A.U.B. **Physics 235** **Feb. 03, 1996**

Physics Department **FINAL EXAM** **Time 2 hours.**

Choose one set out of the following sets: {1,2,3}, {1,3,4}, {2,3,4}

30 marks.

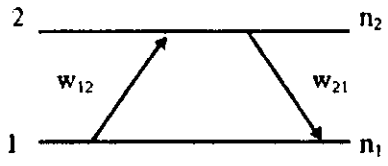
1)- The following quadratic form : $A.dx^2 + 2B.dxdy + C.dy^2$ is always positive when $A>0$, $C>0$, and $\begin{vmatrix} A & B \\ B & C \end{vmatrix} > 0$.

Consider the energy function U(S+dS, V+dV) and call all second order terms of its Taylor expansion d^2U , which is a quadratic form of dS and dV. The stability condition of this thermodynamic system is $d^2U > 0$ for all dS and dV. Show that , this condition is equivalent to: $C_V > 0$ and

$$\frac{\partial(P,T)}{\partial(V,T)} = \left.\frac{\partial P}{\partial V}\right)_T < 0 \quad .$$

30 marks.

2)-A two levels system of N atoms is out of equilibrium state, that is the occupancies $n_1(t)$ and $n_2(t)$ are different from their equilibrium values n°_1 and n°_2 given by the Boltzmann distribution function. Use the master equation to describe how $n = n_1 - n_2$ evolves toward its equilibrium value n° , Knowing that the transition probabilities between the states 1 and 2 are w_{12} and w_{21} with $w_{21}/w_{12} = \exp[(\varepsilon_2 - \varepsilon_1)/\tau]$.



40 marks.

3)-The elasticity of a rubber band can be described in terms of a one-dimensional model of polymer involving N molecules linked together end - to - end. The angle between successive links is equally likely to be 0° or 180° .

a)-Show that the number of arrangements that give an overall length of L = 2md is given by:

$$g(N,m) = \frac{2N1}{\left(\frac{N}{2}+m\right)!\left(\frac{N}{2}-m\right)!}$$

where m is >0 and d is the length of one link. Indicate clearly the reasoning you used to get this result.

b)- For m << N this expression becomes $g(N,m) \approx g(N,0).e^{-2m^2/N}$.

Find the entropy σ of the system as a function of L for N>>1, L<<Nd.

c)- Find the force f required to maintain the length L for L<< Nd.

(Use dF= - σdτ + f dL).

d)- If the energy of the link is $\varepsilon_\pm = \pm f.d$ and the rubber band is at equilibrium at temperature τ , find the average length $\bar{d}$ per link . Plot then the overall length L of the polymer in terms of the relative energy ε/τ. Discuss the analogy of this system with the N-spin 1/2 system in a magnetic field.

30 marks.

4) a)- Using the partition function for N identical particles $Z_N = (n_Q V)^N / N!$, show that the Gibbs sum $\mathcal{Z} = \sum_{AsN} \exp[(N\mu - \varepsilon_s)/\tau]$ is $\exp(\lambda n_Q V)$.

b)- Show that the probability there are N atoms in the gas in volume V in diffusive contact with a reservoir is $P(N) = \langle N \rangle^N . e^{-\langle N \rangle} / N!$ which is just the Poisson distribution function.

c)- Confirm that P(N) above satisfy $\sum_{N=0}^{\infty} P(N) = 1$ and $\sum_{N=0}^{\infty} NP(N) = \langle N \rangle$.