



AUB  
Physics Department

Physics 235  
Final Exam

February 4, 1997  
Time: 2 hours

**40 marks**

- 1) Consider a system of  $N$  independent particles each with spin  $1/2$ , in a magnetic field  $B$ , and at constant  $T$ .
- Find the internal energy, the entropy, the specific heat at constant  $B$ , and the total magnetic moment  $\bar{M}$  of this system, with the help of the canonical distribution. Plot the following quantities :  $U/N\mu B$  ,  $S/Nk$  ,  $C_B/Nk$  in terms of the reduced thermal energy  $kT/\mu B$  where  $\mu$  is the magnetic moment of each particles  $U$  is the internal energy,  $S$  is the entropy and  $C_B$  is the specific heat.
  - Calculate the fluctuation  $(M - \bar{M})^2$  of the total magnetic moment  $M$  of this system and compare your result to  $C_B$  . (This relationship is known as the fluctuation-dissipation theorem).
  - Suppose that  $N$  Ising spins  $\sigma_i$  ( $\sigma_i = \pm 1$ ) are arranged along a ring. Assume that the energy of this system is given by:

$$E = -J \sum_{j=1}^N \sigma_j \cdot \sigma_{j+1} \quad (\sigma_{N+1} = \sigma_1)$$

Calculate the partition function  $Z$ , the free energy  $F$ , the internal energy  $U$  and the specific heat  $C_B$  of this system : (you may use the identity

$$\exp(a \sigma_i \cdot \sigma_j) = \cosh a + \sigma_i \cdot \sigma_j \sinh a$$

Compare your results with the part a) of this problem to deduce the meaning of  $J$  in the thermodynamic limit  $N \rightarrow \infty$ . ( A microstate of this system is defined as  $\ell = \{\sigma_1 = \pm 1, \sigma_2 = \pm 1, \dots, \sigma_j = \pm 1, \dots, \sigma_N = \pm 1\}$  )

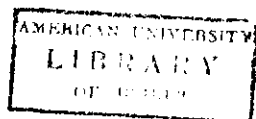
**30 marks**

- 2) a) Show that the Maxwell velocity distribution function can be written as :

$$F(\vec{v}) = F(v_x, v_y, v_z) = \pi^{-3/2} \cdot (v_m)^{-3} \cdot \exp[-(v/v_m)^2]$$

where  $v_m$  is the most probable speed.

- What is the expression of the speed distribution function  $\delta(v)$  in terms of  $v_m$  . Here  $v = |\vec{v}|$  .
- Consider 1 mole of molecules and approximate  $dv$  by  $\Delta v = 0.01 v_m$ . Find the number of molecules with speed  $v$  in  $dv$  at  $v = 0$ ,  $v = v_m$  and  $v = 8 v_m$  .



30 marks

3) a) Estimate the Fermi energy of copper ( $A = 63.5$ ) which has a density of  $9 \times 10^3 \text{ kg/m}^3$ . Assume that copper has a single  $s$  electron;  $hc = 1240 \text{ eV.nm}$ ,  $m_e = 0.511 \text{ MeV}/c^2$ .

b) What is the average energy of a conduction electron in copper at Low temperature.

c) Knowing that for a non-relativistic Fermion gas, the ideal gas equation of state is

$$PV = \frac{2}{3} E.$$

Show that the pressure  $P$  of the conduction electron in a metal is

$$P = \frac{2}{5} n E_F$$

where  $n$  is the density of conduction electrons and  $E_F$  is the Fermi energy.

Using the above result, evaluate the electron pressure for copper and compare your answer to atmospheric pressure.

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