PHYSICS 235
Final Examination

24 hours take-home exam. Due February 2, 2006 at 11 a.m.

1. Thermal photons

a) Find the average number of thermal photons in equilibrium at room temperature in a cavity filled with isotropic dielectric medium of refractive index n_0=1.5. The volume of the cavity is 1 cm^3 . Assume that n_0 does not depend on the wavelength.

b) The cavity is compressed <u>adiabatically</u> (and reversibly) to 1/10 of the original volume.

Find the new values for the following parameters of the radiation inside the cavity:

- temperature ;
- average number of photons;
- energy per photon;
- total energy;
- total entropy

2. Neutrons in a 1D harmonic trap

Neutrons (no mutual interaction) are placed in a spin-independent 1-D harmonic potential $U = \frac{1}{2} m_n w_0^2 x^2$ (assume for simplicity that there are no degrees of freedom associated with y and z coordinates). The frequency of vibration for one neutron in the trap is $\omega_0 = 10^2 s^{-1}$. There are altogether $N=10^{15}$ neutrons in the trap and the whole system is at room temperature T = 300K

a) Find the density of one-particle states
b) Find the Fermi temperature
c) Provide reasonable estimates of the chemical potential, the total energy <E>, and the heat capacity of the system under given conditions (if you are making approximations, specify and justify).

d) Can you reach the semi-classical behavior if you remove some neutrons from the trap keeping the same temperature?
Estimate the upper limit of N below which the behavior is semi-classical
Find the expression for the Helmholtz free energy F(N,T,ω) in this limit.

e) If now $\omega_0 = 10^{15} s^{-1}$ what would change in the way you answer questions a) d)?
Name as many important moments as you can

3. Canonical Partition function of a model system

A single-particle Hamiltonian has only 3 eigenstates of with respective energies 0, 2ε, and 3ε. A system consists of 2 particles (no mutual interaction), and is in contact with a thermostat at temperature T.

Find the canonical partition function Z
a) if the two particles are distinguishable
b) if the two particles are identical fermions
c) if the two particles are identical bosons

You can choose one problem (either 4 or 5)

4. A forgotten light bulb A 100-W light bulb is left burning inside a refrigerator. Can the refrigerator cool below the ambient room temperature, if the power drawn by the cooling unit is 20 W?

Neglecting any heat leaking due to thermal conductivity, write down the condition for the **steady state** of the refrigerator interior.
Find the steady-state temperature inside the refrigerator assuming that the refrigerator is an ideal Carnot unit.

Show the energy and the entropy diagrams for the steady state.

5. Phase transitions in iron
The properties of solid iron at atmospheric pressure and relatively high temperatures can be summarized as follows:

- two different crystalline forms (α and γ) are possible;
- In the temperature range 1100 K < T< 1600 K, γ-iron is the thermodynamically stable phase, while α- iron is only metastable;
- for temperatures below 1100 K and above 1600 K, the reverse is true: α-iron is the stable phase, while γ is metastable;
- the heat capacities of the two phases, C_α and C_γ, are (approximately) independent of temperature, and $C_\alpha > C_\gamma$.

a) How can the stated facts about stability of the two phases be reformulated based on the minimal properties of the relevant thermodynamic potential ? Which thermodynamic potential should you look for ?

b) Sketch qualitatively chemical potentials of the two phases as functions of temperature at fixed atmospheric pressure.
Pay attention to the physical meaning and the sign of the slope and of the second derivative (convexity) of the $\mu(T)$ curve!
Pay attention to the existence of two transition temperatures T_1=1100 K and at T_2 = 1600 K

c) Sketch qualitatively how the entropy of the sample changes with T upon heating from 1000 K to 1700 K assuming that at any given temperature the sample is in true equilibrium (heating is done slowly enough so that only the stable phase survives at any given temperature).

How are the parameters of the curves in parts b) and c) related?