Physics 235
Final Examination
Time : 3 hours

1. Condensation of Bound Fermions 50%
N particles of spin 1/2 and mass m in a volume V are able to form bound pairs of spin zero, the binding energy per pair being $-E_0$ ($E_0 > 0$). The particles are otherwise non-interacting .

a) Use the minimal property of the free energy to show that, in equilibrium wth respect to forming and breaking pairs,

$$\mu_2 = 2\mu_1$$

where μ_1 is the chemical potential of a single particle, and μ_2 is the chemical potential of a pair.

b) Sketch a diagram showing the energy levels for pairs and single particles (The choice of zero level is arbitrary, but it has to be consistent, one and the same for the both phases!)

c) Find the number of pairs in equilibrium at $\tau = 0$. Show the chemical potentials in the diagram.

d) Suppose that at low enough but finite temperatures some pairs form a Bose-Einstein condensate. What is the value of μ_1 in the presence of the Bose-Einstein condensate of pairs? Are the single particles strongly Fermi degenerate at these low temperatures, or not?

e) Write down the conditon relating the Einstein condensation temperature τ_E to N/V, m, and E_0 . Assume $E_0 >> \tau$ to calculate approximately one of the integrals you will encounter.

2. Room air conditioner (25%) A room air conditioner operates as a Carnot cycle refrigerator between an outside temperature T_h and a room at a lower temperature T_l The room gains heat from the outdoors at a rate (proportionate to the temperature difference) $A(T_h - T_l)$; this heat is removed by the air conditioner. The power supplied to the cooling unit is P.

a) Show that the steady-state temperature of the room is

$$T_l = (T_h + P/2A) - \left[(T_h + P/2A)^2 - T_h^2\right]^{1/2}$$

b) Sketch qualitatively the dependence of the room temperature on the power of the cooling unit. Write down the asymptotics in the limits of very small P and very large P.

3. Extracting work from an isolated system. (25%)Two bodies each of mass M and constant specific heat c, are initially at temperatures T_1 and T_2. A Carnot engine operates between them until they are eventually brought to thermal equilibrium.

a) Show that the final temperature of the two bodies is $T_f = \sqrt{T_2 T_1}$

b) Find the maximum amount of work that can be delivered by this Carnot engine