June 25, 2002

Math. 314

Final Examination

Problem 1. 15 pts.

Write how the fundamental group $\pi(X, x_0)$ is defined.

Problem 2. 10 pts.

Use the fact that S^2 is simply connected (no proof is required here for this fact) together with an appropriate corollary to the Seifert - van Kampen theorem to prove that the 3-sphere S^3 is simply connected.

Problem 3. 6 pts.

Remind the statement of the Poincare conjecture. Why is it so important?

Problem 4. 6 pts.

Suppose that X and Y are contractible spaces. Give a detailed proof that $X \times Y$ is contractible, too.

Problem 5. 6 pts.

Give a detailed example of a continuous map $p : \mathbb{R}^2 \to S^1 \times S^1$ such that $(\mathbb{R}^2, p)$ is a covering of the torus $S^1 \times S^1$.

Problem 6. 8 pts.

Let G_1 and G_2 be two non-trivial groups. Show that their free product $G_1 * G_2$ is not abelian and must contain an element of infinite order.

Time: 120 minutes. *Good Luck!*