Mathematics 314, Final Examination, June 15, 2001

Problem 1: 8 pts.

Let $f : X \to Y$ be a continuous function between connected and locally arc-connected spaces. Explain what is the induced homomorphism f_* and outline the proof that f_* is indeed a homomorphism.

Problem 2. 8 pts.

Quote the Seifert - van Kampen theorem. Then give a detailed explanation why it can not be used to obtain $\pi(S^1)$.

Problem 3. 8 pts.

Find $\pi(S^2 - \{x_1 x_2, x_3, x_4\})$, where x_1, x_2, x_3 and x_4 are different points of the sphere S^2. Include all essential details of your reasoning.

Problem 4. 5 pts.

Explain what is an n-dimensional simplex in the m-dimensional Euclidean space $\mathbb{R}^m$, and what are the barycentric coordinates associated with it.

Problem 5. 5 pts.

Quote the classification theorem for closed surfaces and use it to see what is the connected sum $T_3 \# P_3$ of the torus with 3 holes with the non-orientable surface of genus 3.

Problem 6. 5 pts.

Show that a covering space $(\tilde{M}, p)$ of a connected n-dimensional manifold M is another manifold of the same dimension. Is this still true for manifolds with boundary?

Problem 7. 5 pts.

Suppose that presentations are given of groups G_1 and G_2 by means of generators and relations. Show how to obtain from this a presentation of the direct product $G_1 \times G_2$.

Time: 120 minutes.

Good Luck!

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