Mathematics 314, Final Examination, June 6, 2003

Problem 1. 12 pts.

What does it mean that a compact space is a topological polyhedron? Write all relevant definitions (for instance, of an n-simplex and of a simplicial complex).

Problem 2. 12 pts.

Remind the statements of the path-lifting property, the homotopy lifting property and the mapping lifting property for a covering space $(\tilde{X}, p)$ of a given space X.

Problem 3. 7 pts.

Find the fundamental group of the Möbius band.

Problem 4. 7 pts.

Give an example of two compact simply connected spaces which are not of the same homotopy type.

Problem 5. 7 pts.

Let $X = S^2 - \{x_1, x_2, x_3\}$, where x_1, x_2 and x_3 are different points on the sphere S^2. Find a graph (that is, a one-dimensional topological polyhedron) Y which is a deformation retract of X. Then use your Y to obtain $\pi(X)$.

Problem 6. 7 pts.

Let $X = \tilde{X}_1 = \tilde{X}_2 = S^1$ (the unit circle in the complex plane $\mathbb{C}$). Let $p_1 : \tilde{X}_1 \to X$ and $p_2 : \tilde{X}_2 \to X$ be given by $p_1(z) = z^m$ and $p_2(z) = z^n$, where m and n are non-zero integers. Thus, $(\tilde{X}_1, p_1)$ and $(\tilde{X}_2, p_2)$ are covering spaces of X.

Suppose that there exists a homomorphism $\phi : (\tilde{X}_1, p_1) \to (\tilde{X}_2, p_2)$ of covering spaces. What relation must hold between m and n? Prove your claim.

Time: 120 minutes. *Good Luck!*