AUB	Physics 235	February 4,1997
Physics Department	Final Exam	Time: 2 hours

40 marks

1) Consider a system of N independent particles each with spin 1/2, in a magnetic field B, and at constant T.

a) Find the internal energy, the entropy, the specific heat at constant B, and the total magnetic moment $\bar{M}$ of this system, with the help of the canonical distribution. Plot the following quantities : $U/N\mu B$, S/Nk , C_B/Nk in terms of the reduced thermal energy $kT/\mu B$ where μ is the magnetic moment of each particles U is the internal energy, S is the entropy and C_B is the specific heat.

b) Calculate the fluctuation $\overline{(M-\bar{M})^2}$ of the total magnetic moment M of this system and compare your result to C_B . (This relationship is known as the fluctuation-dissipation theorem).

c) Suppose that N Ising spins σ_i ($\sigma_i = \pm 1$) are arranged along a ring. Assume that the energy of this system is given by:

$$E = -J\sum_{j=1}^{N} \sigma_j . \sigma_{j+1} \qquad (\sigma_{N+1} = \sigma_1)$$

Calculate the partition function Z, the free energy F, the internal energy U and the specific heat C_B of this system : (you may use the identity

$$\exp(a\,\sigma_i . \sigma_j) = \cosh a + \sigma_i . \sigma_j \sinh a \;)$$

Compare your results with the part a) of this problem to deduce the meaning of J in the thermodynamic limit $N \to \infty$. (A microstate of this system is defined as $\ell = \{\sigma_1 = \pm 1, \sigma_2 = \pm 1, \ldots, \sigma_j = \pm 1, \ldots, \sigma_N = \pm 1\}$)

30 marks

2) a) Show that the Maxwell velocity distribution function can be written as :

$$F(\vec{v}) = F(v_x, v_y, v_z) = \pi^{-3/2} . (v_m)^{-3} . \exp[-(v/v_m)^2]$$

where v_m is the most probable speed.

b) What is the expression of the speed distribution function $\delta(v)$ in terms of v_m . Here $v = |\vec{v}|$.

c) Consider 1 mole of molecules and approximate dv by $\Delta v = 0.01\, v_m$. Find the number of molecules with speed v in dv at $v = 0$, $v = v_m$ and $v = 8\, v_m$.

30 marks

3) a) Estimate the Fermi energy of copper (A = 63.5) which has a density of 9×10^3 kg/m^3
Assume that copper has a single 4 s electron; hc = 1240 eV.nm, m_e = 0.511 MeV/c^2.

b) What is the average energy of a conduction electron in copper at Low temperature.

c) Knowing that for a non-relativistic Fermion gas, the ideal gas equation of state is

$$PV = \frac{2}{3}E.$$

Show that the pressure P of the conduction electron in a metal is

$$P = \frac{2}{5} n E_F$$

where n is the density of conduction electrons and E_F is the Fermi energy.
Using the above result, evaluate the electron pressure for copper and compare your answer to atmospheric pressure.
