# Mathematics 314, Final Examination, June 4, 2005

Problem 1. 20 pts.

Describe how the fundamental group $\pi(X, x_0)$ is defined.

Problem 2. 20 pts.

Let $M$ and $N$ be connected n-dimensional manifolds without boundary and let $M\#N$ denote their connected sum.

Suppose that $n \geq 3$ and that $\pi(M)$ and $\pi(N)$ are known.

What is $\pi(M\#N)$ ? Why? Why this would not work when $n = 2$ ?

Problem 3. 15 pts.

Remind the definition of a topological polyhedron.

Do not quote the definitions of a simplex or of a linearly independent set of points in $\mathbb{R}^n$.

Problem 4. 15 pts.

Show that the torus $T = S^1 \times S^1$ is homeomorphic to a subset of the 3-dimensional sphere $S^3$.

Then prove that $T$ is not a deformation retract of $S^3$.

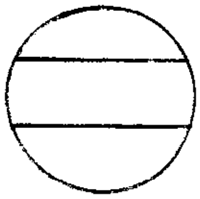
Problem 5. 15 pts.

Let $(\tilde{X}, p)$ be a covering space of $X$ and $(\tilde{Y}, q)$ be a covering space of $Y$.

Show that $\tilde{X} \times \tilde{Y}$ is a covering space for $X \times Y$.

Problem 6. 15 pts.

Find the fundamental group of the following space/figure



Time: 120 minutes.

*Good Luck!*