is the circuit we built up on our breadboard. We used the function generator to input the square voltage and an oscilloscope to visualize the voltage across the capacitor as a function of time. Since the output is taken across the capacitor in the RC circuit then we have a low pass filter(lag network) which allows the passage of low frequencies and does not allow high frequencies to pass (output is equal to zero). The filter is characterized by a cutoff frequency at which the output's signal magnitude ratio is equal to -3dB. (output in dB = -20log (Vout/Vin)

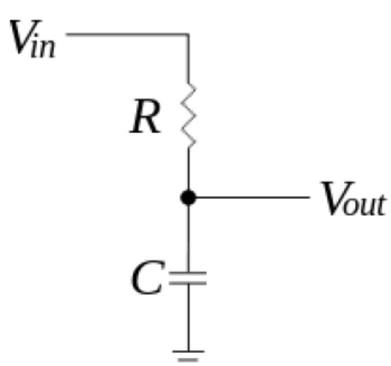


Figure 1: Low pass filter

In this part of the lab we were interested of the variations of the voltage as a function of time (in part 2, we cared about the variation of the magnitude and phase of Vout/Vin versus magnitude and we study Bode plots for second order systems). We can visualize the input and the output by connecting the terminals of channel 1 of the oscilloscope to the input and the terminals of channel 2 to the output. We notice that the frequency obtained for the output voltage is the same as the frequency of the input square voltage.

w = 2*f*pi

1) Derive the transfer function of the system

I(resistor) = I(capacitor)
(Vin(t)-Vout(t))/R = C*(dVout(t)/dt)

Applying laPlace transform to both sides of the equation yields:
Vin(s) – Vout(s) = R*C*s*(Vout(s)) – 0
Assuming that we have to voltage at t = 0.

Vin(s) = Vout(s) [1 + R*C*s]
Therefore,
Vout(s)/Vin(s) = 1 / [1 +R*C*s]

Similarly, we could have worked using the impedances :
Vout/Vin = (1/s*C) / (1/s*C +R) = 1/(1 + R*C*s)

2) Calculate the time constant and find the cut-off frequency (theoretical)

The transfer function obtained is a typical transfer function for a first order system of the form:
Tf = 1/1+ts; where t is the time constant

The break frequency, also called the turnover frequency or cutoff frequency (in hertz), is determined by the time constant:

$$\tau = RC = \frac{1}{2\pi f_c}$$

Tau = R*C =

$$f_c = \frac{1}{2\pi\tau} = \frac{1}{2\pi RC}$$

fc = 1/2*pi*R*C =

or equivalently (in radians per second):

$$\omega_c = \frac{1}{\tau} = \frac{1}{RC}.$$

wc = 1/ (R*C) =

3) **Build the circuit (was done during the lab)**

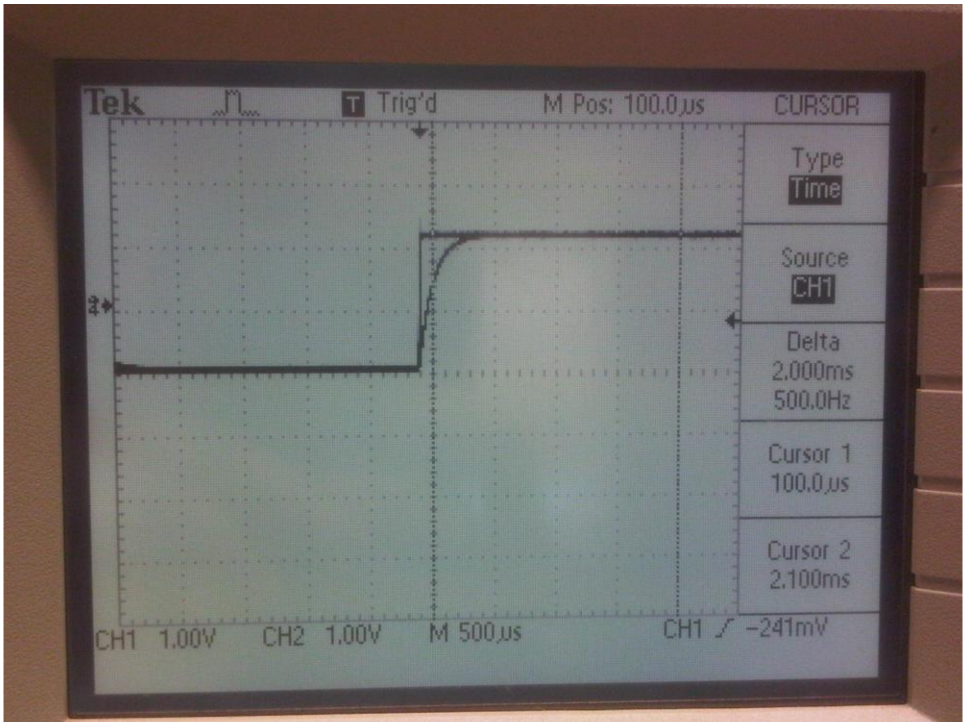
4) **Use a signal generator and an oscilloscope to give the circuit a square wave input and deduce from the generated output the time constant (experimental).**

When we superimposed the input and the output on the oscilloscope, we were able to careful look at the response.

We increased the horizontal sensitivity to get a value of the peak to peak accurately using the horizontal cursor.
The peak to peak value was V.

We can also read it by pressing on the button measure and read the value pk to pk for channel 2.
We then get 0.63*pk to pk = V.

Using the vertical cursor, we get the abscissa of this point. The value obtained is equal to the time constant obtained experimentally tau = sec.



Time response of the input and output as a function of time

5) Measure the rise time and steady state error:

We already got the value of the peak to peak value. We calculate 0.1*pk to pk and place the horizontal cursor on and find the intersection with the curve we then use the vertical cursor to get t1.

t1 = s

0.9*pk to pk and similarly get the corresponding time t2.

t2 = s

Rise time = t2 – t1 = s

The steady state is nearly zero. We can see that the graph of the input and the one of the output overlap at the end of each square of the signal.

6) Compare the experimental and theoretical time constants and explain.

References:

- http://www.allaboutcircuits.com/vol_2/chpt_8/2.html
- http://www.electronics-tutorials.ws/filter/filter_2.html

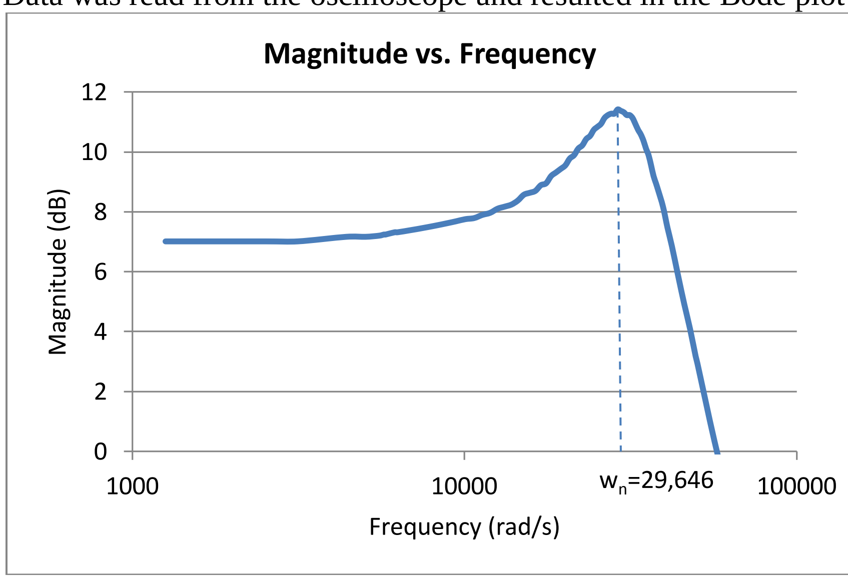
Part II:

Procedure

A sinusoidal input was given to the circuit shown above. An oscilloscope was used to measure the peak to peak voltage of the response as we varied the frequency.

Experimental Results

Data was read from the oscilloscope and resulted in the Bode plot shown below.



Analysis

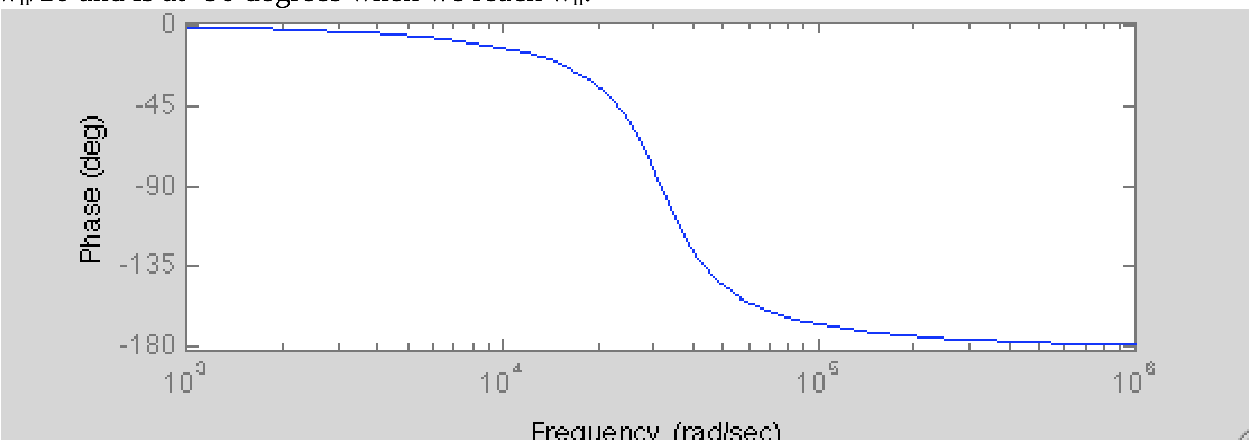
This shows that w_n=29,646 Hz which is to be proven theoretically later on.
System Type: It is a type 2 system since it decreases at a slope of -42 dB/decade.

Zeroes: Since the magnitude is decreasing after w_n, then there are no zeroes.
Poles: The peak to peak voltage starts decreasing at w_n and since it is a Type II system then it has a double pole at $s=w_n$.

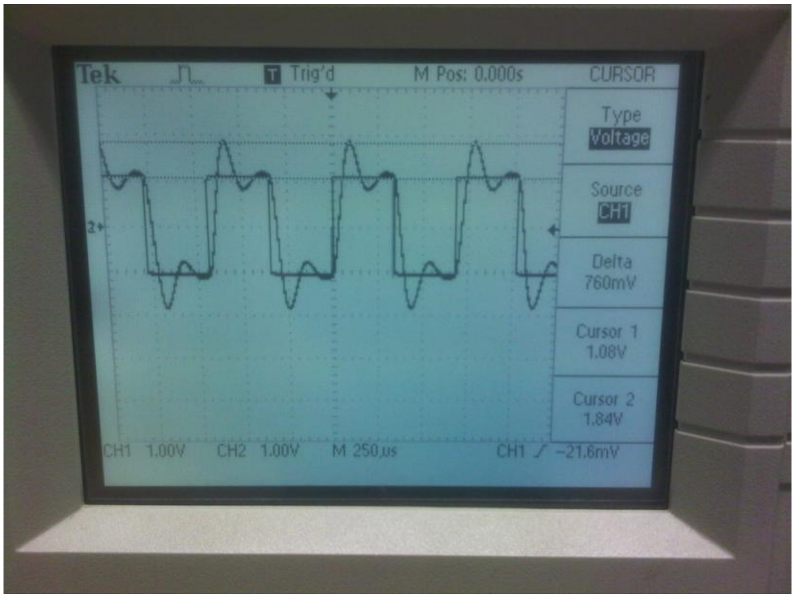
Finding the value of K:

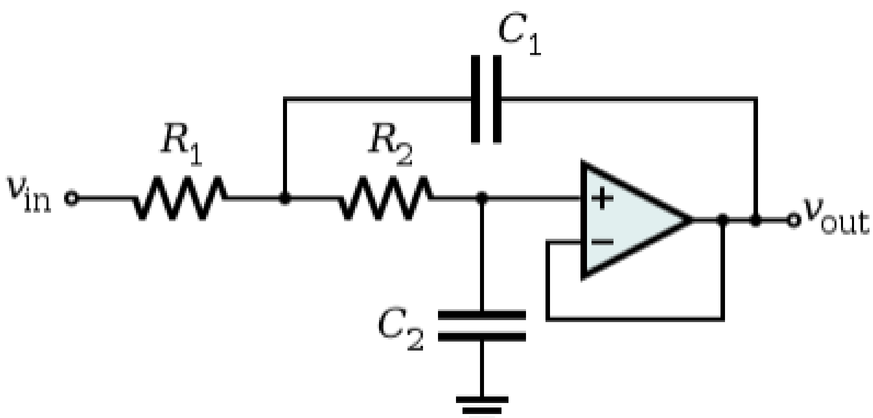
$$7=20\log(K)$$
$$\Rightarrow K=2.238$$

To find the phase angle we used the following relations:
$\Theta = (\Delta T/T)*360$ in degrees
$\Theta = (\Delta T/T)*2pi$ in radians
This results in the graph below, it can be noted that the decrease in phase angle starts at around $w_n/10$ and is at -90 degrees when we reach w_n.



Applying a square wave signal as an input, we get the response shown below.
Experimental results show the following:
Overshoot: 38%
Risetime: 20 microseconds
Steady-state error: 0





Sum of currents on the node to the right of V_{in}

$$i_{c2} = i_{c1} + i_{R1}$$

$$\frac{V_{out}}{1/sC_2} = \frac{V_{out} - V_1}{1/sC_1} + \frac{V_{in} - V_1}{R_1}$$

Where V_1 is the voltage on the node to the right of V_{in}

$$V_1 = i_{C2}R_2 + V_{out} = V_{out}(R_2C_2s + 1)$$

Since by Ohm's law $V_{out} = \frac{i_{C2}}{sC_2}$

Replacing V_1 in the current equation evaluated above:

$$\frac{V_{out}}{1/sC_2} = \frac{V_{out}(1 - R_2C_2s - 1)}{1/sC_1} + \frac{V_{in} - V_{out}(R_2C_2s + 1)}{R_1}$$

$$V_{out}(R_2C_2C_1s^2 + C_2s + \frac{R_2C_2s}{R_1} + \frac{1}{R_1}) = \frac{V_{in}}{R_1}$$

Multiplying both sides by R_1:

$$V_{out}(R_2R_1C_2C_1s^2 + R_1C_2s + R_2C_2s + 1) = V_{in}$$

$$H(s) = \frac{1}{R_2R_1C_2C_1s^2 + (R_1C_2 + R_2C_2)s + 1}$$

$$H(s) = \frac{{}^{1}\!/_{R_2R_1C_2C_1}}{s^2 + \left({}^{1}\!/_{R_2C_1} + {}^{1}\!/_{R_1C_1}\right)s + {}^{1}\!/_{R_2R_1C_2C_1}} \equiv \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}$$

$$\omega_n = \sqrt{{}^{1}\!/_{R_2R_1C_2C_1}}$$

$$\zeta = \frac{R_1 + R_2}{2R_2R_1C_1}\sqrt{R_2R_1C_2C_1}$$

Plugging in the values R_1 = 10kohm, R_2 = 10kohm, C_1=0.01uF, C_2= 0.001uF:

$$H(s) = \frac{10^9}{s^2 + 2(10^4)s + 10^9}$$

$$\omega_n = 31{,}622.77\ rd/s$$

w_n is close to the experimental value obtained.

$$\zeta = 0.316$$

ζ <0.7, This explains the hump before the decrease in peak to peak voltage.